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Energy absorption of a Bose gas in a periodically modulated optical lattice

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We compute the energy absorbed by a one-dimensional system of cold bosonic atoms in an optical lattice subjected to lattice amplitude modulation periodic with time. We perform the calculation for the superfluid and the Mott insulator created by a weak lattice, and the Mott insulator in a strong lattice potential. For the latter case we show results for three-dimensional systems as well. Our calculations, based on bosonization techniques and strong-coupling methods, go beyond standard Bogoliubov theory. We show that the energy absorption rate exhibits distinctive features of low-dimensional systems and Luttinger liquid physics. We compare our results with experiments and find good agreement.

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Cold atoms provide a remarkable laboratory to study the physics of strongly correlated quantum systems. Cold atomic gases loaded in optical lattices [1,2] and Feshbach resonances [3] allow for an unprecedented control of many parameters of the system, including the interactions, both for bosons and fermions. However, the ability to probe the properties of such systems, and in particular, to measure momentum and frequency dependent correlation functions, still remains very limited. One of the most common experimental probe is time-of-flight (TOF) imaging, which under most conditions give access to the momentum distribution [4]. More recently other spectroscopies such as Bragg [5,6], energy absorption rate (EAR) [2,7], radio frequency [8], and shot noise [9] have been demonstrated. Thus in order to properly characterize these systems it is of primary importance to have a good understanding as well as theoretical predictions for such experimental probes.

In a series of recent experiments, the Zurich group [2,7] measured the EAR of Bose gases of ultracold ^{87}Rb atoms loaded in optical lattices. The system was heated by periodically modulating the lattice along one direction, and the energy absorption rate as a function of the modulation frequency was subsequently estimated from the width of the atom distribution around zero momentum measured in TOF. Interestingly, of all the system dimensionalities reported in [2], the one-dimensional (1D) ones exhibit the broadest continuous spectrum away from the Mott regime, which has no simple explanation in standard Bogoliubov theory [2,10], and has led some to consider other schemes [11–14].

In this work, we show that treating the 1D interacting Bose gas in the framework of the Luttinger liquid (LL) [15–18] correctly takes into account short distance correlations that are neglected in the Bogoliubov theory, and leads naturally to a continuum of excitations, hence a continuous absorption spectrum. Furthermore, these correlations are responsible for the transition to the Mott insulating (MI) phase for sufficiently strong interactions.

In contrast to the broad spectra observed for the superfluid (SF), in Refs. [2,7] two much narrower peaks were observed for the Mott phase. Here we also study this regime and consider two physically distinct situations: First, a sufficiently strongly interacting Bose gas in one dimension, as in Ref. [19] becomes Mott insulating for a weak *commensurate* periodic potential [15,20,21]. In this case, we present results for the EAR spectrum obtained using the form factors for the effective low-energy sine-Gordon (sG) model [22]. These results also apply to the MI phase of the 1D Bose-Hubbard model near the SF-MI transition. We find that the spectrum exhibits a finite threshold, corresponding precisely to the Mott gap. Second, we consider a very deep optical lattice, so that hopping is strongly suppressed and the Mott gap is large. This situation is well described by the Bose-Hubbard model, for which we have computed in one dimension, the shape of the lowest excitation peak in the absorption spectrum, which occurs at an excitation frequency $\omega \approx U/\hbar$, with U being the on-site boson-boson repulsion. We also give the results for the peak width in higher dimensions.

Consider a system of interacting bosons in a lattice created by an optical potential $V(x,y,z)=V_{0x}\sin^2(kx)+V_{0y}\sin^2(ky)+V_{0z}\sin^2(kz)$, where the wave vector is $k=2\pi/\lambda$, with λ being the laser wavelength. We first consider an optical lattice that is very deep in the y and z directions (e.g., $V_{0x} \ll V_\perp \equiv V_{0y}=V_{0z}$). Atoms accumulate in the minima of this potential, where they experience strong transverse confinement, thus forming 1D gas tubes with a weaker periodic potential along the axis. For large enough V_{0y} and V_{0z} each 1D system becomes isolated from each other [23]: this is the 1D limit.

To obtain analytical results, we neglect trapping and finite size effects.[1] If $V_{0x} \ll \mu$, where μ is the chemical potential of the 1D interacting gas, the system in the presence of the lattice is well described by the following sine-Gordon model [15,18,20,21]:

[1]The trapping will lead to a coexistence of the Mott phase with a small superfluid fraction at the edges and near the center of the trap [34]. This will contribute to the energy absorption at low frequencies, but this part of the spectrum is not well resolved in the experiments [7]. It also has marked consequences for higher resonances in the spectrum (peak at $\omega \approx 2U$) which is beyond the scope of the present paper.

$$H_{\text{eff}} = \frac{\hbar v_s}{2\pi} \int dx[K(\pi\Pi)^2 + K^{-1}(\partial_x\phi)^2] + g_0 \int dx \cos[2\phi(x)], \tag{1}$$

where $\phi(x)$ and $\Pi(x)$ are canonically conjugate fields; ϕ represents density fluctuations and $\theta = \pi\int_{-\infty}^{x} dx'\Pi(x')$ corresponds to phase fluctuations. v_s is the speed of sound and the coupling to the axial potential is $g_0 \sim \rho_0 V_{0x}(V_{0x}/\mu)^{n_0-1}$, with n_0 the number of bosons per potential well. K is a dimensionless parameter determined by the strength of the boson-boson interactions and the linear density ρ_0 [17,24]: the SF-MI transition occurs at the universal value $K_c = 2$ (in terms of $\gamma = mg/\hbar^2\rho_0$, the dimensionless interaction strength of the Lieb-Liniger model with m the atom mass and g proportional to the scattering length, it corresponds to $\gamma \approx 3.5$).

A weak time-dependent modulation of the lattice [2,7] $V_{0x} \to V_{0x} + \delta V_x \cos(\omega t)$, leads to a perturbation that can be written in the above low-energy description as $H'(t) = \delta V_x \cos(\omega t)\mathcal{O}$, with $\mathcal{O} = f_0 \int dx \cos(2\phi)$, where $f_0 = dg_0/dV_{0x}$. Within linear response theory, the EAR per particle at frequency ω is given by

$$\dot{\epsilon}(\omega) = \frac{2\delta V_x^2}{N} \omega \, \text{Im}[-\chi_{\mathcal{O}}(\omega)], \tag{2}$$

where $\chi_{\mathcal{O}}(\omega)$ is the Fourier transform of the retarded correlation function $-i\hbar^{-1}\Theta(t)\langle[\mathcal{O}(t), \mathcal{O}(0)]\rangle$, with Θ the step function. Note that, in the bosonization technique [17,18], the density $\rho(x) = \rho_0 - \partial_x\phi(x)/\pi + \cos[2\phi(x) - 2\pi\rho_0 x] + \cdots$, and hence the EAR probes the $q \approx 2\pi\rho_0 = 2n_0 k$ part of the excitation spectrum. Standard Bogoliubov theory has no low-energy excitation near this momentum, hence no absorption, and one has to resort to a nonlinear response to account for the observed spectrum [14]. But in one dimension, the correct excitation spectrum has in fact a continuum at $q \approx 2\pi\rho_0$ [25,26], which is taken fully into account by the bosonization method.

In the 1D SF (namely, LL) phase, where $K > 2$, the cosine term of (1) is irrelevant and the system is gapless. $\chi_{\mathcal{O}}(\omega)$ for $\hbar\omega \ll \mu$ can be obtained [27] by means of bosonization techniques [17,18], and we get

$$\dot{\epsilon}(\omega) = \frac{A}{\hbar}\left(\frac{f_0 \delta V_x}{\rho_0}\right)^2 \left(\frac{\hbar\omega}{\mu}\right)^{2K-1}, \tag{3}$$

where A is a nonuniversal prefactor depending on the microscopic details of the model (Bose-Hubbard or Lieb-Liniger). Equation (3) directly shows that the continuum of low-energy excitations at $q \approx 2\pi\rho_0$ leads to this continuous absorption curve.[2] As $\hbar\omega \to \mu$, bosonization ceases to be valid. However, standard sum rules for the density response function [28] imply that the integrated absorption spectrum is finite, and thus the spectral weight must decrease for $\omega \sim \mu/\hbar$ (or $\omega \sim J/\hbar$ for the Bose-Hubbard model). One thus expects a rather broad spectrum in one dimension, as observed in the experiments [7].

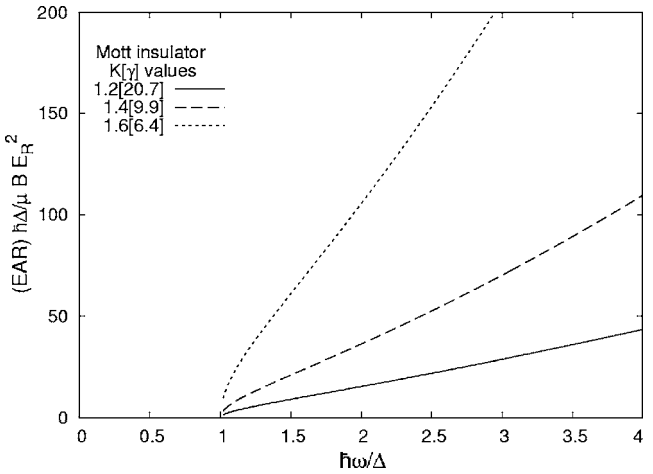

FIG. 1. Energy absorption rate (EAR) for 1D interacting bosons in a weak optical lattice, for the MI phase (with small Mott gap) with different values of the parameter K (in brackets are the corresponding values of γ [17]).

On the other hand, in dimensions higher than one, continuous absorption must start from a finite frequency, as required by Landau's criterion for superfluidity. We speculate that the relatively broad absorption spectrum observed for the SF in the "1D to three-dimensional (3D) crossover" regime [2] may be due to the possible existence of a broad rotonlike feature. Since this particular lattice is highly anisotropic, a naive extrapolation of the dimensional crossover theory of Ref. [23] suggests that at high frequencies, the 1D-like excitation continuum persists, while at low frequencies, the excitation is more 3D-like, with a broad rotonlike minimum being the vestige of the 1D continuum at $q \approx 2\pi\rho_0$, but this minimum must not be at zero frequency to satisfy Landau's criterion (see, also, Ref. [29]).

Next we turn to the MI phase in one dimension, which occurs for sufficiently repulsive interactions, $K < 2$. Now, the cosine term in (1) leads to an excitation gap, with gapped solitons and antisolitons in the $1 < K < 2$ range of interest here [16,18]. Using the form factor approach, and keeping only the one soliton-one antisoliton contribution to the absorption, we find [27]

$$\dot{\varepsilon}(\omega) = \frac{B}{\hbar}\left(\frac{f_0 \delta V_x}{\rho_0}\right)^2 \frac{\mu\Theta[(\hbar\omega)^2 - \Delta^2]}{\sqrt{(\hbar\omega)^2 - \Delta^2}} |f[\theta_0(\omega)]|^2, \tag{4}$$

where B is a nonuniversal prefactor, $\Delta = 2M_s v_s^2$ is the Mott gap, and $M_s \sim \mu(g_0/\rho_0\mu)^{1/(2-K)}/v_s^2$ $(K < 2)$ is the soliton gap (mass). $\theta_0(\omega) = 2\,\text{arccosh}(\hbar\omega/\Delta)$ is the relative rapidity of the soliton and the antisoliton, and $f(\theta_0) = [\sinh\theta_0/\sinh(\theta_0 + i\pi/2\xi)]e^{T(\theta_0)}$, where $T(\theta_0) = \int_0^\infty (dt/t)[\sinh^2 t(1 - (i\theta_0/\pi))\sinh[t(\xi-1)]/\sinh(2t)\cosh(t)\sinh(t\xi)]$, is the (unnormalized) form factor of the operator $\cos 2\phi$ [22], with $\xi = K/(2-K)$. The EAR is plotted in Fig. 1 for various K, showing the Mott gap to energy absorption. For $\hbar\omega \gtrsim \Delta$ the EAR increases monotonically in a way slower than in the SF.

[2] If the 1D potential $V_{0x}\sin^2(kx)$ is not commensurate with the density, then $\dot{\epsilon}(\omega > v_s|q|) = \mathcal{A}\omega[\omega^2 - v_s^2 q^2]^{K-1}$, where $q = 2n_0 k - 2\pi\rho_0$ is the incommensurability.

We now consider the case where the lattice potential is very deep ($\mu \ll V_{0x}$), for a d-dimensional hypercubic lattice. The system is then described [30] by the Bose-Hubbard model $H_{\rm BH}=H_J+H_U$, where

$$H_J=-\sum_{\boldsymbol{R},\boldsymbol{x}_\alpha}\frac{J_\alpha}{2}b^\dagger_{\boldsymbol{R}}b_{\boldsymbol{R}+\boldsymbol{x}_\alpha},\quad H_U=\frac{U}{2}\sum_{\boldsymbol{R}}(\delta n_{\boldsymbol{R}})^2, \tag{5}$$

$b^\dagger_{\boldsymbol{R}}$ creates a boson at lattice site $\boldsymbol{R}$, $\boldsymbol{x}_\alpha$ are lattice vectors joining the site $\boldsymbol{R}$ to its two nearest neighbors along direction $\alpha=1,\ldots,d$; $\delta n_{\boldsymbol{R}}=b^\dagger_{\boldsymbol{R}}b_{\boldsymbol{R}}-n_0$. For $V_{0\alpha}\gg E_R$ lattices, $J_\alpha=(8/\sqrt{\pi})E_R(V_{0\alpha}/E_R)^{3/4}\exp[-2(V_{0\alpha}/E_R)^{1/2}]$ and $U=4\sqrt{2\pi}(a_s/\lambda)E_R(V_{0x}V_{0y}V_{0z}/E_R^3)^{1/4}$ [31], with a_s the scattering length, can be controlled by varying the laser intensity $V_{0\alpha}$ (measured in units of the recoil energy $E_R=\hbar^2k^2/2m$). In one dimension (i.e., $d=1$, or effectively when $J_1\gg J_\alpha$, for $\alpha=2,\ldots,d$), the SF-MI transition occurs at $(U/J)_c=1.92$ [32], while in a $d=3$ square lattice, $(U/J)_c=5.8$, with $J=\Sigma_\alpha J_\alpha$. Near the transition, on the MI side, the above description in terms of the sG model (1), still applies [18]. Thus, the EAR is also given by (4), and the Mott gap Δ is exponentially small. As U/J_1 grows, eventually $\Delta\approx U$.

Since $J_\alpha=J_\alpha(V_{0\alpha})$ and $U=U(\{V_{0\alpha}\})$ are functions of the optical potential strength $V_{0\alpha}$, the modulation along one direction $V_{0x}=V_{01}\to V_{0x}+\delta V_x\cos(\omega t)$ induces the following perturbation to (5) [33]:

$$H'(t)=\delta V_x F_U H_{BH}+\tilde{\mathcal{O}}\cos\omega t, \tag{6}$$

where $\tilde{\mathcal{O}}=-\frac{1}{2}\Sigma_{\boldsymbol{R},\boldsymbol{x}_\alpha}\delta J_\alpha\, b^\dagger_{\boldsymbol{R}}b_{\boldsymbol{R}+\boldsymbol{x}_\alpha}$, with $\delta J_1=J_1(F_J-F_U)\delta V_x$, $\delta J_\alpha=-J_\alpha F_U\delta V_x$ $(\alpha>1)$, $F_J=d\ln J_1/dV_{0x}$, and $F_U=d\ln U/dV_{0x}$. The first term in (6) is $\propto H_{BH}$ and does not contribute to the absorption.

The EAR can thus be computed by studying the linear response to the last term of (6), via a strong-coupling expansion in J_α/U. We sketch here the derivation in the 1D case; full details and the more involved higher-dimensional case can be found in [27]. In the MI phase with n_0 bosons per site (n_0 is an integer), the ground state of H_U is $|\Phi_0\rangle=|n_1=n_0,\ldots,n_M=n_0\rangle$, while the first excited state $|\Phi(R,r)\rangle$ with the *same* number of bosons has an extra boson ("particle") at site $R=1,\ldots,M$ and one fewer boson ("hole") at $R+r$ $(r=1,\ldots,M-1)$ and costs an energy U. To obtain the linear response for $\hbar\omega\approx U$, we only need the ground state and the states with one particle and one hole. To take into account H_J, one needs to diagonalize the kinetic energy in a subspace where the particle and the hole hop, but no additional pair is created or destroyed. Thus, at *arbitrary* filling n_0, we employ the ansatz $|\Phi(Q,q)\rangle=\sqrt{2}/M\Sigma_{R,r}e^{iQR}e^{i\theta(Q)r}\sin(qr/2)|\Phi(R,r)\rangle$ ($Q=2\pi j/M$ and $q=2\pi l/M$ for M lattice sites with periodic boundary conditions). The eigenvalues of the kinetic energy are $\epsilon(Q,q)=U-J\rho(Q)\cos(q/2)$. Here $\rho(Q)$ and $\theta(Q)$ are the modulus and the argument of $n_0+e^{iQ}(1+n_0)$. Using the spectral decomposition of the correlation function $\chi_{\tilde{\mathcal{O}}}(\omega)$ (to leading order in perturbation theory in δJ_1),

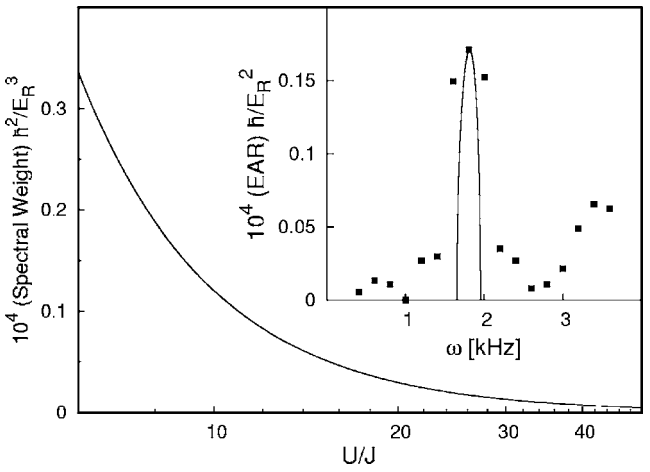

FIG. 2. Spectral weight of the first resonance peak in the MI phase in one dimension. In the inset we show the predicted 1D shape of the peak for $U/J=36$, and the comparison with (normalized in the vertical axis) experimental results from Ref. [2].

$$\dot{\varepsilon}(\omega)\approx\frac{2\pi\omega}{N}\sum_{Q,q}\left|\langle\Phi(Q,q)|\tilde{\mathcal{O}}|\Phi_0\rangle\right|^2\delta[\hbar\omega-\epsilon(Q,q)], \tag{7}$$

we get the EAR in one dimension to be

$$\dot{\varepsilon}(\omega)=\frac{\delta J_1^2}{2J_1}\left(\frac{n_0+1}{2n_0+1}\right)\omega\sqrt{1-\left[\frac{\hbar\omega-U}{(2n_0+1)J_1}\right]^2}. \tag{8}$$

There is thus a resonance at $\hbar\omega\approx U$ with a width at the *base* of the peak $2W=2(2n_0+1)J_1$. At lowest order in δJ_1 the absorption is zero for $\hbar\omega<U-W$ or $\hbar\omega>U+W$. Note that the shape of the resonance peak is not symmetric around $\hbar\omega=U$.

This is shown in the inset of Fig. 2, where the absorption maximum is at $\hbar\omega\approx U[1+(2n_0+1)^2J_1^2/U^2]$. The square-root form in Eq. (8) is a nonperturbative result, i.e., an effective resumation of a certain class of diagrams [27]. The peak spectral weight, f, can readily be computed,

$$f=\frac{\pi(n_0+1)}{4\hbar^2}U(\delta J_1)^2, \tag{9}$$

and is shown in Fig. 2. In $d>1$ there is no simple formula for the precise shape of the resonance peak. However, the width of the peak still has the same form, provided that J_1 is replaced by J. Moreover, it does not depend on δJ_α, and therefore it is unaffected by the lattice modulation in one or more dimensions. In Fig. 3, we plot $2W$ as a function of U/J, for the same setup as in Ref. [2]. The line shape shown in the inset of Fig. 3 is in good agreement with the experimental findings reported in Refs. [2,7].

Finally, we compare our results with the experimental findings. In the strong-coupling MI phase, our results are in good agreement with the experimental observations: (i) In one dimension, the line shape shown in the inset of Fig. 2 is in good agreement with the experimental findings reported in Refs. [2,7]. (ii) As shown in Fig. 3, for $U/J\gtrsim 10$, the width of the peak at U decreases with U/J in all dimensions, which is again in agreement with the experimental observation. Our

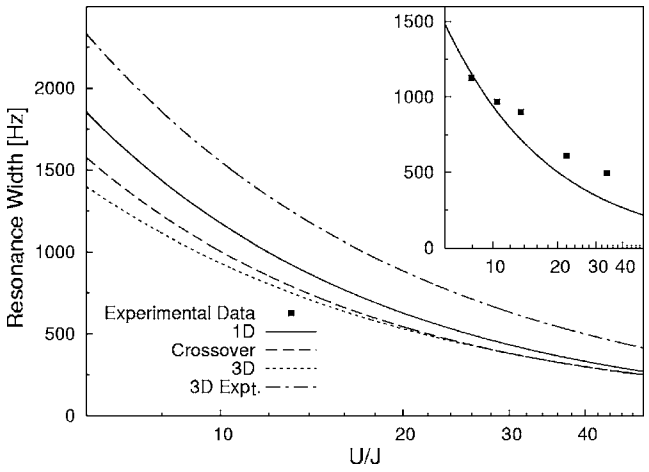

FIG. 3. Width at the *base* of the first resonance peak in the MI phase. In the 1D, 1D-3D crossover, and 3D cases we take $V_\perp = 30E_R$, $V_\perp = 20E_R$, and $V_\perp = V_{x0}$, respectively, and $n_0 = 1$. The 3D Expt. case is the same as 3D but with $n_0 = 2$, which is closer to the experimental values. The inset compares the predicted *half* widths with the experimental data from Ref. [2] for one dimension.

values for the half width in one dimension are also in good agreement with experiments, as shown in the inset, except for a slight broadening that is due to the trapping, and also to the fact that the experiments are done in a multiple tubes setup, which changes the filling from one tube to another. (iii) In experiments, for fixed U/J, the width of the resonance becomes *smaller* as one goes from the 1D to 3D lattices. Our calculations capture this effect in the 1D and 1D-3D crossover cases, although a larger value of the width at the base is predicted for three dimensions.[3] However, we would like to emphasize that here we compute the width at the base whereas the half width is fitted in the experiments. Clearly, more experimental data in the MI regime are needed to test not only for the width of the resonance peak,[4] but also its spectral weight. In the weak-coupling regime, as noted above, our results are also able to explain qualitatively the broad absorption spectra measured in the experiments for 1D systems, with a continuous absorption curve for the SF, starting from $\omega = 0$. Accurate measurements of the low-frequency part of the spectrum should allow, in principle, to determine the parameter K, which characterizes long-range correlations in the LL.

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[3]We are not able to compute the half widths in 3D, which makes comparisons with experiments difficult. Note also the difference in fillings in 1D and 3D: in experiments the average boson occupancy per site is 1.2 (1D), and 2.4 (3D) whereas we have taken $n_0 = 1$ (1D) and 2 (2D).

[4]In particular, comparing an estimate of the peak width to our linear response result can give an idea of the importance of nonlinear effects.

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