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ELECTRON - HELIUM ATOM SCATTERING

(SECOND BORN APPROXIMATION)

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CONTENTS

CONTENTS, continued

Page

ABSTRACT

In chapter I of this thesis the Born series solution of the Schrödinger equation for the scattering of a structureless particle by a static potential is derived and known results about its radius and rate of convergence given. The derivation is generalised to electron-atom scattering in chapter II. It is proved that the second Born scattering amplitude satisfies relationships analagous to the optical theorem and the dispersion relation conjectured by Gerjuoy and Krall. Various approximate methods of evaluating the second Born correction to the scattering amplitude are discussed.

In chapter III the method used to reduce the second Born correction to the scattering amplitude to a sum of known integrals is given and chapter IV contains the method of evaluating these known integrals.

The results obtained from applying various forms of the simplified second Born approximation to elastic and inelastic collisions of electrons with Helium and Hydrogen atoms are presented in Chapter V. At incident electron energies of up to twenty times threshold a correction of between 5% and 10% to the first Born total cross sections is obtained for the following collisions:-

$$e + He\ (1s,1s) \rightarrow e + He\ (1s,1s)$$
$$\rightarrow e + He\ (1s,2s)$$
$$\rightarrow e + He\ (1s,2p)$$

For transitions between the ground state and excited d states of both Hydrogen and Helium atoms induced by electrons with impact energy of twenty times threshold the correction to the first Born total cross section is less than 3%, apart from the excitation of the 3^1D state of Helium when it is 15%. Differential and total cross sections are given for the excitation of model doubly excited states of Helium.

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CORRECTIONS

Page 8, line 8	$e^{i\underline{K}.\underline{r}}$.
Page 23, line 17	is polynomial bounded.
Page 26, last line	If $\ell_i^2 < \Delta_n^2$ ...... $Q_{in}^{B1}(\ell_i^2) = 0$ for $\ell_i^2 < \Delta_n^2$.
Page 27, line 2	$\frac{1}{4\pi^2} P \int_0^\infty \frac{\ell_i Q_i^{B1}(\ell_i^2)}{\ell_i^2 - k_i^2} d\ell_i^2$
Page 48, line 6	$\frac{t^\lambda Y(\lambda, n\ell, n'\ell', r)}{(t^2+\alpha^2)^r}$ where $Y(\lambda, n\ell, n'\ell', r)$ is independent of t and obtained by substituting (3.31) into (3.23), (3.24) and (3.25). r takes integer values up to some maximum value M, dependent on the wavefunctions and states under consideration.
Page 48, line 16	Orthognality
Page 49, Top line	where $N_j \neq 0$ for $j=1 \ldots N$ and the λ of (3.33) has been transformed to j.
Page 49, line 6	$Y(0, n\ell, n'\ell, k)$.
Page 52, line 3	substituting
Page 54, line 14	(3.35)
Page 55, line 8	of this $\underline{q}$ integral by the usual method,
Page 65, Penultimate line	Consider the expression
Page 66, line 2	To use this as an identity.

Page 80, line 8 $I(\theta', k_i^2)$

Page 120, New Paragraph

It was shown in section 3.3 that $I(ns, m\ell; n's, m'\ell'; \underline{t})$ could be expressed as a linear combination of terms of the form

$$E(n\ell m, n'\ell'm';\underline{t}) - E(n\ell m, n'\ell'm'; 0) = \sum_{j=1}^{N} \hat{\mathcal{S}}_{\lambda_j,\ell m,\ell'm'}(\underline{t}) \Big/ \sum_{k=1}^{M}$$

$$\frac{t^{N_j}\, Y(j,n\ell,n'\ell',k)}{(t^2 + \alpha_j^2)^k}$$

(c.f. 3.16 and 3.37). On performing this substitution, the coefficients of $\frac{t^{N_j}}{(t^2 + \alpha_j^2)^k}$ in this new expression for $I(ns,m\ell;n's,m'\ell';t)$ are relabelled as $Y(j;ns,m\ell;n's,m'\ell';k)$.

CHAPTER I

POTENTIAL SCATTERING

1.1 Introduction

The simplest non-trivial problem in the quantum theory of scattering is that of the scattering of a monoenergetic beam of particles of constant flux by a fixed static conservative potential. Let an incident particle have mass μ and initial momentum $\underline{k}$. The wave function describing such a beam of particles has the form $e^{i\underline{k}.\underline{r}}$. Suppose the potential $\lambda V(\underline{r})$ is such that its effect is negligible at large values of r. λ is a dimensionless constant introduced for convenience. In this so called asymptotic region the wave function representing the outgoing scattered particles has the form $\frac{e^{ikr}}{r}$, and since the magnitude of this scattered part depends on the angle through which the particles are scattered, the wave function describing the system will have the asymptotic form

$$\Psi(\underline{r}) \underset{r\to\infty}{\sim} e^{i\underline{k}.\underline{r}} + \frac{e^{ikr}}{r} f(\theta,\phi). \tag{1.1}$$

$f(\theta,\phi)$ is called the scattering amplitude. For (1.1) to be valid the potential $\lambda V(r)$ must satisfy the condition

$$\lim_{r\to\infty} r\left|\lambda V(\underline{r})\right| = 0 , \tag{1.2}$$

and as a result the coulomb potential is excluded. The coulomb potential distorts spherical and plane waves even at large distances from its centre.

The number of particles per unit time scattered into solid angle $d\Omega$, about the direction (θ,ϕ), is proportional to the incident flux of particles. The factor of proportionality is called the differential cross section $I(\theta,\phi)$ and has the dimensions of area. By comparing the fluxes of the incident and scattered waves in the asymptotic region it follows that

$$I(\theta,\phi) = |f(\theta,\phi)|^2 . \tag{1.3}$$

The total cross section Q is defined as

$$Q = \int I(\theta,\phi)\, d\Omega . \tag{1.4}$$

Since the potential is conservative the total energy of the system remains constant and equals the initial kinetic energy of the particle ($E = \frac{\hbar^2 k^2}{2\mu}$). The unperturbed Hamiltonian of the particle is $H_o(\underline{r}) = -\frac{\hbar^2}{2\mu} \nabla_{\underline{r}}^2$ and the wave function $\Psi(\underline{r})$ satisfies the time-independent Schrödinger equation

$$[H_o(\underline{r}) + V(\underline{r})]\, \Psi(\underline{r}) = E\, \Psi(\underline{r}) . \tag{1.5}$$

1.2 The Born Series

Consider the equations

$$(E - H_o \pm i\varepsilon)\, \Psi_\varepsilon^{\pm} = \lambda V \Psi_\varepsilon^{\pm} \tag{1.6}$$

where the dependence of H_o, V and $\Psi_\varepsilon^{\pm}$ on $\underline{r}$ is understood. Since H_o is an hermitian operator its eigen values are real and it follows that if W is complex then the operator $(W-H_o)$ has no zero eigen values. Consequently if $W=E\pm i\varepsilon$, where E and ε are both real, then $(W-H_o)^{-1}$ is well defined.

Operating on both sides of (1.6) with $(E-H_o \pm i\,\varepsilon)^{-1}$ gives

$$\Psi_\varepsilon^{\pm} = \varphi_\varepsilon + \frac{\lambda}{E-H_0 \pm i\varepsilon} V \Psi_\varepsilon^{\pm} \tag{1.7}$$

where ϕ_ε is any solution of

$$(H_0 - E \mp i\,\varepsilon)\,\varphi_\varepsilon = 0. \tag{1.8}$$

Let ϕ_n be a member of the set of eigen functions of

$$(H_0 - E_n)\,\varphi_n = 0. \tag{1.9}$$

Then

$$\frac{1}{E-H_0 \pm i\,\varepsilon}\,\varphi_n = \frac{1}{E-E_n \pm i\,\varepsilon}\,\varphi_n$$

so, using the closure property of the eigen functions of an hermitian operator, (1.7) becomes

$$\Psi_\varepsilon^{\pm}(\underline{r}) = \varphi_\varepsilon(\underline{r}) + \lambda \sum_n \frac{\varphi_n(\underline{r})}{E-E_n \pm i\varepsilon} \int \varphi_n(\underline{r}')\,V(\underline{r}')\,\Psi_\varepsilon^{\pm}(\underline{r}')\,d\underline{r}' \tag{1.10}$$

or

$$= \varphi_\varepsilon(\underline{r}) + \lambda \sum_n \frac{\varphi_n(\underline{r})\rangle\langle\varphi_n(\underline{r}')|\,V(\underline{r}')\,|\Psi_\varepsilon^{\pm}(\underline{r}')\rangle}{E-E_n \pm i\,\varepsilon}.$$

in Dirac's notation. In the limit as $\varepsilon \to 0$ (1.6) reduces to the Schrödinger equation (1.5). We define the two Green's functions of the operator $(E-H_o)$ by

$$G_0^{\pm} = \lim_{\varepsilon \to 0} \frac{1}{E-H_0 \pm i\varepsilon} = \lim_{\varepsilon \to 0} \sum_n \frac{\varphi_n\rangle\langle\varphi_n}{E-E_n \pm i\varepsilon} \tag{1.11}$$

and obtain as the two independent solutions of (1.5)

$$\Psi^{\pm} = \varphi_0 + \lambda G_0^{\pm} V \Psi^{\pm} \tag{1.12}$$

where ϕ_o is any solution of the homogeneous equation

$$(H_0 - E)\varphi_0 = 0. \tag{1.13}$$

A second order differential equation has two independent solutions which depend on the boundary conditions of the problem. When transforming a differential equation to an integral equation, the boundary conditions of the differential equation become implicitly contained in the integral operators of the integral equation. Consequently the two solutions of (1.5) become the solutions of the two integral equations (1.12) with different integral operators G_o^+ and G_o^-. It will be shown in the next section that G_o^+ represents an outgoing spherical wave boundary condition and G_o^- an incoming spherical wave boundary condition.

In general it is not possible to find an exact solution of (1.12), so a method of approximating to the correct solution must be used. In this thesis an iterative method of solving (1.12) is investigated. The solution of (1.12) to zeroth order in λ is ϕ_o. Substituting ϕ_o for $\Psi^{\pm}$ in the right hand side of (1.12) gives a first order solution. Repeating this process produces a sequence of wave functions $\Psi_n^{\pm}$ defined by

$$\Psi_o^{\pm} = \phi_o$$

$$\Psi_{n+1}^{\pm} = \phi_o + \lambda G_o^{\pm} V \Psi_n^{\pm}$$

which, hopefully, converges to the exact solution of the integral equation (1.12) i.e.

$$\Psi^{\pm} = \varphi_0 + \lambda G_0^{\pm} V \varphi_0 + \lambda^2 G_0^{\pm} V G_0^{\pm} V \varphi_0 + \cdots \tag{1.14}$$

This is called the Born series for $\Psi^{\pm}$ and its convergence and rate of convergence are discussed in section four of this chapter.

1.3 The Scattering Amplitude

The scattering amplitude is determined by the asymptotic form of the wave functions $\Psi^{\pm}$. This asymptotic form is simply derived from the exact expression for $G_o^{\pm}$.

The eigen values E_n and eigen functions ϕ_n of (1.9) are $\frac{\hbar^2 K^2}{2\mu}$ and $(2\pi)^{-3/2} e^{i\,\underline{k}.\,\underline{r}}$ where $\underline{K}$ can take any value. Consequently the summation over n in (1.11) must be replaced by an integration over the entire $\underline{K}$ space, giving

$$G_o^{\pm}(\underline{r},\underline{r}') = \lim_{\varepsilon\to 0} \frac{2\mu}{(2\pi)^3\hbar^2}\int \frac{e^{i\underline{k}.(\underline{r}-\underline{r}')}}{k^2-K^2\pm i2\mu\varepsilon}\, d\underline{k} \,.$$

Let $\underline{K}$ have co-ordinates (K, θ, ϕ) in spherical polar co-ordinates whose polar axis is parallel to $(\underline{r} - \underline{r}')$. The angular integrations are easily evaluated to give

$$G_o^{\pm}(\underline{r},\underline{r}') = \lim_{\varepsilon\to 0} \frac{\mu}{2\pi^2 i\hbar^2|\underline{r}-\underline{r}'|}\int_{-\infty}^{\infty} \frac{e^{iK|\underline{r}-\underline{r}'|}}{k^2-K^2\pm i2\mu\varepsilon}\, K\,dK \,.$$

Consider the contour integral in the complex plane

$$\int_C \frac{e^{iK|\underline{r}-\underline{r}'|}}{k^2-K^2+i2\mu\varepsilon}\, K\,dK \,.$$

Let the contour C be along the real axis from -R to +R , then around the semicircle of radius R, centre the origin, in the upper half plane. As $R \to \infty$ the contribution to this integral from the semicircle vanishes. There is a simple pole of the integrand at $K = \sqrt{k^2+i2\mu\varepsilon}$

with residue $-\frac{1}{2} \exp\left[i \sqrt{k^2+i\varepsilon}\ |\underline{r} - \underline{r}'| \right]$, and so by the residue theorem

$$\int_{-\infty}^{\infty} \frac{e^{iK|\underline{r}-\underline{r}'|}}{k^2-K^2+i2\mu\varepsilon} K\,dK = -i\pi \exp\left[i\sqrt{k^2+i\varepsilon}\ |\underline{r}-\underline{r}'| \right].$$

Taking the limit as $\varepsilon \to 0$

$$G_o^+(\underline{r},\underline{r}') = -\frac{\mu}{2\pi\hbar^2} \frac{e^{ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}. \qquad (1.15)$$

Similarly one can show that $G_o^-(\underline{r},\underline{r}') = -\frac{\mu}{2\pi\hbar^2} \frac{e^{-ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}$.

To determine the scattering amplitude, the asymptotic form of $G_o^{\pm}(\underline{r},\underline{r}')$ as $r \to \infty$ is required. When $r > r'$

$$|\underline{r}-\underline{r}'| = r - \frac{\underline{r}.\underline{r}'}{r} + O\left(\frac{r'}{r}\right),$$

thus

$$\frac{e^{\pm ik|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|} \underset{r\to\infty}{\sim} \frac{e^{\pm ikr}\, e^{\mp i\underline{k}'.\underline{r}'}}{r}$$

where $\underline{k}' = \frac{k\underline{r}}{r}$. The term ϕ_o occuring in (1.12) is any solution of (1.13), and since we are considering an incoming particle of momentum $\underline{k}$ we choose $\phi_o = e^{i\underline{k}.\underline{r}}$. The asymptotic form of $\Psi^{\pm}(\underline{r})$ is then

$$\Psi^{\pm}(\underline{r}) \underset{r\to\infty}{\sim} e^{i\underline{k}.\underline{r}} - \frac{\mu\lambda}{2\pi\hbar^2} \frac{e^{\pm ikr}}{r} \int e^{\mp i\underline{k}'.\underline{r}'} V(\underline{r}')\, \Psi^{\pm}(\underline{r}')\, d\underline{r}'. \qquad (1.16)$$

We can now clearly see the difference between the two solutions $\Psi^{\pm}$. Ψ^- represents a scattered incoming spherical wave, which is physically unreasonable, whilst Ψ^+ represents an outgoing spherical wave, which is our required solution. By comparison with (1.1)

$$f(\theta,\phi) = -\frac{\mu\lambda}{2\pi\hbar^2} \int e^{-i\underline{k}'.\underline{r}'} V(\underline{r}')\, \Psi^+(\underline{r}')\, d\underline{r}' \qquad (1.17)$$

where $e^{i\,\underline{k}'.\underline{r}'}$ is a solution of (1.13) and shall be denoted by $\phi_f(\underline{r}')$. It represents a scattered particle of momentum k moving in the direction $\underline{k}'$. If we replace Ψ^+ by its Born series (1.14) we obtain a Born series for the scattering amplitude:-

$$f(\theta,\phi) = -\frac{\mu}{2\pi\hbar^2}\left\{\lambda\langle\varphi_f|V|\varphi_0\rangle + \lambda^2\langle\varphi_f|VG_0^+V|\varphi_0\rangle + \cdots\right\}. \quad (1.18)$$

The first term in this series is called the first Born scattering amplitude, and the second term, the second Born correction to the scattering amplitude.

An alternative method of defining the scattering amplitude is in terms of the transition operator T, defined by the equation

$$T\varphi_0 = \lambda V \Psi^+.$$

Then

$$f(\theta,\phi) = -\frac{\mu}{2\pi\hbar^2}\langle\varphi_f|T|\varphi_0\rangle,$$

T satisfies the integral equation

$$T = \lambda V + \lambda V G_0^+ T$$

and has a Born series expansion

$$T = \lambda V + \lambda^2 V G_0^+ V + \lambda^3 V G_0^+ V G_0^+ V + \cdots .$$

1.4 Convergence of the Born Series

In this section it is shown that under suitable conditions the infinite series (1.14) converges to the solution of the integral equation (1.12). One approach is to solve the integral equation by the Fredholm method, obtaining a solution of the form

$$\Psi^+(\underline{r}) = \varphi_0(\underline{r}) + \frac{\lambda}{d(\lambda)}\int D(\lambda;\underline{r},\underline{r}')\,\varphi_0(\underline{r}')\,d\underline{r}'$$

where $d(\lambda)$ and $D(\lambda; \underline{r}, \underline{r}')$ are power series in λ . This solution converges for all λ (except in "pathological" cases), and it can be shown that the radius of convergence of the Born series for $\Psi^+(\underline{r})$ is $|\lambda_c|$ where λ_c is the smallest zero of $d(\lambda)$. (c.f. Jost and Pais (1951))

The approach described in this section is that of Manning (1965). Essentially, it involves a term by term comparison of the Born series with a series that is known to converge. It gives a simple derivation of the earlier results of Zemach and Klein (1958) and Kohn (1954), who used similar methods. For convenience, we shall set λ to unity and only consider the Ψ^+ wave function. Consequently we are investigating the convergence properties of the Born series solution to the integral equation

$$\Psi = \varphi_0 + G_0^+ V \Psi. \tag{1.19}$$

The basis of this approach is the Banach-Weissinger theorem for iterative processes. In its most general form it is concerned with elements of a Banach space S. If the elements of S are denoted by f_1, f_2 ... then they satisfy the following three Axioms:-

1. They form an abelian group with respect to an operation which we shall call addition and denote by +.

11. The set is "normed". That is for every element $f \in S$ there is defined a real non-negative number $||f||$, called the norm of f, which has the following properties:-

    (i) $|| af || = |a| \times || f ||$ where $f \in S$ and a is an arbitary complex number.

    (ii) $|| f + g || \leq || f || + || g ||$ where $f, g \in S$.

(iii) $|| f || > 0$ for $f \neq \theta$ where $f \in S$ and θ is the null element of S.

111. S is complete. That is for every sequence $\{f_n\} \in S$ with the property that

$$\lim_{m,n\to\infty} || f_m - f_n || = 0 , \tag{1.20}$$

there exists an element $f \in S$ such that

$$\lim_{n\to\infty} || f - f_n || = 0 . \tag{1.21}$$

The Banach Weissinger theorem can be stated as follows:-
Let F be a single valued mapping of a Banach space S into itself which satisfies for every f and $g \in S$

$$|| F(f) - F(g) || \leq \alpha || f - g || \tag{1.22}$$

with $\alpha < 1$. Let f_o be an element of S. f_1 is defined as $f_1 = F(f_o)$. Suppose that $|| f_1 - f_o ||$ is finite and S contains all elements h in the sphere

$$|| h - f_1 || \leq \frac{\alpha}{1-\alpha} || f_1 - f_0 || . \tag{1.23}$$

Then the sequence $f_o, f_1, f_2 \ldots$ defined by

$$f_{n+1} = F(f_n) \tag{1.24}$$

is contained in the sphere (1.23) and converges (in the sense of (1.21)) to a unique element f_∞ which also lies in the sphere and has the property

$$f_\infty = F(f_\infty) . \tag{1.25}$$

Furthermore

$$\begin{aligned} || f_\infty - f_n || &\leq \frac{\alpha}{1-\alpha} || f_n - f_{n-1} || \\ &\leq \frac{\alpha^n}{1-\alpha} || f_1 - f_0 || . \end{aligned} \tag{1.26}$$

Consider the set $\{\psi_n(x)\}$ of continuous functions of $x = (x_1 \ldots x_n)$. They form an abelian group under addition and a convenient choice of norm is

$$\|\Psi_n\| = \max_x \left(\frac{|\Psi_n(x)|}{W(x)} \right) \tag{1.27}$$

where W(x) is a fixed, finite, continuous, real, positive function of x. With this norm the set $\{\psi_n(x)\}$ forms a Banach space and satisfies the condition (1.23). Define the functional F to be

$$F(\Psi_n) = \varphi_0 + G_0^+ V \Psi_n . \tag{1.28}$$

Then the Born series of approximations to the solution of the integral equation $\Psi = F(\Psi)$ is generated by

$$\Psi_{n+1} = F(\Psi_n) \quad , \quad \Psi_0 = \varphi_0 .$$

Consider two arbitary elements f, g ε $\{\psi_n\}$. Then

$$\begin{aligned} \|F(f) - F(g)\| &= \|G_0^+ V[f-g]\| \\ &= \left\| \int G_0^+(x,x') V(x') [f(x') - g(x')] \, dx' \right\| \\ &\leqslant \alpha \|f-g\| \end{aligned}$$

where $$\alpha = \max_x \left[\frac{1}{W(x)} \int |G_0^+(x,x') V(x')| W(x') \, dx' \right] . \tag{1.29}$$

Consequently if $\alpha < 1$ the condition (1.22) of the Banach-Weissinger theorem is satisfied and the Born series converges to the exact solution. Since $\psi_1 = \phi_0 + G_0^+ V\psi_0$ and $\psi_0 = \phi_0$ using (1.26)

$$\begin{aligned} \|\Psi - \Psi_n\| &\leqslant \frac{\alpha^n}{1-\alpha} \|\Psi_1 - \Psi_0\| \\ &\leqslant \frac{\alpha^{n+1}}{1-\alpha} \|\varphi_0\| \end{aligned} \tag{1.30}$$

and so we have a bound on the error of stopping the iteration after n steps. Similar results can be obtained for the scattering amplitude and the transition operator.

Substituting the value of $G_o^+(\underline{r}, \underline{r}^1)$ given by (1.15) into (1.29) we obtain

$$\alpha = \max_r \frac{\mu}{2\pi W(r)\hbar^2} \int \frac{|V(\underline{r}')|}{|\underline{r}-\underline{r}'|} W(r')\, d\underline{r}'.$$

Making the choice $W(r) = 1$ gives a sufficient condition for the convergence of the Born series, namely that

$$\max_r I(\underline{r}) < 1 \tag{1.31}$$

where

$$I(\underline{r}) = \frac{\mu}{2\pi\hbar^2} \int \frac{|V(\underline{r}')|}{|\underline{r}-\underline{r}'|} d\underline{r}'.$$

Another convergence criterion is obtainable from the functional

$$F'(\psi) = \varphi_0 + G_0^+ V \varphi_0 + G_0^+ V G_0^+ V \psi \tag{1.32}$$

which generates the Born series $\psi_{n+1} = F'(\psi_{n-1})$, $\psi_o = \phi_o$. Consider two arbitary elements f, g ε $\{\psi_n\}$. Then

$$\| F'(f) - F'(g) \| = \left\| \iint G_0^+(\underline{r},\underline{r}') V(\underline{r}') G_0^+(\underline{r}',\underline{r}'') V(\underline{r}'') [f(\underline{r}'') - g(\underline{r}'')] d\underline{r}' d\underline{r}'' \right\|.$$

Let

$$g(\underline{r},\underline{r}'') = \frac{\int G_0^+(\underline{r},\underline{r}') V(\underline{r}') G_0^+(\underline{r}',\underline{r}'')\, d\underline{r}'}{G_0^+(\underline{r},\underline{r}'')}, \tag{1.33}$$

then

$$\| F'(f) - F'(g) \| = \left\| \int g(\underline{r},\underline{r}'') G_0^+(\underline{r},\underline{r}'') V(\underline{r}'') [f(\underline{r}'') - g(\underline{r}'')] d\underline{r}'' \right\|$$

$$\leqslant \|g\| \max_r I(\underline{r}) \|f - g\|$$

where we have set W(r) = 1 and defined

$$\|g\| = \max_{r, r''} |g(\underline{r},\underline{r}'')|. \tag{1.34}$$

Thus a sufficient condition for the convergence of the Born series is that

$$\alpha' = \|g\| \max_{r} I(\underline{r}) < 1 . \tag{1.35}$$

The importance of this result is that Zemach and Klein (1958) have shown that $\lim_{k\to\infty} \| g \| = 0$ provided the potential $V(\underline{r})$ satisfies the conditions:-

(i) $I(\underline{r}) < \infty$ for all $\underline{r}$

(ii) $I(\underline{r})$ is a continuous function of $\underline{r}$

(iii) $I(\underline{r}) = O(\frac{1}{r})$ as $r \to \infty$.

Consequently $\lim_{k\to\infty} \alpha' = 0$ and since $\psi_o = \phi_o$, (1.30) implies

$$\lim_{k\to\infty} \| \Psi - \varphi_0 \| = 0 .$$

Thus in the high energy limit the Born series converges to its first term ϕ_o.

1.5 Padé Approximants and the Schwinger Variational Principle

The $[N,M]$ Padé approximant to a series

$$g(\lambda) = g_o + \lambda g_1 + \lambda^2 g_2 + \ldots .$$

is defined (Wall 1948) as the ratio

$$[N,M] = \frac{P_N(\lambda)}{Q_M(\lambda)} ,$$

where $P_N(\lambda)$ and $Q_M(\lambda)$ are polynomials in λ of order N and M respectively. They are uniquely defined by the equations

$$Q_M(\lambda)\, g(\lambda) = P_N(\lambda) + O(\lambda^{N+M+1}) ,$$

$$Q_M(0) = 1 .$$

We can write the Born series (1.18) for the scattering amplitude as

$$f = \lambda f_1 + \lambda^2 f_2 + \cdots\cdots \tag{1.36}$$

where

$$f_1 = -\frac{\mu}{2\pi\hbar^2} \langle \varphi_f | V | \varphi_0 \rangle , \quad f_2 = -\frac{\mu}{2\pi\hbar^2} \langle \varphi_f | V G_0^+ V | \varphi_0 \rangle .$$

The $[1, 1]$ Padé approximant to this infinite series (1.36) is

$$f^{P1} = \frac{\lambda^2 f_1^2}{\lambda f_1 + \lambda^2 f_2} . \tag{1.37}$$

There is, however, an independent method of obtaining the approximation (1.37) to the scattering amplitude. Schwinger has shown that the expression (1.38) below for the scattering amplitude is stationary with respect to small variations in the wave functions ψ_i and ψ_{-s}.

$$f(\theta,\phi) = \frac{\mu}{2\pi\hbar^2} \frac{\left[\int \psi_{-s}(\underline{r}) V(\underline{r}) e^{i\underline{k}_i\cdot\underline{r}} d\underline{r}\right]\left[\int e^{-i\underline{k}_s\cdot\underline{r}} V(\underline{r}) \psi_i(\underline{r}) d\underline{r}\right]}{\int \psi_{-s}(\underline{r}) V(\underline{r}) \psi_i(\underline{r}) d\underline{r} + \iint \psi_{-s}(\underline{r}) V(\underline{r}) G_0^+(\underline{r},\underline{r}') V(\underline{r}') \psi_i(\underline{r}') d\underline{r}\, d\underline{r}'} \tag{1.38}$$

ψ_i and ψ_{-s} are both solutions of the Schrödinger equation (1.5) and have asymptotic forms

$$\psi_i(\underline{r}) \underset{r\to\infty}{\sim} e^{i\underline{k}_i\cdot\underline{r}} + \frac{e^{ikr}}{r} f(\theta,\phi)$$

$$\psi_{-s}(\underline{r}) \underset{r\to\infty}{\sim} e^{-i\underline{k}_s\cdot\underline{r}} + \frac{e^{ikr}}{r} f(\theta,\phi) \qquad (k = |\underline{k}_i| = |\underline{k}_s|)$$

If trial plane waves are chosen for ψ_i and ψ_{-s} i.e. $\psi_i = e^{i\underline{k}_i\cdot\underline{r}}$ and $\psi_{-s} = e^{-i\underline{k}_s\cdot\underline{r}}$ then (1.38) reduces to (1.37).

CHAPTER 2

ELECTRON-ATOM SCATTERING

2.1 Introduction

Throughout the remainder of this thesis the frame of reference for the study of collisions will be the centre of mass frame. That is the centre of mass of the system of bodies will be assumed fixed and the bodies moving relative to it. Also only collisions between an atom (A) and a structureless charged particle (B) will be considered. The effects of exchange will be ignored since they are expected to be small in the energy range to be investigated.

The Hamiltonian of this system can be divided into three parts. One part represents the relative motion of the particle and the atom. This has the form $-\frac{\hbar^2}{2\mu}\nabla^2_{\underline{R}}$ where $\underline{R}$ denotes the relative position of the bodies and μ the reduced mass of the two bodies i.e. if m_A and m_B are the masses of the atom and particle respectively, $\mu = \frac{m_A m_B}{m_A+m_B}$. Secondly a part representing the internal motion of the atom. This is the Hamiltonian of the unperturbed atom H_A and satisfies the Schrödinger equation

$$\left[H_A(\underline{r}_A) - \varepsilon_n\right]\varphi_n(\underline{r}_A) = 0 \qquad (2.1)$$

where $\underline{r}_A$ denotes the coordinates of the bound electrons relative to the nucleus, ε_n and ϕ_n the energy and wavefunction of the n^{th} state of the atom. Lastly there is the interaction potential between the particle and the atom. If the particle has charge $Z_B e$ and the nucleus of the atom charge $Z_A e$, e being the charge on an electron, then this potential is

$$V(\underline{R},\underline{r}_A) = -Z_B e^2\left(\frac{Z_A}{R} - \sum_{\substack{\text{bound}\\\text{electrons}}}\frac{1}{|\underline{R}-\underline{r}_A|}\right). \qquad (2.2)$$

Consequently, the Schrödinger equation of the system is

$$\left[-\frac{\hbar^2}{2\mu}\nabla_{\underline{R}}^2 + H_A(\underline{r}_A) + V(\underline{R},\underline{r}_A)\right]\Psi(\underline{R},\underline{r}_A) = E\,\Psi(\underline{R},\underline{r}_A). \tag{2.3}$$

In the asymptotic region the interaction potential $V(\underline{R},\underline{r}_A)$ is assumed to be zero so asymptotically the wavefunction $\Psi(\underline{R},\underline{r}_A)$ must represent an unperturbed atom and a freely moving particle. If the atom is initially in state i and the particle has momentum $\underline{k}_i$ then initially the asymptotic form of the wavefunction is $e^{i\underline{k}_i\cdot\underline{R}}\varphi_i(\underline{r}_A)$ and the total energy of the system $E = \frac{\hbar^2 k_i^2}{2\mu} + \varepsilon_i$. After the collision the atom will be in some arbitrary but accessible final state f, the particle will have been scattered into some arbitrary direction, denoted by the angles (θ,ϕ) and by the conservation of energy will have momentum k_f, where $E = \frac{h^2 k_f^2}{2\mu} + \varepsilon_f$. By an accessible state of an atom it is meant that the incident particle has sufficient energy to excite that state of the atom. The wavefunction of the scattered particle will be a spherical wave centered on the centre of mass of the colliding particles. However the probability of the particle being scattered into a certain direction will be a function of that direction so the asymptotic form of the total wavefunction is

$$\Psi(\underline{R},\underline{r}_A)\underset{R\to\infty}{\sim} e^{i\underline{k}_i\cdot\underline{R}}\varphi_i(\underline{r}_A) + \sum_{\substack{\text{all accessible}\\ \text{final states}}} f_{if}(\theta,\phi)\frac{e^{ik_f R}}{R}\varphi_f(\underline{r}_A) \tag{2.4}$$

and $f_{if}(\theta,\phi)$ is the scattering amplitude for the transition from state i to state f. The number of particles per unit time which excite state f of the atom and are scattered into solid angle $d\Omega$, about the direction (θ,ϕ) is proportional to the incident flux of particles. The factor of proportionality is called the differential cross section for the transition from state i to state f, $I_{if}(\theta,\phi)$. By comparing the fluxes of the incident and scattered waves in the asymptotic region it follows that

$$I_{if}(\theta,\phi) = \frac{k_f}{k_i}\left|f_{if}(\theta,\phi)\right|^2. \tag{2.5}$$

The total cross section for the transition from state i to f is

$$Q_{if} = \int I_{if}(\theta,\phi)\,d\Omega \tag{2.6}$$

and the total cross section

$$Q = \sum_f Q_{if} \tag{2.7}$$

where the summation is over all accessible states of the atom.

2.2 The Born Series for Structureless Particle-Atom Scattering.

The derivation of the Born series for the total wavefunction of a structureless particle-atom collision is very similar to that for potential scattering. Consider the two differential equations

$$\left(E - H_A(\underline{r}_A) + \frac{\hbar^2}{2\mu}\nabla^2_{\underline{R}} \pm i\epsilon\right)\Psi^{\pm}_{\epsilon}(\underline{R},\underline{r}_A) = V(\underline{R},\underline{r}_A)\,\Psi^{\pm}_{\epsilon}(\underline{R},\underline{r}_A) \tag{2.8}$$

where E and ϵ are real. Operating on both sides of (2.8) with the well-defined operator $\left(E - H_A(\underline{r}_A) + \frac{\hbar^2}{2\mu}\nabla^2_{\underline{R}} \pm i\epsilon\right)^{-1}$ gives

$$\Psi^{\pm}_{\epsilon}(\underline{R},\underline{r}_A) = \psi^{\pm}_{\epsilon}(\underline{R},\underline{r}_A) + \frac{1}{E - H_A(\underline{r}_A) + \frac{\hbar^2}{2\mu}\nabla^2_{\underline{R}} \pm i\epsilon}\,V(\underline{R},\underline{r}_A)\,\Psi^{\pm}_{\epsilon}(\underline{R},\underline{r}_A) \tag{2.9}$$

where $\psi^{\pm}_{\epsilon}$ is any solution of

$$\left(E - H_A(\underline{r}_A) + \frac{\hbar^2}{2\mu}\nabla^2_{\underline{R}} \pm i\epsilon\right)\psi^{\pm}_{\epsilon}(\underline{R},\underline{r}_A) = 0. \tag{2.10}$$

Consider the separable equation

$$\left(H_A(\underline{r}_A) - \frac{\hbar^2}{2\mu} \nabla_{\underline{R}}^2 - E_{n,\underline{q}} \right) \Psi_{n,\underline{q}}(\underline{R}, \underline{r}_A) = 0. \tag{2.11}$$

By comparison with (2.1) and the Schrödinger equation of a free particle

$$\left(\frac{\hbar^2}{2\mu} \nabla_{\underline{R}}^2 - \frac{\hbar^2 q^2}{2\mu} \right) (2\pi)^{-3/2} e^{i\underline{q}.\underline{R}} = 0,$$

it is clear that the eigenfunctions and eigenvalues of (2.11) are

$$\Psi_{n,\underline{q}}(\underline{R}, \underline{r}_A) = (2\pi)^{-3/2} e^{i\underline{q}.\underline{R}} \varphi_n(\underline{r}_A)$$

and

$$E_{n,\underline{q}} = \varepsilon_n + \frac{\hbar^2 q^2}{2\mu}. \tag{2.12}$$

Using the closure properties of eigenfunctions

$$\frac{\delta(\underline{R}, \underline{R}')\delta(\underline{r}_A, \underline{r}_A')}{E - H_A(\underline{r}_A) + \frac{\hbar^2}{2\mu}\nabla_{\underline{R}}^2 \pm i\varepsilon} = \mathop{S\!\!\!\!\int}_n \int d\underline{q} \, \frac{1}{E - H_A(\underline{r}_A) + \frac{\hbar^2}{2\mu}\nabla_{\underline{R}}^2 \pm i\varepsilon} \Psi_{n,\underline{q}}(\underline{R}, \underline{r}_A) \Psi^*_{n,\underline{q}}(\underline{R}', \underline{r}_A')$$

$$= \mathop{S\!\!\!\!\int}_n \int d\underline{q} \, \frac{\Psi_{n,\underline{q}}(\underline{R}, \underline{r}_A) \Psi^*_{n,\underline{q}}(\underline{R}', \underline{r}_A')}{E - \varepsilon_n - \frac{\hbar^2 q^2}{2\mu} \pm i\varepsilon}$$

where $\mathop{S\!\!\!\!\int}_n$ denotes a summation over all the discrete states of the atom and an integration over all the continuum states. Energy is conserved in this collision, so if the atom is in state n the particle will have momentum $\underline{k}_n$ whose magnitude is given by $E = \varepsilon_n + \frac{\hbar^2 k_n^2}{2\mu}$.

In Dirac's notation (2.9) is

$$\Psi_\varepsilon^{\pm}(\underline{R}, \underline{r}_A) = \Psi_\varepsilon^{\pm}(\underline{R}, \underline{r}_A) + \mathop{S\!\!\!\!\int}_n \int d\underline{q} \, \frac{\Psi_{n,\underline{q}}(\underline{R}, \underline{r}_A) \rangle \langle \Psi_{n,\underline{q}}(\underline{R}', \underline{r}_A') | V(\underline{R}', \underline{r}_A') | \Psi_\varepsilon^{\pm}(\underline{R}', \underline{r}_A') \rangle}{\frac{\hbar^2 k_n^2}{2\mu} - \frac{\hbar^2 q^2}{2\mu} \pm i\varepsilon}. \tag{2.13}$$

In the limit as $\varepsilon \to 0$ (2.8) reduces to the Schrödinger equation (2.3) and (2.10) reduces to (2.11). Since the atom is initially in state i and the particle has momentum $\underline{k}_i$ we choose $\lim_{\varepsilon \to 0} \Psi_\varepsilon^\pm(\underline{R},\underline{r}_A) = e^{i\underline{k}_i\cdot\underline{R}}\,\varphi_i(\underline{r}_A)$. Consequently (2.13) becomes

$$\Psi^\pm(\underline{R},\underline{r}_A) = e^{i\underline{k}_i\cdot\underline{R}}\varphi_i(\underline{r}_A) + \iint G_0^\pm(\underline{R},\underline{r}_A;\underline{R}',\underline{r}_A')V(\underline{R}',\underline{r}_A')\Psi^\pm(\underline{R}',\underline{r}_A')\,d\underline{R}'\,d\underline{r}_A' \tag{2.14}$$

$$\text{where } G_0^\pm(\underline{R},\underline{r}_A;\underline{R}',\underline{r}_A') = \lim_{\varepsilon\to 0}\frac{2\mu}{(2\pi)^3\hbar^2}\,\mathop{S}_n\!\!\!\!\!\!\int d\underline{q}\,\frac{e^{i\underline{q}\cdot(\underline{R}-\underline{R}')}\varphi_n(\underline{r}_A)\varphi_n^*(\underline{r}_A')}{k_n^2-q^2\pm i\varepsilon} \tag{2.15}$$

$$= -\frac{\mu}{2\pi\hbar^2}\mathop{S}_n\left(|\underline{R}-\underline{R}'|\right)^{-1}e^{\pm ik_n|\underline{R}-\underline{R}'|}\,\varphi_n(\underline{r}_A)\varphi_n^*(\underline{r}_A')$$

$$\underset{R\to\infty}{\sim} -\frac{\mu}{2\pi\hbar^2}\mathop{S}_n\frac{e^{\pm ik_nR}}{R}\varphi_n(\underline{r}_A)\,e^{\mp i\underline{k}_n\cdot\underline{R}'}\varphi_n^*(\underline{r}_A') \tag{2.16}$$

using results contained in section 1.3. Substituting G_0^+ from (2.16) into (2.14) gives the wavefunction with the correct asymptotic form (i.e. outgoing scattered particles), namely

$$\Psi^+(\underline{R},\underline{r}_A) \underset{R\to\infty}{\sim} e^{i\underline{k}_i\cdot\underline{R}}\varphi_i(\underline{r}_A) - \frac{\mu}{2\pi\hbar^2}\mathop{S}_n\frac{e^{ik_nR}}{R}\varphi_n(\underline{r}_A)$$
$$\times\iint e^{-i\underline{k}_n\cdot\underline{R}'}\varphi_n^*(\underline{r}_A')V(\underline{R}',\underline{r}_A')\Psi^+(\underline{R}',\underline{r}_A')\,d\underline{R}'\,d\underline{r}_A' \,. \tag{2.17}$$

Comparing (2.17) with (2.4) gives the expression for the scattering amplitude from state i to state f

$$f_{if}(\theta,\phi) = -\frac{\mu}{2\pi\hbar^2}\iint e^{-i\underline{k}_f\cdot\underline{R}}\varphi_f^*(\underline{r}_A)V(\underline{R},\underline{r}_A)\Psi^+(\underline{R},\underline{r}_A)\,d\underline{R}\,d\underline{r}_A \,. \tag{2.18}$$

The zeroth order solution of (2.14) is $\Psi_0^+(\underline{R},\underline{r}_A) = e^{i\underline{k}_i\cdot\underline{R}}\phi_i(\underline{r}_A)$ and substituting this into (2.18) gives the first Born approximation to the scattering amplitude,

$$f_{if}^{B1}(\theta,\phi) = -\frac{\mu}{2\pi\hbar^2}\iint e^{i(\underline{k}_i-\underline{k}_f)\cdot\underline{R}}\varphi_f^*(\underline{r}_A)V(\underline{R},\underline{r}_A)\varphi_i(\underline{r}_A)\,d\underline{R}\,d\underline{r}_A \,. \tag{2.19}$$

The first order solution of (2.14) is

$$\Psi_1^+(\underline{R},\underline{r}_A) = \Psi_0^+(\underline{R},\underline{r}_A) + \iint G_0^+(\underline{R},\underline{r}_A;\underline{R}',\underline{r}_A') V(\underline{R}',\underline{r}_A') e^{i\underline{k}_i\cdot\underline{R}'} \varphi_i(\underline{r}_A') d\underline{R}' d\underline{r}_A' .$$

Substituting the second term of the right hand side into (2.18) gives the second Born correction to the scattering amplitude

$$f_{if}^{B2}(\theta,\phi) = -\frac{\mu}{2\pi\hbar^2}\iint e^{-i\underline{k}_f\cdot\underline{R}}\varphi_f^*(\underline{r}_A)V(\underline{R},\underline{r}_A)G_0^+(\underline{R},\underline{r}_A;\underline{R}',\underline{r}_A')V(\underline{R}',\underline{r}_A')e^{i\underline{k}_i\cdot\underline{R}'}\varphi_i(\underline{r}_A') \times d\underline{R}\,d\underline{r}_A\,d\underline{R}'\,d\underline{r}_A'$$

$$= \lim_{\epsilon\to 0} -\frac{2\mu^2}{(2\pi)^4\hbar^4} \mathop{S}_n \int \frac{d\underline{q}}{k_n^2-q^2+i\epsilon} \iint e^{i(\underline{q}-\underline{k}_f)\cdot\underline{R}}\varphi_f^*(\underline{r}_A)V(\underline{R},\underline{r}_A)\varphi_n(\underline{r}_A)\,d\underline{R}\,d\underline{r}_A \quad (2.20)$$
$$\times \iint e^{i(\underline{k}_i-\underline{q})\cdot\underline{R}'}\varphi_n^*(\underline{r}_A')V(\underline{R}',\underline{r}_A')\varphi_i(\underline{r}_A')\,d\underline{R}'\,d\underline{r}_A'$$

where (2.15) has been used. The expressions (2.19) and (2.20) can be further simplified by using Bethe's integral

$$\int \frac{e^{i\underline{K}\cdot\underline{R}'}}{|\underline{R}-\underline{R}'|}\,d\underline{R}' = \frac{4\pi}{K^2}\,e^{i\underline{K}\cdot\underline{R}} . \quad (2.21)$$

Using (2.2) for the interaction potential

$$\int e^{i\underline{K}\cdot\underline{R}}V(\underline{R},\underline{r}_A)\,d\underline{R} = -Z_B e^2 \int e^{i\underline{K}\cdot\underline{R}}\left[\frac{Z_A}{R} - \sum_{\substack{\text{bound}\\ \text{electrons}}}\frac{1}{|\underline{R}-\underline{r}_A|}\right]d\underline{R}$$

$$= Z_B e^2 \frac{4\pi}{K^2}\left[\sum_{\substack{\text{bound}\\ \text{electrons}}} e^{i\underline{K}\cdot\underline{r}_A} - Z_A\right] .$$

The integral $I(f,i;\underline{K})$ is defined as

$$I(f,i;\underline{K}) = \int \varphi_f^*(\underline{r}_A)\left[\sum_{\substack{\text{bound}\\ \text{electrons}}} e^{i\underline{K}\cdot\underline{r}_A} - Z_A\right]\varphi_i(\underline{r}_A)\,d\underline{r}_A \quad (2.22)$$

and the expressions for the Born scattering amplitudes become

$$f_{if}^{B1}(\theta,\phi) = \frac{-2}{(\underline{k}_i-\underline{k}_f)^2}\,\frac{\mu e^2 Z_B}{\hbar^2}\,I(f,i;\underline{k}_i-\underline{k}_f) \quad (2.23)$$

and $$f_{if}^{B2}(\theta,\phi) = \frac{2}{\pi^2}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2 \lim_{\varepsilon\to 0} \mathop{\mathrm{S}\!\!\!\int}_n \int d\underline{q}\, \frac{I(f,n;\underline{q}-\underline{k}_f)\, I(n,i;\underline{k}_i-\underline{q})}{(q^2-k_n^2-i\varepsilon)(\underline{q}-\underline{k}_f)^2(\underline{q}-\underline{k}_i)^2} . \tag{2.24}$$

2.3 Identities satisfied by the Second Born Scattering Amplitude.

The fact that particles are neither created nor destroyed during a collision produces a relationship called the optical theorem. It states that the imaginary part of the forward elastic scattering amplitude is proportional to the total cross section (see (2.7)).

$$\mathrm{Im}\, f_{ii}(0,k_i^2) = \frac{k_i}{4\pi}\, Q(k_i^2) , \tag{2.25}$$

where we now explicitly show the dependence of f_{ii} and Q on the relative energy of the colliding bodies.

It is generally believed that the forward elastic scattering amplitude for electron-atom collisions has the following analytic properties in the complex k^2 plane. (Gerjuoy and Krall (1960)). It is an analytic function of k^2 with the exception of poles at the positions of the bound states of the electron-atom system and a cut along the real k^2 axis. Also the combination

$$\left[f_{ii}(0,k_i^2) - f_{ii}^{B1(D)}(0,k_i^2) + f_{ii}^{B1(E)}(0,k_i^2) \right]$$

is a polynomial bounded in the complex plane where $f_{ii}^{B1(D)}$ and $f_{ii}^{B1(E)}$ are the first Born approximations to the direct and exchange scattering amplitudes respectively. If there are no bound states of the electron-atom system below the threshold for elastic scattering (as with Helium) these properties give one the dispersion relation

$$\mathrm{Re}\, f_{ii}(0,k_i^2) = f_{ii}^{B1(D)}(0,k_i^2) - f_{ii}^{B1(E)}(0,k_i^2) + \frac{1}{4\pi^2} P\int_0^\infty \frac{k' Q(k'^2)}{k'^2 - k_i^2}\, dk'^2 . \tag{2.26}$$

It will now be shown that the second Born scattering amplitude satisfies identities analagous to these two relationships. The basis of the derivation is the identity

$$\lim_{\varepsilon\to 0}\int_0^\infty \frac{dq}{q^2-k_n^2-i\varepsilon} = P\int_0^\infty \frac{dq}{q^2-k_n^2} + \frac{i\pi}{2k_n}\int_0^\infty \delta(q-k_n)\,dq \qquad (2.27a)$$

where P denotes the principle part of the integral and $\delta(r)$ is the dirac delta function. k_n is the wavenumber of the incident electron when the target atom is in some intermediate state n. If the energy of this state n relative to the initial state of the atom is Δ_n^2 rydbergs, then $k_n^2 = k_i^2 - \Delta_n^2$. If $k_i^2 > \Delta_n^2$ then the state n is accessible, k_n^2 is positive and (2.27a) holds. However, if the state n is inaccessible (i.e. $k_i^2 < \Delta_n^2$) then k_n^2 is negative and (2.27b) holds

$$\lim_{\varepsilon\to 0}\int_0^\infty \frac{dq}{q^2+|k_n^2|-i\varepsilon} = P\int_0^\infty \frac{dq}{q^2+|k_n^2|}\,. \qquad (2.27b)$$

The first Born total cross section for the transition from state i to state n is (cf (2.5) and (2.6))

$$Q_{in}^{B1}(k_i^2) = \frac{k_n}{k_i}\int \left|f_{in}^{B1}(\theta,\phi)\right|^2 d\Omega\,.$$

Using the expression (2.23) for $f_{in}^{B1}(\theta,\phi)$ gives

$$Q_{in}^{B1}(k_i^2) = 4\frac{k_n}{k_i}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2\int \frac{|I(n,i;\underline{k}_i-\underline{k}_n)|^2}{(\underline{k}_i-\underline{k}_n)^4}\,d\hat{\underline{k}}_n \qquad (2.28)$$

$$= \frac{8\pi}{k_i^2}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2\int_{k_i-k_n}^{k_i+k_n}\frac{|I(n,i;\underline{K})|^2}{K^3}\,dK \qquad (2.29)$$

where $K^2 = k_i^2 + k_n^2 - 2k_ik_n\cos\theta = (\underline{k}_i - \underline{k}_n)^2$. The axis of quantisation of the atom is chosen parallel to $\underline{K}$ so that the matrix element $I(n,i;\underline{K})$ is independent of the azimuthal angle ϕ of $\underline{K}(K,\theta,\phi)$.

Using the identity (2.27a) the imaginary part of the second Born correction to the forward elastic scattering amplitude from the n^{th} intermediate state is

$$
\begin{aligned}
\mathrm{Im}\, f_{ii}^{B2(n)}(0,k_i^2) &= \frac{2}{\pi^2}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2 \frac{\pi}{2k_n}\int \frac{I(i,n;\underline{q}-\underline{k}_i)\, I(n,i;\underline{k}_i-\underline{q})\,\delta(q-k_n)}{(\underline{q}-\underline{k}_i)^4}\, q^2\, dq\, d\hat{\underline{q}} \\
&= \frac{k_n}{\pi}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2 \int \frac{|I(n,i;\underline{k}_i-\underline{k}_n)|^2}{(\underline{k}_n-\underline{k}_i)^4}\, d\hat{\underline{k}}_n \\
&= \frac{k_i}{4\pi}\, Q_{in}^{B1}(k_i^2)\,.
\end{aligned}
\qquad (2.30)
$$

If the n^{th} state is inaccessible then the incident electron has insufficient energy to excite that state and hence the first Born approximation to the total cross section for the excitation of that state is zero. Also in this case the imaginary part of the second Born correction vanishes since the identity (2.27b) holds rather than (2.27a) and the former has no imaginary part. Consequently (2.30) holds whether the n^{th} state of the atom is accessible or not. The first Born elastic scattering amplitude is purely real. Summing over all states of the atom gives

$$
\mathrm{Im}\left[f_{ii}^{B1}(0,k_i^2) + f_{ii}^{B2}(0,k_i^2)\right] = \frac{k_i}{4\pi}\, Q^{B1}(k_i^2) \qquad (2.31)
$$

which is analagous to the optical theorem, (2.25).

From (2.24) and (2.27a)

$$
\mathrm{Re}\, f_{ii}^{B2(n)}(0,k_i^2) = \frac{2}{\pi^2}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2 P\int_0^\infty dq \int_0^\pi d\theta \int_0^{2\pi} d\phi\, \frac{|I(n,i;\underline{q}-\underline{k}_i)|^2}{(q^2-k_n^2)(\underline{q}-\underline{k}_i)^4}\, q^2 \sin\theta
$$

where $\underline{q}$ has coordinates (q,θ,ϕ) with respect to spherical polar coordinates with polar axis $\underline{k}_i$. The axis of quantisation is chosen parallel to $(\underline{q} - \underline{k}_i)$. Consider the vector $\underline{K}(K,\alpha,\phi)$ defined by

$$
\underline{K} = \underline{q} - \underline{k}_i\,.
$$

It can be easily shown that

$$\frac{q^2 \sin\theta}{q^2 - k_n^2}\, dq\, d\theta = \frac{K^2 \sin\alpha}{K^2 + 2Kk_i\cos\alpha + \Delta_n^2}\, dK\, d\alpha$$

where $\Delta_n^2 = k_i^2 - k_n^2$ and that as $\underline{q}$ varies over all space so does $\underline{K}$.

If the state n is inaccessible then the identity (2.27b) must be used instead of (2.27a) and this gives

$$Rl\, f_{ii}^{B2(n)}(0,k_i^2) = \frac{2}{\pi^2}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2 P\int_0^\infty dq \int_0^\pi d\theta \int_0^{2\pi} d\phi\, \frac{|I(n,i;\underline{q}-\underline{k}_i)|^2}{(q^2+|k_n^2|)(\underline{q}-\underline{k}_i)^4}\, q^2 \sin\theta \, .$$

Applying the same transformation as above one obtains

$$\frac{q^2 \sin\theta}{q^2 + |k_n^2|}\, dq\, d\theta = \frac{K^2 \sin\alpha}{K^2 + 2Kk_i\cos\alpha + \Delta_n^2}\, dK\, d\alpha$$

and consequently the same expression for $Rl\, f_{ii}^{B2(n)}$ is obtained whether the intermediate state n is accessible or not. Replacing q and θ by K and ν (= cos α) and integrating over ϕ gives

$$Rl\, f_{ii}^{B2(n)}(0,k_i^2) = \frac{4}{\pi}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2 P\int_0^\infty dK \int_{-1}^{1} d\nu\, \frac{|I(n,i;\underline{K})|^2}{K^2(K^2+2Kk_i\nu+\Delta_n^2)} \, .$$

Now substitute for ν in terms of l_i defined by

$$\nu = \frac{l_i K - K^2 - Kk_i - \Delta_n^2}{K(k_i + l_i)}$$

to obtain

$$Rl\, f_{ii}^{B2(n)}(0,k_i^2) = \frac{4}{\pi}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2 P\int_0^\infty dK \int_{\frac{K^2+\Delta_n^2}{2K}}^\infty dl_i\, \frac{|I(n,i;\underline{K})|^2}{K^3(l_i^2 - k_i^2)} \, .$$

Changing the order of integration

$$Rl\, f_{ii}^{B2(n)}(0,k_i^2) = \frac{4}{\pi}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2 P\int_{\Delta_n}^\infty dl_i \int_{l_i - l_n}^{l_i + l_n} dK\, \frac{|I(n,i;\underline{K})|^2}{K^3(l_i^2 - k_i^2)}$$

$$= \frac{1}{4\pi^2} P\int_{\Delta_n^2}^\infty \frac{l_i Q_{in}^{B1}(l_i^2)}{l_i^2 - k_i^2}\, dl_i^2 \qquad (2.32)$$

where

$$l_n^2 = l_i^2 - \Delta_n^2 \, .$$

If $\ell_i^2 < \Delta_n^2$ then that state is inaccessible and hence $Q_{in}^{B1}(\ell_i^2) = 0$ for $\ell_i^2 < \Delta_n^2$.

Summing over all states of the atom gives

$$Re\left[f_{ii}^{B1}(0,k_i^2)+f_{ii}^{B2}(0,k_i^2)\right]=f_{ii}^{B1}(0,k_i^2)+\frac{1}{4\pi^2}P\int_0^\infty \frac{l_i\,Q^{B1}(l_i^2)}{l_i^2-k_i^2}\,dl_i^2 \qquad (2.33)$$

which is analagous to the dispersion relation (2.26).

Bransden and McDowell (1970) have used (2.25) and (2.26) to calculate the forward elastic scattering amplitude for electron-Helium atom scattering. The values of the total cross section they adopted are in close agreement with the first Born total cross section (derived from the accurate results of Bell, Kennedy and Kingston (1969) and Bell and Kingston (1969)) above 200eV. These authors used many-parameter correlated wavefunctions in their calculations. The principle contribution to the integral (2.26) comes from the region about the pole at k_i, and also above 200eV the exchange scattering amplitude is negligible. It follows that Bransden and McDowell's forward scattering amplitudes are in close agreement with the exact forward second Born scattering amplitude, $f_{ii}^{B1} + f_{ii}^{B2}$, above 200eV.

2.4 Simplification Approximations

To exactly evaluate the second Born correction to the scattering amplitude (2.24) it is necessary to sum over all states of the atom. In practice this is very difficult (c.f. Newstein (1955)) and some method of approximating to the infinite summation must be used. To date, at least six methods have been used in the literature.

Much of the earlier work on the second Born approximation involved truncating the infinite summation after including a few states. (c.f. Kingston, Moiseiwitsch and Skinner (1960 a,b), Kingston and Skinner (1961), Pomilla and Shapiro (1964), Taylor and Burke (1964), Moiseiwitsch and Perrin (1965), Hertel and Rost (1971))

For the forward elastic scattering of electrons by Hydrogen atoms, the summation over all states of the atom may be evaluated exactly.

The wave functions of Hydrogen are known and considerable simplification occurs because the initial and final momentum of the incident electron are equal and parallel. Holt (1972) has calculated the exact second Born scattering amplitude in this case. His results are presented in Table 1 together with the results of including only states up to and including those with principle quantum number three. It is clear that the truncation approximation inadequately represents the infinite sum. This is because the contribution from the continuum states to the infinite sum is large and this contribution is not accounted for by the truncation approximation.

Massey and Mohr (1934) replaced k_n in the denominator of (2.24) by a constant, k_{av}, independent of n and used the closure property of wavefunctions to evaluate the infinite sum. This type of approximation is called a simplifying approximation. Massey and Mohr chose k_{av} to be k_i, the initial momentum of the incident particle. If one writes

$$k_{av}^2 = k_i^2 - 2\Delta_{av} \tag{2.34}$$

then Δ_{av} is the energy, in atomic units, of some "average" state relative to the initial state of the atom. Massey and Mohr chose Δ_{av} to be zero. However, this choice gives rise to an infinite imaginary part for the forward elastic scattering amplitude.

Rothenstein (1954) used this approach to calculate total cross sections for the excitation of the 2p state of Hydrogen and Helium atoms by electron impact.

Moiseiwitsch and Williams (1959) chose k_{av} to be the momentum that the incident particle would have if the atom was in a state of principle quantum number two (i.e. $\Delta_{av} = \varepsilon_2$).

Moiseiwitsch (1963) showed that for elastic electron-Hydrogen atom collisions the Massey Mohr approximation is equivalent to taking an infinite value for the polarizability of the target atom. He further showed that if one set Δ_{av} to the energy of the level with principle quantum number three then the correct value for the diple polarizability of the target atom is obtained.

Holt and Moiseiwitsch (1968) suggest an amalgamation of the truncation and simplifying approximations, which will be called the H M approximation. (cf Holt and Moiseiwitsch (1969), Holt (1969 and 1972), Holt, Hunt and Moiseiwitsch (1971 a,b)). The contributions to the infinite sum from the lowest states are evaluated exactly and the simplifying approximation used to evaluate the contribution from higher states. If all states up to and including those of principle quantum number N are included explicitly then k_{av} is set to k_{N+1} in the simplifying approximation.

From Table 1 the H M approximation is a great improvement over the truncation approximation, being about 5% from the exact answer. For electron-Helium atom scattering it was shown in section 2.3 that the Bransden and McDowell dispersion relation real part of the forward elastic scattering amplitude was a close approximation to the exact second Born scattering amplitude. In Table 2 the dispersion relation values are compared with those given by the H M approximation. They differ by at least 30%.

Birman and Rosendorff (1968) used a method, similar to the H M approximation, to calculate elastic electron-Helium atom differential cross sections. The ground state was included explicitly and closure used to approximate to the contribution from the remaining states. However Δ_{av} was considered to be a free parameter chosen to fit the experimental data of Hughes et al. (1932). It was found that Δ_{av} varied with incident energy but was equivalent to a state well into the continuum of the Helium atom. (i.e. for 700eV electrons Δ_{av} = 3.5au).

In this thesis a variant on the H M approximation is proposed. The lower lying states are included explicitly and k_{av} chosen so as to reproduce the forward elastic scattering amplitudes of Bransden and McDowell. Consequently this approximation yields the exact second Born forward elastic scattering amplitude. Since k_{av} is determined from the total cross section, it is not inconsistent to assume it to be independent of the transition considered or the angle through which the particle is scattered.

It is found that Δ_{av} is a function of the energy of the incident particle and that for any given energy k_i^2, different values of Δ_{av} are required to fit $Re\, f(0,k_i^2)$ and $Im f(0,k_i^2)$. The variation of Δ_{av} with the number of intermediate states explicitly included has also been investigated. In Table 3 are presented the values of Δ_{av} required to fit the scattering amplitudes of Bransden and McDowell. With N = 1 only the $1\,^1S$ intermediate state is explicitly included, N = 2 the $1\,^1S$, $2\,^1S$, $2\,^1P$ and N = 3 the $1\,^1S$, $2\,^1S$, $3\,^1S$, $2\,^1P$ and $3\,^1P$ states. In figures 1 and 2 the variation of the real and imaginary parts of the forward elastic scattering amplitude with Δ_{av} at one energy is shown.

Bonham (1971) used a modified Massey-Mohr approximation. He chose k_{av} to be $(k_i^2 - \Delta)^{\frac{1}{2}} \cos\theta$ where θ is the angle through which the particle is scattered. He suggests various choices of Δ.

2.5 Padé Approximants and the Schwinger Variational Principle

The $[1,1]$ Padé approximant (cf section (1.5)) to the scattering amplitude is

$$f^{P1}(\theta,k_i^2) = \frac{\left[f^{B1}(\theta,k_i^2)\right]^2}{f^{B1}(\theta,k_i^2) - f^{B2}(\theta,k_i^2)} \quad . \tag{2.38}$$

As with potential scattering this form can also be derived from the Schwinger variational principle using trial wavefunctions of the form $e^{i\underline{k}_n \cdot \underline{R}}\ \varphi_n(\underline{r}_A)$.

This approximation to the scattering amplitude (to be denoted by P1) has been used by Garibotti and Massara (1971 a,b) to investigate elastic electron-Hydrogen and Helium atom collisions. In Garibotti and Massaro (1971 a), $f_{ii}^{B2}(\theta,k_i^2)$ is evaluated using the truncation approximation, only including the states 1^1S, 2^1S and 2^1P. In Garibotti and Massaro (1971 b) a variation of the H M approximation is used. The contributions from the 1^1S, 2^1S and 2^1P states are explicitly included (only the 1^1S state in the Helium atom case) and Δ_{av} is chosen so as to make $|f_{ii}^{SP1}(0,k_i^2)|^2$ stationary.

For electron - Hydrogen atom scattering Garibotti and Massaro give the values of Δ_{av} that make $|f_{ii}^{SP1}(0,k_i^2)|^2$ stationary. The variation with Δ_{av} of the real and imaginary parts of the second Born correction to the forward elastic scattering amplitude has been calculated and the results are displayed in Figure 3. The wavenumber of the incident electron is 4 and the 1^1S, 2^1S and 2^1P states are explicitly included. The exact values are taken from Holt (1972) and the value of Δ_{av} used by Garibotti and Massaro is indicated.

It is clear that their choice of Δ_{av} gives very poor values of Re $f_{ii}^{SB2}(0,k_i^2)$ and Im $f_{ii}^{SB2}(0,k_i^2)$ in this case.

For electron - Helium atom scattering Garibotti and Massaro do not give the values of Δ_{av} that make $|f_{ii}^{SP1}(0,k_i^2)|^2$ stationary. However, using a different Helium ground state wavefunction (3.6) to Garibotti and Massaro, we find that for the scattering of 500 eV electrons the stationary value of $|f_{ii}^{SP1}(0,k_i^2)|^2$ occurs when Δ_{av} is 9.6 atomic units. In this case only the 1^1S state has been explicitly included. The values of the real and imaginary parts of $f_{ii}^{SB2}(0,k_i^2)$ are 0.516 and 0.296 respectively, which may be compared with the exact values (Bransden and McDowell (1970)) of 0.498 and 0.7. The real part of this simplified second Born correction is in good agreement with the exact value, but the imaginary part is in very poor agreement.

2.6 Differential Cross Sections

We can write the Born series for the scattering amplitude as

$$f_{if}(\theta,k_i^2) = \lambda f_{if}^{B1}(\theta,k_i^2) + \lambda^2 f_{if}^{B2}(\theta,k_i^2) + \lambda^3 f_{if}^{B3}(\theta,k_i^2) + \cdots$$

where λ is a dimensionless parameter introduced to show the order to which the interaction potential occurs in each term. If we truncate the series after the second term then the differential cross section has the form

$$I_{if}(\theta,k_i^2) = \frac{k_f}{k_i}\left|\lambda f_{if}^{B1}(\theta,k_i^2) + \lambda^2 f_{if}^{B2}(\theta,k_i^2)\right|^2$$

$$= \frac{\lambda^2 k_f}{k_i}\left|f_{if}^{B1}(\theta,k_i^2)\right|^2 \left|1 + \lambda \mathrm{Re}\frac{f_{if}^{B2}(\theta,k_i^2)}{f_{if}^{B1}(\theta,k_i^2)} + i\lambda \mathrm{Im}\frac{f_{if}^{B2}(\theta,k_i^2)}{f_{if}^{B1}(\theta,k_i^2)}\right|^2,$$

since the first Born scattering amplitude is either purely real or imaginary. Definining

$$\alpha = \mathrm{Re}\frac{f_{if}^{B2}(\theta,k_i^2)}{f_{if}^{B1}(\theta,k_i^2)} \qquad \beta = \mathrm{Im}\frac{f_{if}^{B2}(\theta,k_i^2)}{f_{if}^{B1}(\theta,k_i^2)}$$

then

$$I_{if}^{FB2}(\theta,k_i^2) = \frac{\lambda^2 k_f}{k_i}\left|f_{if}^{B1}(\theta,k_i^2)\right|^2\left\{1 + 2\lambda\alpha + \lambda^2\alpha^2 + \lambda^2\beta^2\right\} \qquad (2.39)$$

and this will be called the full second Born approximation (denoted by FB2). However a further term of order λ^4 arises from the third Born correction term, and this may be of the same order of magnitude as the α^2 and β^2 terms. The third order approximation to the differential cross section (denoted by RB2) neglects all terms of order λ^4,

$$I_{if}^{RB2}(\theta, k_i^2) = \frac{\lambda^2 k_f}{k_i} \left| f_{if}^{B1}(\theta, k_i^2) \right|^2 \left\{ 1 + 2\lambda\alpha \right\}. \tag{2.40}$$

A prefix S denotes that a simplifying approximation is used to calculate the second Born scattering amplitude. i.e. SRB2 denotes that a simplifying approximation is used to calculate the second Born correction and (2.40) used to calculate the differential cross section, SP1 denotes that a simplifying approximation is used to calculate the second Born correction and the [1,1] Padé approximant (2.38) is used to calculate the scattering amplitude.

2.7 Other Theoretical Approximations.

There are many different approximations used in the theory of electron-atom collisions. A detailed discussion of the Born approximation has been given in the previous sections of this chapter. In this section three other approximations are briefly discussed.

The most suitable approximation to use for a particular collision depends, to a large extent, on the energy of the incident electron. When the incident electron only has sufficient energy to excite a few states of the atom then a close-coupling type of approximation is required. When the speed of the incident electron is high relative to the speed of the bound electrons of the atom, then the first Born approximation gives good results.

However there is an intermediate range of incident electron energies where both the first Born and the close-coupling approximations are expected to be invalid. The second Born approximation and the three approximations discussed below are all expected to be valid in both the intermediate and high incident electron energy ranges.

The first Born approximation to the scattering amplitude is (cf (2.19) and (2.2))

$$f_{if}^{B1}(\theta,\phi) = -\frac{\mu Z_B e^2}{2\pi\hbar^2}\left\langle e^{i\underline{k}_f\cdot\underline{R}}\,\varphi_f(\underline{r}_A)\right|\sum_{\substack{\text{bound}\\ \text{electrons}}}\frac{1}{|\underline{R}-\underline{r}_A|} - \frac{Z_A}{R}\left|e^{i\underline{k}_i\cdot\underline{R}}\,\varphi_i(\underline{r}_A)\right\rangle . \quad (2.41)$$

It should be noted that the interaction between the incident electron and the nucleus gives a vanishing contribution due to the orthogonality of the initial and final state wavefunctions. The scattering amplitude in the coulomb projected Born approximation is

$$f_{if}^{CPB1}(\theta,\phi) = -\frac{\mu Z_B e^2}{2\pi\hbar^2}\left\langle \varphi_{\underline{k}_f}(\underline{R})\,\varphi_f(\underline{r}_A)\right|\sum_{\substack{\text{bound}\\ \text{electrons}}}\frac{1}{|\underline{R}-\underline{r}_A|}\left|e^{i\underline{k}_i\cdot\underline{R}}\,\varphi_i(\underline{r}_A)\right\rangle \quad (2.42)$$

where $\phi_{\underline{k}_f}(\underline{R})$ is the coulomb wavefunction. (cf McDowell and Coleman (1970) p 239). The charge of the coulombic potential is chosen to be $-\frac{2e^2m}{\hbar^2 k_f}$ which represents the attractive field between the incident electron and the atomic nucleus. The electron-nucleus interaction is now contained in the final state wavefunction and no longer gives a vanishing contribution to the scattering amplitude. To date, this approximation has been used to investigate charge transfer collisions (Geltman (1971)), excitation of atomic Hydrogen by electrons (Geltman and Hidalgo (1971)) and the excitation of atomic Helium by electrons (Hidalgo and Geltman (1972)).

Bransden and Coleman (1972) suggested an approximation which is similar to the close-coupling approximation. The total wavefunction of the system may be expanded in the form

$$\Psi(\underline{R},\underline{r}_A) = \mathop{S}_{n} F_n(\underline{R})\, \varphi_n(\underline{r}_A) \tag{2.43}$$

where the sum over n includes an integration over the continuum states. Using this expansion, the Schrödinger equation for the total system (2.3) may be reduced to the infinite set of coupled equations

$$\left(\nabla^2 + k_n^2\right) F_n(\underline{R}) = 2 \mathop{S}_{m} V_{nm}(\underline{R})\, F_m(\underline{R}) \tag{2.44}$$

where the matrix potentials $V_{nm}(\underline{R})$ are defined by

$$V_{nm}(\underline{R}) = \left\langle \varphi_n(\underline{r}_A) \middle| V(\underline{R},\underline{r}_A) \middle| \varphi_m(\underline{r}_A) \right\rangle . \tag{2.45}$$

In many cases only a few states, say n = 0,1,2 ... N, will be of importance in the expansion (2.43). For these states equations (2.44) are retained whilst for all other states $n > N$ the equations (2.44) are replaced by

$$(\nabla^2 + k_n^2) F_n(\underline{R}) = 2 \sum_{m=0}^{N} V_{nm}(\underline{R}) F_m(\underline{R}) \qquad (n > N) \tag{2.46}$$

These equations have the solution

$$F_n(\underline{R}) = 2 \sum_{m=0}^{N} \int G_0^+(\underline{R},\underline{R}') V_{nm}(\underline{R}') F_m(\underline{R}') \, d\underline{R}' \qquad (n > N) \tag{2.47}$$

where G_o^+ $(\underline{R},\underline{R}')$ is the Green's function for the operator $(\nabla^2 + k_n^{\ 2})$.
(cf section 1.3). Substituting these values for $F_n(\underline{R})$, $n > N$, into (2.44) gives the $N + 1$ coupled equations

$$(\nabla^2 + k_n^2) F_n(\underline{R}) = 2 \sum_{m=0}^{N} V_{nm}(\underline{R}) F_m(\underline{R}) + 4 \sum_{m=0}^{N} \int K_{nm}(\underline{R},\underline{R}') F_m(\underline{R}') \, d\underline{R}' \qquad (n = 0, 1 \cdots N) \tag{2.48}$$

where

$$K_{nm}(\underline{R},\underline{R}') = \sum_{j=N+1}^{\infty} V_{nj}(\underline{R}) \, G_0^+(\underline{R},\underline{R}') \, V_{jm}(\underline{R}'). \tag{2.49}$$

The potentials $K_{nm}(\underline{R},\underline{R}')$ are very complicated and are evaluated by using a simplification approximation similar to those discussed in section 2.4. The average energy parameter, Δ_{av}, is chosen to give the correct long range interaction between the incident electron and the atom in its ground state. An impact parameter form of this approximation has been used to study electron and proton scattering by atomic Hydrogen (Bransden et al 1972) and by atomic Helium (Berrington et. al. 1972).

Lastly there is the Glauber approximation. This approximation is the simplest of the so called Eikonal approximations and is expected to be valid for small angle scattering at high energies. However, from the results it gives for electron - Hydrogen atom scattering, it would appear that its range of validity may be larger than expected. Since the derivation of the Glauber approximation is very complex for particle - atom collisions (cf Glauber (1959)), the expression for the scattering amplitude is quoted but not derived.

$$f_{if}^{G}(\theta,\phi) = \frac{ik_i}{2\pi}\int \varphi_f^*(\underline{r}_A)\,\Gamma(\underline{b},\underline{r}_A)\,\varphi_i(\underline{r}_A)\,e^{i\underline{k}.\underline{b}}\,d\underline{r}_A\,d^2b$$

where $\Gamma(\underline{b},\underline{r}_A) = 1 - \exp\left(\frac{-i}{\hbar v_i}\int_{-\infty}^{\infty} V(\underline{b},\underline{r}_A,z)\,dz\right)$

where V is the potential seen by the incoming particle, whose coordinates are ($\underline{b}$,z), $\underline{b}$ being the impact parameter.

The Glauber approximation is, unfortunately, very difficult to apply in practice, since if one is considering the scattering of a particle by an N-electron atom then it is necessary to evaluate a (3N + 2)-dimensional integral. It is only recently (Thomas and Gerjuoy (1971)) that the scattering amplitude for collisions of electrons and protons with atomic Hydrogen has been obtained in a closed form.

CHAPTER 3

REDUCTION OF THE SECOND BORN CORRECTION TO A SUM OF KNOWN INTEGRALS.

3.1 Introduction

To calculate the first Born scattering amplitude

$$f_{if}^{B1}(\theta, k_i^2) = -\frac{2\mu e^2 Z_B}{\hbar^2 K^2} I(f,i;\underline{K}), \tag{3.1}$$

where $\underline{K} = \underline{k}_i - \underline{k}_f$ and

$$I(f,i;\underline{K}) = \int \varphi_f^*(\underline{r}_A) \left[\sum_{\text{bound electrons}} e^{i\underline{K}.\underline{r}_A} - Z_A \right] \varphi_i(\underline{r}_A)\, d\underline{r}_A \tag{3.2}$$

the axis of quantisation of the atom is chosen parallel to $\underline{K}$. The effect of this choice is to considerably simplify the evaluation of (3.2) and also to make the integral (3.2) vanish unless the initial and final states of the atom have the same magnetic quantum number. For example, considering the transition from the ground state to the $(1s, 2p)^1P$ level of Helium, only the $(1s, 2p_0)^1P$ state gives a non-zero result.

However McDowell and Coleman (1970, p . 317 .) have shown that one can deduce the value of (3.2) with an arbitary axis of quantisation from the result with the axis of quantisation parallel to $\underline{K}$. One finds that (3.2) is a function of the angle between $\underline{K}$ and the axis of quantisation and does not necessarily vanish if the initial and final states have different magnetic quantum numbers. If the initial state has zero angular quantum number (i.e. is an s state) then the sum of $|I(f,i;\underline{K})|^2$ over all magnetic substates of the final state f is independent of the direction of the axis of quantisation. Consequently the first Born differential cross section is independent of the choice of the axis of quantisation of the atom and so we may choose it to be parrallel to $\underline{K}$.

When evaluating the second Born correction

$$f_{if}^{B2}(\theta,k_i^2) = \frac{2}{\pi^2}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2 \lim_{\epsilon\to 0} \sum_n \int d\underline{q}\, \frac{I(f,n;\underline{q}-\underline{k}_f)\, I(n,i;\underline{k}_i-\underline{q})}{(q^2-k_n^2-i\epsilon)(\underline{q}-\underline{k}_f)^2(\underline{q}-\underline{k}_i)^2} , \tag{3.3}$$

for consistency, the axis of quantisation is also chosen parallel to $\underline{K}$. This chapter is concerned with the reduction of (3.3) to a linear combination of known integrals of the form

$$\lim_{\epsilon\to 0} \int \frac{d\underline{q}}{(q^2-k^2-i\epsilon)[(\underline{q}-\underline{k}_i)^2+\lambda^2]^n[(\underline{q}-\underline{k}_f)^2+\mu^2]^m} \tag{3.4}$$

and

$$\lim_{\epsilon\to 0} \int \frac{(\underline{q}-\underline{k}_i)^2\, d\underline{q}}{(q^2-k^2-i\epsilon)(\underline{q}-\underline{k}_f)^2[(\underline{q}-\underline{k}_f)^2+\mu^2]^m} . \tag{3.5}$$

When the matrix element $I(n\ell,n'\ell';\underline{t})$ is expressed in terms of elementary functions it contains a multiplicative factor of the form i^λ $(\lambda = |\ell-\ell'|, |\ell-\ell'+2| \ldots . \ell+\ell')$, the remainder of the expression being purely real. Hence the expression $i^{\ell+\ell'}$ determines whether the matrix element $I(n\ell,n'\ell';\underline{t})$ is purely real ($\ell+\ell'$ even) or purely imaginary ($\ell+\ell'$ odd). Consider the product of two such matrix elements $I(n\ell,n''\ell'';\underline{t})\ I(n''\ell'',n'\ell';\underline{s})$. Whether this product is real or imaginary depends on the factor $(i^{\ell+\ell''} \times i^{\ell''+\ell'})$ which in turn depends on the factor $i^{\ell+\ell'}$. Consequently the two expressions $I(f,i;\underline{K})$ and $I(f,n;\underline{q}-\underline{k}_f)\ I(n,i;\underline{k}_i-\underline{q})$ are both either real or imaginary.

In section 2.6 it was shown that to calculate differential cross sections one needed to know the two ratios

$$\alpha = Re\,\frac{f_{if}^{B2}(\theta,k_i^2)}{f_{if}^{B1}(\theta,k_i^2)} \quad \text{and} \quad \beta = Im\,\frac{f_{if}^{B2}(\theta,k_i^2)}{f_{if}^{B1}(\theta,k_i^2)} .$$

It has been shown above that the ratio $\dfrac{I(f,n;\underline{q}-\underline{k}_f)\,I(n,i;\underline{k}_i-\underline{q})}{I(f,i;\underline{k})}$

is real and hence to calculate α the real part of the $\underline{q}$ integrals (3.4) and (3.5) is required whereas to calculate β the imaginary part is needed.

3.2 Wave Functions

For the singly excited s and p states of the Helium atom analytic Hartree-Fock wave functions are available in the literature. The ground state wave function used is that of Green et al. (1954)

$$\varphi_{1s^2}(\underline{r}_1,\underline{r}_2) = N\left(e^{-Zr_1}+c\,e^{-2Zr_1}\right)\left(e^{-Zr_2}+c\,e^{-2Zr_2}\right) \tag{3.6}$$

where $c = 0.6$ and $Z = 1.4558$. McEachran has given analytic Hartree-Fock functions for the $(1_s,n\ell)$ $n^1\ell$ $(\ell = 0,\ 1;\ n \leq 6)$ states of Helium (Cohen and McEachran (1969), Crothers and McEachran (1970)), in the form

$$\varphi_{1s,n\ell m}(\underline{r}_1,\underline{r}_2) = u(\underline{r}_1)\,v_{n\ell m}(\underline{r}_2) + u(\underline{r}_2)\,v_{n\ell m}(\underline{r}_1) \tag{3.7}$$

where

$$u(\underline{r}) = 2e^{-r}\,Y_{00}(\hat{\underline{r}}) \tag{3.8a}$$

$$v_{n\ell m}(\underline{r}) = \sum_{j=2\ell+1}^{N} a_j\, e^{-\alpha_n r}\, r^{\ell}\, L_j^{2\ell+1}(2\alpha_n r)\, Y_{\ell m}(\hat{\underline{r}}) \tag{3.8b}$$

and $\alpha_n = \frac{1}{n}$. The $L_j^{2\ell+1}(x)$ are associated Laguerre functions and the $Y_{\ell m}(\hat{\underline{r}})$ spherical harmonics. Scaled atomic units are used (the unit of length is Za_o, that of energy Z^2a.u.). Owing to the difficulties in evaluating the known integrals (3.4) it was found to be impracticable to include more than the first four terms of the sum in (3.8b).

The overlap integral $\langle (1s)^2 | (1s,2s) \rangle$ using (3.6) and (3.7) was -0.015, an unacceptably large value, since the 2^1S excitation cross section is of order 10^{-3} times the elastic scattering cross section. An improved 2s orbital was obtained by minimising

$$F = 100 \times \left| \langle (1s)^2 | (1s,2s) \rangle \right|^2 + \left\{ Q^{Bl\,(accurate)}_{1^1S-2^1S}(200eV) - Q^{Bl}_{1^1S-2^1S}(200eV) \right\}^2$$

where Q^{Bl}_{if} is the trial first Born total cross section for the transition from state i to state f and $Q^{Bl\,(accurate)}_{i-f}$ the accurate first Born value of Bell et al. (1969). The minimisation was carried out at 200 eV and the resulting "improved" wave function gave good first Born cross sections at all energies considered. The coefficients of the Cohen and McEachran function and the modified function are given in Table 4. The corresponding first Born total cross sections together with the accurate values of Bell et al. (1969) are given in Table 5. When calculating the elastic, 2^1S and 2^1P cross sections the eigen energies ε_n adopted are the Cohen and McEachran Hartree-Fock values.

For the singly excited d state and the model doubly excited states suitably symmetrised products of hydrogenic orbitals were chosen as the wave functions

$$\varphi_{ns,ml}(\underline{r}_1,\underline{r}_2) = \frac{1}{\sqrt{2}} \left\{ v_{ns}(\alpha|1)\, v_{ml}(\beta|2) + v_{ns}(\alpha|2)\, v_{ml}(\beta|1) \right\} \qquad (3.9)$$

The nuclear charges α and β and eigen energies ε_n of the states are shown in Table 6. The nuclear charges used for the model doubly excited states are those of Gillespie (1971). Since the electron in the n^1D orbital is "a long way" from the nucleus the 1^1S electron will "see" a nuclear charge $\alpha=2$ and the n^1D electron a nuclear charge β of approximately 1. The value of β is chosen to reproduce the Bell et al. (1969) accurate first Born total cross sections for the excitation of the n^1D states. When calculating cross sections for the excitation of

(1s,nd) and model doubly excited states the eigen energies adopted are the experimental values. (c.f. Cohen & McEachran, Crothers and McEachran and Table 6).

When computing the second Born correction for the excitation of the 3^1D and 4^1D states of Helium it was found that the contributions from the higher ^1P intermediate states were anomalous. That is the contribution from the $(1s,np)^1P$ intermediate state diverged as n increased. The calculations were repeated using wave functions for the 3, 4, 5 and 6^1P states of the form (3.9) with $\alpha = 2.0$, $\beta = 1.0$ and eigen energies equal to the experimental values. This value of β was found to give best agreement with the generalised oscillator strengths of Bell et al (1969). The results using the two 3^1P wave functions were consistent for both the $(1s)^{2\,1}S \rightarrow (1s,3d)^1D$ and $(1s,4d)^1D$ transitions but those using the 5^1P and 6^1P wave functions were inconsistent. For the $(1s)^{2\,1}S \rightarrow (1s,3d)^1D$ transition the two 4^1P wave functions gave inconsistent results but for the $(1s)^{2\,1}S \rightarrow (1s,4d)^1D$ transition the results were consistent.

This effect was found to be due to the truncation of the series (3.8b), for the Cohen and McEachran wave functions, after four terms. The size of the maxima of the electron densities of the excited orbitals were significantly different, in the case of the 4, 5 and 6^1P wave functions, from the hydrogenic wave function ones. The 3^1P maxima were in reasonable agreement. The maxima of the electron densities of the 3^1D and 4^1P wave functions occur at approximately the same radial distances and hence the 4^1P contribution using Cohen and McEachran wave functions is significantly affected by the truncation. However the maxima of the 4^1P and 4^1D wave functions do not occur at the same radial distances and hence the contributions from the two 4^1P state wave functions are consistent.

In the calculations reported here the following types of wave function were used. For excitation of the 3^1D state Cohen and McEachran wave functions were used for the 2 and 3^1P states and a product of hydrogenic orbitals for the 4, 5 and 6^1P states. For excitation of the 4^1D state the 2, 3 and 4^1P state wave functions were Cohen and McEachran's, the 5 and 6^1P state wave functions a product of hydrogenic orbitals.

All but the ground state wave function may be written in the form

$$\varphi_{n's,nlm}(\underline{r}_1,\underline{r}_2) = u_{n's}(\underline{r}_1)\,u_{nlm}(\underline{r}_2) + u_{n's}(\underline{r}_2)\,u_{nlm}(\underline{r}_1) \tag{3.10}$$

where

$$u_{nlm}(\underline{r}) = u_{nl}(r)\,Y_{lm}(\hat{\underline{r}}) \tag{3.11a}$$

and

$$u_{nl}(r) = \sum_{i=1}^{4} A_{nl,i}\, e^{-\alpha_{nl} r}\, r^{i+l-1} \,. \tag{3.11b}$$

The ground state wave function has a similar form except that

$$u_{1s}(r) = \sqrt{2N\pi}\,\left(e^{-Zr} + c\,e^{-2Zr}\right). \tag{3.12}$$

It should be noted that the radial parts of all the wave functions are purely real.

3.3 Reduction of Matrix Elements

$$I(ns,ml;n's,m'l';\underline{t}) = \int \varphi^*_{ns,ml}(\underline{r}_1,\underline{r}_2)\left[\sum_{i=1}^{2} e^{i\underline{t}.\underline{r}_i} - Z_A\right]\varphi_{n's,m'l'}(\underline{r}_1,\underline{r}_2)\; d\underline{r}_1\, d\underline{r}_2 \tag{3.13}$$

Since the wave functions used are symmetric in the bound electrons and for Helium atoms $Z_A = 2$, (3.13) reduces to

$$I(ns,ml;n's,m'l';\underline{t}) = \int\left[u^*_{ns}(\underline{r}_1)\,u^*_{ml}(\underline{r}_2) + u^*_{ns}(\underline{r}_2)\,u^*_{ml}(\underline{r}_1)\right]\left[2e^{i\underline{t}.\underline{r}_1} - 2\right] \times \left[u_{n's}(\underline{r}_1)\,u_{m'l'}(\underline{r}_2) + u_{n's}(\underline{r}_2)\,u_{m'l'}(\underline{r}_1)\right] d\underline{r}_1\, d\underline{r}_2 \,. \tag{3.14}$$

The integral E is defined as

$$E(nlm, n'l'm'; \underline{t}) = \int u^*_{nlm}(\underline{r})\, e^{i\underline{t}.\underline{r}}\, u_{n'l'm'}(\underline{r})\, d\underline{r} \tag{3.15a}$$

$$E(nlm, n'l'm'; 0) = \int u^*_{nlm}(\underline{r})\, u_{n'l'm'}(\underline{r})\, d\underline{r}\,. \tag{3.15b}$$

Then (3.13) reduces to

$$\begin{aligned} I(ns, ml; n's, m'l'; \underline{t}) = 2\{ & E(ml, m'l'; 0)[E(ns, n's; \underline{t}) - E(ns, n's; 0)] \\ & + E(ml, n's; 0)[E(ns, m'l'; \underline{t}) - E(ns, m'l'; 0)] + E(ns, m'l'; 0)[E(ml, n's; \underline{t}) \\ & - E(ml, n's; 0)] + E(ns, n's; 0)[E(ml, m'l'; \underline{t}) - E(ml, m'l'; 0)]\}\,. \end{aligned} \tag{3.16}$$

The evaluation of the E integrals will now be considered, firstly (3.15b). If the two orbitals have different angular quantum numbers then this integral vanishes due to the orthonormality of spherical harmonics. If they have the same angular quantum numbers then the integration over angles gives a contribution of unity.

$$\begin{aligned} E(nlm, n'lm; 0) &= \int_0^\infty u_{nl}(r)\, u_{n'l}(r)\, r^2\, dr \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} A_{nl,i} A_{n'l,j} \int_0^\infty r^{i+j+2l}\, e^{-(\alpha_{nl}+\alpha_{n'l})r}\, dr \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} A_{nl,i} A_{n'l,j} \frac{(i+j+2l)!}{[\alpha_{nl}+\alpha_{n'l}]^{i+j+2l+1}} \end{aligned} \tag{3.17}$$

$$E(1s, ns; 0) = \sqrt{2N\pi} \sum_{i=1}^{N} A_{ns,i}\, (i+1)! \left[\frac{1}{(\alpha_{ns}+Z)^{i+2}} + \frac{c}{(\alpha_{ns}+2Z)^{i+2}}\right] \tag{3.18}$$

$$E(1s, 1s; 0) = \frac{1}{2} \tag{3.19}$$

The integrals (3.15a) are more complex to evaluate since they contain the dependence on t and hence must have the form $\dfrac{1}{(t^2+\lambda^2)^n}$.

The axis of quantisation of the atoms has been chosen to be parallel to $\underline{K}$ ($=\underline{k}_i - \underline{k}_f$) and this direction is also used as the polar axis for the spherical co-ordinates of $\underline{r}$. The identity (c.f. Edmonds (1960))

$$e^{i\underline{t}.\underline{r}} = 4\pi \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} i^{\lambda}\, j_{\lambda}(tr)\, Y_{\lambda\mu}(\hat{\underline{r}})\, Y^*_{\lambda\mu}(\hat{\underline{t}}) \tag{3.20}$$

where $j_\lambda(x)$ are spherical Bessel functions of order λ and $Y_{\lambda\mu}$ spherical harmonics, is used. $\hat{\underline{r}}$ (or $\hat{\underline{t}}$) denotes the angles between $\underline{r}$ (or $\underline{t}$) and the polar axis, $\underline{K}$. An asterisk means that the complex conjugate should be taken. Hence

$$E(nlm, n'l'm'; \underline{t}) = \int u^*_{nlm}(\underline{r})\, e^{i\underline{t}.\underline{r}}\, u_{n'l'm'}(\underline{r})\, d\underline{r}$$

$$= 4\pi \sum_{\lambda=0}^{\infty} i^\lambda \left[\sum_{\mu=-\lambda}^{\lambda} Y^*_{\lambda\mu}(\hat{\underline{t}}) \int Y^*_{lm}(\hat{\underline{r}})\, Y_{\lambda\mu}(\hat{\underline{r}})\, Y_{l'm'}(\hat{\underline{r}})\, d\hat{\underline{r}}\right]$$

$$\times \int_0^\infty u_{nl}(r)\, j_\lambda(tr)\, u_{n'l'}(r)\, r^2 dr$$

$$= \sum_{\lambda=0}^{\infty} \mathcal{Y}_{\lambda, lm, l'm'}(\hat{\underline{t}})\; R_{\lambda, nl, n'l'}(t)$$

where

$$\mathcal{Y}_{\lambda, lm, l'm'}(\hat{\underline{t}}) = 4\pi i^\lambda \sum_{\mu=-\lambda}^{\lambda} Y^*_{\lambda\mu}(\hat{\underline{t}}) \int Y^*_{lm}(\hat{\underline{r}})\, Y_{\lambda\mu}(\hat{\underline{r}})\, Y_{l'm'}(\hat{\underline{r}})\, d\hat{\underline{r}}$$

and

$$R_{\lambda, nl, n'l'}(t) = \int_0^\infty u_{nl}(r)\, j_\lambda(tr)\, u_{n'l'}(r)\, r^2 dr$$

Using the two relations

$$Y^*_{lm}(\hat{\underline{r}}) = (-1)^m\, Y_{l\,-m}(\hat{\underline{r}})$$

and

$$\int Y_{l_1 m_1}(\hat{\underline{r}})\, Y_{l_2 m_2}(\hat{\underline{r}})\, Y_{l_3 m_3}(\hat{\underline{r}})\, d\hat{\underline{r}} = \left[\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}\right]^{1/2} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

where $\begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ are the 3-j symbols of Wigner (c.f. Edmonds (1960)),

one obtains

$$\mathcal{Y}_{\lambda, lm, l'm'}(\hat{\underline{t}}) = \left[4\pi(2l+1)(2\lambda+1)(2l'+1)\right]^{1/2} i^\lambda (-1)^m \begin{pmatrix} l & \lambda & l' \\ 0 & 0 & 0 \end{pmatrix} \sum_{\mu=-\lambda}^{\lambda} Y^*_{\lambda\mu}(\hat{\underline{t}}) \begin{pmatrix} l & \lambda & l' \\ -m & \mu & m' \end{pmatrix}.$$

The 3-j symbols are only non-zero if they satisfy the following three conditions:- $m_1 + m_2 + m_3 = 0$, $\ell_1\ \ell_2$ and ℓ_3 satisfy the triangle inequality and if $m_1 = m_2 = m_3 = 0$ then $\ell_1+\ell_2+\ell_3$ must be even. Consequently $\mu = m - m'$ and $\lambda = |\ \ell - \ell'\ |,\ |\ \ell - \ell'\ | + 2, \ldots\ \ell + \ell'$. Therefore

$$\mathcal{Y}_{\lambda, lm, l'm'}(\hat{\underline{t}}) = \left[4\pi(2l+1)(2\lambda+1)(2l'+1)\right]^{1/2} i^\lambda (-1)^m\, Y^*_{l, m-m'}(\hat{\underline{t}}) \begin{pmatrix} l & \lambda & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l & \lambda & l' \\ -m & m-m' & m' \end{pmatrix} \qquad (3.21)$$

and

$$E(nlm, n'l'm'; \underline{t}) = \sum_{\lambda} \mathcal{J}_{\lambda,lm,l'm'}(\hat{\underline{t}}) \; R_{\lambda,nl,n'l'}(t) \qquad \left(\lambda = |l-l'|, |l-l'|+2 \cdots l+l'\right) \tag{3.22}$$

A list of the $\mathcal{J}_{\lambda,lm,l'm'}(\hat{\underline{t}})$ used in this thesis is given in Appendix A.

$$\begin{aligned} R_{\lambda,nl,n'l'}(t) &= \int_0^\infty u_{nl}(r)\, j_\lambda(tr)\, u_{n'l'}(r)\, r^2\, dr \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} A_{nl,i}\, A_{n'l',j} \int_0^\infty r^{i+j+l+l'} e^{-(\alpha_{nl}+\alpha_{n'l'})r} j_\lambda(tr)\, dr \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} A_{nl,i}\, A_{n'l',j}\; I\left(i+j+l+l', \lambda, \alpha_{nl}+\alpha_{n'l'}, t\right). \end{aligned} \tag{3.23}$$

Similarly

$$R_{\lambda,1s,nl}(t) = \sqrt{2N\pi}\, \sum_{i=1}^{N} A_{nl,i} \left\{ I\left(i+l+1, \lambda, Z+\alpha_{nl}, t\right) + c\, I\left(i+l+1, \lambda, 2Z+\alpha_{nl}, t\right) \right\} \tag{3.24}$$

$$R_{\lambda,1s,1s}(t) = 2N\pi \left\{ I(2,\lambda,2Z,t) + 2c\, I(2,\lambda,3Z,t) + c^2 I(2,\lambda,4Z,t) \right\} \tag{3.25}$$

where in (3.23) → (3.25) the integral I is

$$I(i,\lambda,\alpha,t) = \int_0^\infty r^i e^{-\alpha r} j_\lambda(tr)\, dr \tag{3.26}$$

$$= (-1)^{i-\lambda-1}\, 2^\lambda\, \lambda!\, t^\lambda \left(\frac{\partial}{\partial\alpha}\right)^{i-\lambda-1} \left(t^2+\alpha^2\right)^{-\lambda-1}. \tag{3.27}$$

By inserting the sine and cosine expansion of $j_\lambda(tr)$ in the integral (3.26) it can be shown that (3.27) holds for $\lambda = 0$ and 1, for all positive integers i. Assuming (3.27) true for $\lambda = n-1$ and n and using the recurrence relation $j_{n-1}(x) + j_{n+1}(x) = \frac{2n+1}{x} j_n(x)$ it also follows that (3.27) is true for $\lambda = n+1$. Consequently by induction (3.27) is true for all positive integers, λ.

Now consider the expression $\left(\frac{\partial}{\partial\alpha}\right)^i (t^2+\alpha^2)^{-j}$. If it is written in the form $\sum_{k=1}^{[\frac{i+2}{2}]} \frac{(j+i-k)!}{(j-1)!} (-2)^{i+1-k} \alpha^{i+2-2k}\, {}^{i}F_k \left(t^2+\alpha^2\right)^{-j-i-1+k}$ where $[\frac{i+2}{2}]$ is the largest integer less than or equal to $\frac{i+2}{2}$, then the following recursion relations are obtained for ${}^{i}F_k$.

$$\begin{aligned} &{}^{0}F_1 = 1 \qquad {}^{i+1}F_1 = {}^{i}F_1 \qquad \text{If } i \text{ is odd } {}^{i+1}F_{[\frac{i+3}{2}]} = {}^{i}F_{[\frac{i+1}{2}]} \\ &{}^{i+1}F_k = {}^{i}F_k + (i+4-2k)\, {}^{i}F_{k-1} \quad \text{for } 2 \leq k \leq [\tfrac{i+2}{2}]. \end{aligned} \tag{3.28}$$

Defining $$F(i,j,k,\alpha) = \frac{(j+i-k)!\,(-2)^{i+1-k}\,\alpha^{i+2-2k}\,{}^{i}F_k}{(j-1)!} \tag{3.29}$$

then
$$\left(\frac{\partial}{\partial\alpha}\right)^{i}\left(t^2+\alpha^2\right)^{-j} = \sum_{k=1}^{\left[\frac{i+2}{2}\right]} \frac{F(i,j,k,\alpha)}{(t^2+\alpha^2)^{j+i+1-k}} \tag{3.30}$$

and (3.26) becomes

$$I(i,\lambda,\alpha,t) = (-1)^{i-\lambda-1}\,2^{\lambda}\,\lambda!\,t^{\lambda} \sum_{k=1}^{\left[\frac{i-\lambda+1}{2}\right]} \frac{F(i-\lambda-1,\lambda+1,k,\alpha)}{(t^2+\alpha^2)^{i+1-k}} \tag{3.31}$$

Consequently $R_{\lambda}(t)_{n\ell,\, n'\ell'}$ can be written as a sum of terms of the form $\dfrac{t^{\lambda}}{(t^2+\alpha^2)^r}$.

Referring to (3.16) it is clear that associated with every t dependent term there is a t independent term i.e.

$$\int u^{*}_{n\ell m}(\underline{r})\left[e^{i\underline{t}.\underline{r}}-1\right]u_{n'\ell'm'}(\underline{r})\,d\underline{r} = E(n\ell m, n'\ell'm'; \underline{t}) - E(n\ell m, n'\ell'm'; 0). \tag{3.32}$$

In the limit as $t \to 0$ the left hand side of (3.32) vanishes for any continuous functions $u_{n\ell m}$ and $u_{n'\ell'm'}$ and consequently so must the right hand side. Using (3.22) and (3.31)

$$E(n\ell m, n'\ell'm'; \underline{t}) = \sum_{\lambda=|\ell-\ell'|}^{\ell+\ell'} \mathcal{J}_{\lambda,\ell m,\ell'm'}(\hat{\underline{t}}) \sum_{k=1}^{M} \frac{t^{\lambda}\,Y(\lambda, n\ell, n'\ell', k)}{(t^2+\alpha_{\lambda}^2)^k} \tag{3.33}$$

where $Y(\lambda, n\ell, n'\ell', k)$ is independent of t.

If $\ell \neq \ell'$ or $m \neq m'$ then $E(n\ell m, n'\ell'm'; 0)$ vanishes owing to the orthorgnality of the angular parts of the orbitals. From Appendix A

$\mathcal{J}_{0,\ell m,\ell'm'} = \delta^{\ell}_{\ell'}\,\delta^{m}_{m'}$ and so there is no $\lambda = 0$ term in (3.33).

Consequently (3.32) may be written in the form

$$\sum_{j=1}^{N} \Upsilon_{\lambda_j, \ell m, \ell' m'}(\hat{t}) \sum_{k=1}^{M} \frac{t^{N_j} Y(j, n\ell, n'\ell', k)}{(t^2+\alpha_j^2)^k} \text{ where } N_j \neq 0 \text{ for } j = 1 \ldots N. \tag{3.32a}$$

If $\ell=\ell'$ and $m=m'$ then $E(n\ell m, n'\ell' m'; 0)$ need not vanish. In this case λ can take the values 0, 2, 4 ... 2ℓ and

$$\lim_{t\to 0} E(n\ell m, n'\ell m; \underline{t}) = \sum_{k=1}^{M} Y(0, n\ell, n'\ell, k)\alpha_0^{-2k} = E(n\ell m, n'\ell m; 0) \tag{3.34}$$

using (3.33) and the fact that (3.32) $\to 0$ as $t \to 0$.

A transformation of the coefficients $Y(0, n\ell, n'\ell', k)$ is now found so that (3.32) can be written in the same form as (3.32a).

$$(t^2+\alpha^2)^k = t^2(t^2+\alpha^2)^{k-1} + \alpha^2(t^2+\alpha^2)^{k-1}$$

$$= t^2 \sum_{i=1}^{k} \alpha^{2(i-1)} (t^2+\alpha^2)^{k-i} + \alpha^{2k}.$$

Dividing by $\alpha^{2k}(t^2+\alpha^2)^k$ gives

$$(t^2+\alpha^2)^{-k} = \alpha^{-2k} - t^2 \sum_{i=1}^{k} \frac{\alpha^{-2(k+1-i)}}{(t^2+\alpha^2)^i}. \tag{3.35}$$

$$\therefore \Upsilon_{0,\ell m,\ell m}(\hat{t}) \sum_{k=1}^{M} \frac{Y(0,n\ell,n'\ell,k)}{(t^2+\alpha_0^2)^k} - E(n\ell m, n'\ell m; 0) = \Upsilon_{0,\ell m,\ell m}(\hat{t}) \sum_{k=1}^{M} Y(0,n\ell,n'\ell,k)\alpha_0^{-2k}$$

$$- E(n\ell m, n'\ell m; 0) - \Upsilon_{0,\ell m,\ell m}(\hat{t}) \sum_{k=1}^{M} Y(0,n\ell,n'\ell,k) t^2 \sum_{i=1}^{k} \frac{\alpha_0^{-2(k+1-i)}}{(t^2+\alpha_0^2)^i}$$

$$= -t^2\, \Upsilon_{0,\ell m,\ell m}(\hat{t}) \sum_{i=1}^{M} (t^2+\alpha_0^2)^{-i} \sum_{k=i}^{M} Y(0,n\ell,n'\ell,k)\alpha_0^{-2(k+1-i)}$$

$$= t^2\, \Upsilon_{0,\ell m,\ell m}(\hat{t}) \sum_{i=1}^{M} \frac{Y'(0,n\ell,n'\ell,i)}{(t^2+\alpha_0^2)^i} \tag{3.36}$$

where

$$Y'(0,n\ell,n'\ell,i) = -\sum_{k=i}^{M} Y(0,n\ell,n'\ell,k)\,\alpha_0^{-2(k+1-i)}.$$

$$\sum_{j=1}^{N} \Upsilon_{\lambda_j, lm, l'm'}(\hat{t}) \sum_{k=1}^{M} \frac{t^{N_j} Y(j, nl, n'l', k)}{(t^2+\alpha_j^2)^k} \text{ where } N_j \neq 0 \text{ for } j = 1 \ldots N. \tag{3.32a}$$

If $\ell=\ell'$ and $m=m'$ then $E(n\ell m, n'\ell'm';0)$ need not vanish. In this case λ can take the values 0, 2, 4 ... 2ℓ and

$$\lim_{t\to 0} E(nlm, n'lm; \underline{t}) = \sum_{k=1}^{M} Y(0, nl, n'l, k)\,\alpha_0^{-2k} = E(nlm, n'lm; 0) \tag{3.34}$$

using (3.33) and the fact that (3.32) $\to 0$ as $t \to 0$.

A transformation of the coefficients $Y(0,n\ell, n'\ell', k)$ is now found so that (3.32) can be written in the same form as (3.32a).

$$(t^2+\alpha^2)^k = t^2(t^2+\alpha^2)^{k-1} + \alpha^2(t^2+\alpha^2)^{k-1}$$

$$= t^2 \sum_{i=1}^{k} \alpha^{2(i-1)} (t^2+\alpha^2)^{k-i} + \alpha^{2k}.$$

Dividing by $\alpha^{2k}(t^2+\alpha^2)^k$ gives

$$(t^2+\alpha^2)^{-k} = \alpha^{-2k} - t^2 \sum_{i=1}^{k} \frac{\alpha^{-2(k+1-i)}}{(t^2+\alpha^2)^i}. \tag{3.35}$$

$$\therefore \Upsilon_{0,lm,lm}(\hat{t}) \sum_{k=1}^{M} \frac{Y(0,nl,n'l,k)}{(t^2+\alpha_0^2)^k} - E(nlm,n'lm;0) = \Upsilon_{0,lm,lm}(\hat{t}) \sum_{k=1}^{M} Y(0,nl,n'l,k)\,\alpha_0^{-2k}$$

$$- E(nlm,n'lm;0) - \Upsilon_{0,lm,lm}(\hat{t}) \sum_{k=1}^{M} Y(0,nl,n'l,k)\, t^2 \sum_{i=1}^{k} \frac{\alpha_0^{-2(k+1-i)}}{(t^2+\alpha_0^2)^i}$$

$$= -t^2\, \Upsilon_{0,lm,lm}(\hat{t}) \sum_{i=1}^{M} (t^2+\alpha_0^2)^{-i} \sum_{k=i}^{M} Y(0,nl,n'l,k)\,\alpha_0^{-2(k+1-i)}$$

$$= t^2\, \Upsilon_{0,lm,lm}(\hat{t}) \sum_{i=1}^{M} \frac{Y'(0,nl,n'l,i)}{(t^2+\alpha_0^2)^i} \tag{3.36}$$

where

$$Y'(0,nl,n'l,i) = -\sum_{k=i}^{M} Y(0,nl,n'l,k)\,\alpha_0^{-2(k+1-i)}.$$

Summarising it has been shown that

$$E(nlm, n'l'm'; \underline{t}) - E(nlm, n'l'm'; 0) = \sum_{j=1}^{N} \mathcal{I}_{\lambda_j, lm, l'm'}(\hat{\underline{t}}) \sum_{k=1}^{M} \frac{t^{N_j} Y(j, nl, n'l', k)}{(t^2 + \alpha_j^2)^k} \qquad (3.37)$$

where $N_j \neq 0$ for $j = 1 \ldots N$ and $\lambda_j = |\ell-\ell'|, \; |\ell-\ell'| + 2 \ldots \ell+\ell'$.
In Appendix B the form of the integrals $I(f,i;\underline{t})$ are given.

3.4 Angular dependence of the product of two matrix elements.

In section 3.3 the matrix element $I(f,i;\underline{t})$ was reduced to the product of a function of t and a function of the angles between $\underline{t}$ and the axis of quantisation of the atom $\underline{K}(=\underline{k}_i-\underline{k}_f)$. In this section the product of two such matrix elements is considered and its reduction to a form suitable for computation described.

The vectors $\underline{k}_i-\underline{q}$ and $\underline{q}-\underline{k}_f$ are expressed in spherical coordinates with polar axis parallel to $\underline{K}$ and some arbitrary fixed plane containing $\underline{K}$ having zero azimuthal angle. Then $\underline{q}-\underline{k}_f$ has coordinates $(|\underline{q}-\underline{k}_f|, \alpha, \phi)$ and $\underline{k}_i-\underline{q}$ coordinates $(|\underline{k}_i-\underline{q}|, \beta, \phi+\pi)$ where α and β are the angles shown on Figure 4. The cosine rule gives the following expressions for α and β :-

$$\cos\alpha = \frac{(\underline{q}-\underline{k}_f)^2 + K^2 - (\underline{q}-\underline{k}_i)^2}{2K|\underline{q}-\underline{k}_f|} \qquad \cos\beta = \frac{(\underline{q}-\underline{k}_i)^2 + K^2 - (\underline{q}-\underline{k}_f)^2}{2K|\underline{q}-\underline{k}_i|} \qquad (3.38)$$

$$\cos(\alpha+\beta) = \frac{K^2 - (\underline{q}-\underline{k}_i)^2 - (\underline{q}-\underline{k}_f)^2}{2|\underline{q}-\underline{k}_i||\underline{q}-\underline{k}_f|} .$$

There are six products of two matrix elements to consider. It will be assumed that the initial state is always the ground state of the atom, $(1s)^2\,{}^1S$, although the angular dependences hold for other initial s states. Since all intermediate states are summed over, if the state has a degeneracy in its angular part (i.e. $2\,{}^1P_{0,+1}$) then these degeneracies will be summed over. Using B 1 (see Appendix B)

$$I(n's, m's; ns, ms; \underset{\sim}{q} - \underline{k}_f)\, I(ns, ms; ls, ls; \underline{k}_i - \underset{\sim}{q})$$
$$= (\underset{\sim}{q} - \underline{k}_i)^2 (\underset{\sim}{q} - \underline{k}_f)^2 \sum_{jkj'k'} \frac{Y(j; n's, m's; ns, ms; k)\, Y(j'; ns, ms; ls, ls; k')}{[(\underset{\sim}{q} - \underline{k}_f)^2 + \alpha_j^2]^k [(\underset{\sim}{q} - \underline{k}_i)^2 + \alpha_{j'}^2]^{k'}} \quad (3.39)$$

Using B2 and B3

$$\sum_{\mu=-1}^{1} I(n's, m's; ns, mp_\mu; \underset{\sim}{q} - \underline{k}_f)\, I(ns, mp_\mu; ls, ls; \underline{k}_i - \underset{\sim}{q})$$
$$= -3\cos(\alpha+\beta)\, |\underset{\sim}{q} - \underline{k}_f|\, |\underset{\sim}{q} - \underline{k}_i| \sum_{jkj'k'} \frac{Y(j; n's, m's; ns, mp; k)\, Y(j'; ns, mp; ls, ls; k')}{[(\underset{\sim}{q} - \underline{k}_f)^2 + \alpha_j^2]^k [(\underset{\sim}{q} - \underline{k}_i)^2 + \alpha_{j'}^2]^{k'}} \quad (3.40)$$

Using B1 and B3

$$I(n's, m'p; ns, ms; \underset{\sim}{q} - \underline{k}_f)\, I(ns, ms; ls, ls; \underline{k}_i - \underset{\sim}{q})$$
$$= \sqrt{3}\, i \cos\alpha\, |\underset{\sim}{q} - \underline{k}_f|\, (\underset{\sim}{q} - \underline{k}_i)^2 \sum_{jkj'k'} \frac{Y(j; n's, m'p; ns, ms; k)\, Y(j'; ns, ms; ls, ls; k')}{[(\underset{\sim}{q} - \underline{k}_f)^2 + \alpha_j^2]^k [(\underset{\sim}{q} - \underline{k}_i)^2 + \alpha_{j'}^2]^{k'}} \quad (3.41)$$

Using B3 and B5

$$\sum_{\mu=-1}^{1} I(n's, m'p; ns, mp_\mu; \underset{\sim}{q} - \underline{k}_f)\, I(ns, mp_\mu; ls, ls; \underline{k}_i - \underset{\sim}{q})$$
$$= \sqrt{3}\, i [\cos\beta - 3\cos\alpha\cos(\alpha+\beta)\, \delta^j_3]\, (\underset{\sim}{q} - \underline{k}_f)^2\, |\underset{\sim}{q} - \underline{k}_i| \quad (3.42)$$
$$\times \sum_{j=1,2,3,kj'k'} \frac{Y(j; n's, m'p; ns, mp; k)\, Y(j'; ns, mp; ls, ls; k')}{[(\underset{\sim}{q} - \underline{k}_f)^2 + \alpha_j^2]^k [(\underset{\sim}{q} - \underline{k}_i)^2 + \alpha_{j'}^2]^{k'}}$$

Using B1 and B4

$$I(n's, m'd; ns, ms; \underset{\sim}{q} - \underline{k}_f)\, I(ns, ms; ls, ls; \underline{k}_i - \underset{\sim}{q})$$
$$= -\frac{\sqrt{5}}{2} (3\cos^2\alpha - 1) (\underset{\sim}{q} - \underline{k}_f)^2 (\underset{\sim}{q} - \underline{k}_i)^2 \sum_{jkj'k'} \frac{Y(j; n's, m'd; ns, ms; k)\, Y(j'; ns, ms; ls, ls; k')}{[(\underset{\sim}{q} - \underline{k}_f)^2 + \alpha_j^2]^k [(\underset{\sim}{q} - \underline{k}_i)^2 + \alpha_{j'}^2]^{k'}} \quad (3.43)$$

Using B3 and B6

$$\sum_{\mu=-1}^{1} I(n's, m'd; ns, mp_\mu; \underset{\sim}{q} - \underline{k}_f)\, I(ns, mp_\mu; ls, ls; \underline{k}_i - \underset{\sim}{q})$$
$$= \frac{3}{\sqrt{5}} \{\cos(\alpha+\beta) - 3\cos\alpha\cos\beta\}\, |\underset{\sim}{q} - \underline{k}_f|\, |\underset{\sim}{q} - \underline{k}_i| \sum_{kj'k'} \frac{Y(1; n's, m'd; ns, mp; k)\, Y(j'; ns, mp; ls, ls; k')}{[(\underset{\sim}{q} - \underline{k}_f)^2 + \alpha_1^2]^k [(\underset{\sim}{q} - \underline{k}_i)^2 + \alpha_{j'}^2]^{k'}} \quad (3.44)$$

$$+\frac{q}{2\sqrt{5}}\left\{\cos(\alpha+\beta)\left[5\cos^2\alpha-1\right]-2\cos\alpha\cos\beta\right\}|\underline{q}-\underline{k}_f|^3|\underline{q}-\underline{k}_i|$$
$$\times\sum_{k_j'k'}\frac{Y(2;n's,n'd;ns,mp;k)\,Y(j';ns,mp;ls,ls;k')}{[(\underline{q}-\underline{k}_f)^2+\alpha_2^2]^k\,[(\underline{q}-\underline{k}_i)^2+\alpha_{j'}^2]^{k'}}\ .$$

Subsisting for $\cos\alpha$, $\cos\beta$ and $\cos(\alpha+\beta)$ from (3.38) into (3.39) → (3.44) and using partial fractions one can obtain (3.3) in the form of a linear combination of the integrals (3.4) and (3.5).

3.5 The Simplifying Approximation

The second Born correction to the scattering amplitude is (c.f. 2.24)

$$f_{if}^{B2}(\theta,\phi)=\frac{2}{\pi^2}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2\lim_{\varepsilon\to 0}\ \mathop{S}_{n}\int d\underline{q}\,\frac{I(f,n;\underline{q}-\underline{k}_f)\,I(n,i;\underline{k}_i-\underline{q})}{(q^2-k_n^2-i\varepsilon)(\underline{q}-\underline{k}_f)^2(\underline{q}-\underline{k}_i)^2}$$
$$=\frac{2}{\pi^2}\left(\frac{\mu e^2 Z_B}{\hbar^2}\right)^2\mathop{S}_{n} I_n(k_n)\ .$$

The simplifying approximation is the evaluation of the first N states of this summation exactly and approximating the remainder by replacing k_n by a constant k_{av} . Hence

$$\mathop{S}_{n} I_n(k_n)\simeq\sum_{n=1}^{N} I_n(k_n)+\sum_{n=N+1}^{\infty} I_n(k_{av})$$
$$\simeq\ \mathop{S}_{n} I_n(k_{av})+\sum_{n=1}^{N}\left\{I_n(k_n)-I_n(k_{av})\right\}. \qquad (3.45)$$

Here,

$$I_n(k_n)-I_n(k_{av})=\lim_{\varepsilon\to 0}\int d\underline{q}\left\{\frac{1}{q^2-k_n^2-i\varepsilon}-\frac{1}{q^2-k_{av}^2-i\varepsilon}\right\}\frac{I(f,n;\underline{q}-\underline{k}_f)\,I(n,i;\underline{k}_i-\underline{q})}{(\underline{q}-\underline{k}_f)^2(\underline{q}-\underline{k}_i)^2}$$

which can be expressed, using (3.39) → (3.44), as a sum of the known integrals

$$\lim_{\varepsilon\to 0}\int d\underline{q}\left\{\frac{1}{q^2-k_n^2-i\varepsilon}-\frac{1}{q^2-k_{av}^2-i\varepsilon}\right\}\frac{1}{[(\underline{q}-\underline{k}_i)^2+\lambda^2]^n\,[(\underline{q}-\underline{k}_f)^2+\mu^2]^m}\ .$$

The first term of (3.45 is

$$\mathop{S}_{n} I_n(k_{av})=\lim_{\varepsilon\to 0}\int d\underline{q}\,\frac{X}{(q^2-k_{av}^2-i\varepsilon)(\underline{q}-\underline{k}_i)^2(\underline{q}-\underline{k}_f)^2}$$

where

$$X=\mathop{S}_{n}\langle f|2e^{i(\underline{q}-\underline{k}_f).\underline{r}_1}-2|n\rangle\langle n|2e^{i(\underline{k}_i-\underline{q}).\underline{r}_1}-2|i\rangle\ .$$

Using the closure property of wave functions

$$X=\langle f|\{2e^{i(\underline{q}-\underline{k}_f).\underline{r}_1}-2\}\{2e^{i(\underline{k}_i-\underline{q}).\underline{r}_1}-2\}|i\rangle$$
$$=2\left\{\langle f|2e^{i\underline{K}.\underline{r}_1}-2|i\rangle-\langle f|2e^{i(\underline{q}-\underline{k}_f).\underline{r}_1}-2|i\rangle-\langle f|2e^{i(\underline{k}_i-\underline{q}).\underline{r}_1}-2|i\rangle\right\}.$$

Therefore $\underset{n}{S}\, I_n(k_{av}) = \langle f|4e^{i\underline{K}.\underline{r}_1}-4|i\rangle \lim_{\epsilon\to 0}\int \frac{dq}{(q^2-k_{av}^2-i\epsilon)(q-\underline{k}_i)^2(q-\underline{k}_f)^2}$

$$-2\int dq\, \frac{I(f,i;q-\underline{k}_f)+I(f,i;\underline{k}_i-q)}{(q^2-k_{av}^2-i\epsilon)(q-\underline{k}_i)^2(q-\underline{k}_f)^2} \qquad (3.46)$$

The axis of quantisation of the atom is parallel to $\underline{K}$ so the first term of (3.46) is easily evaluated knowing the value of

$$\lim_{\epsilon\to 0}\int \frac{1}{(q^2-k_{av}^2-i\epsilon)(q-\underline{k}_i)^2(q-\underline{k}_f)^2}\,dq \,.$$

The second term is evaluated using the results of Appendix B and the integrals of Chapter 4. There are three cases to consider depending on the angular quantum numbers of the final state f. It is assumed that the initial state is the ground state, but the results hold for any s state.

For a (ns, ms) final state using B 1 the second term of (3.46) is

$$-2\sum_{jk}\lim_{\epsilon\to 0}\int dq\, \frac{Y(j;ns,ms;1s,1s;k)}{(q^2-k_{av}^2-i\epsilon)}\left\{\frac{1}{(q-\underline{k}_i)^2[(q-\underline{k}_f)^2+\alpha_j^2]^k}+\frac{1}{(q-\underline{k}_f)^2[(q-\underline{k}_i)^2+\alpha_j^2]^k}\right\} \qquad (3.47)$$

For a (ns, mp) final state using B 2 the second term of (3.46) is

$$-2\sum_{jk}\lim_{\epsilon\to 0}\int dq\, \frac{Y(j;ns,mp;1s,1s;k)}{(q^2-k_{av}^2-i\epsilon)(q-\underline{k}_i)^2(q-\underline{k}_f)^2}\left\{\frac{\cos\alpha\,|q-\underline{k}_f|}{[(q-\underline{k}_f)^2+\alpha_j^2]^k}+\frac{\cos\beta\,|q-\underline{k}_i|}{[(q-\underline{k}_i)^2+\alpha_j^2]^k}\right\} \qquad (3.48)$$

For a (ns,md) final state using B 4 the second term of (3.46) is

$$-2\sum_{jk}\lim_{\epsilon\to 0}\int dq\, \frac{Y(j;ns,md;1s,1s;k)}{(q^2-k_{av}^2-i\epsilon)}\left\{\frac{3\cos^2\alpha-1}{(q-\underline{k}_i)^2[(q-\underline{k}_f)^2+\alpha_j^2]^k}+\frac{3\cos^2\beta-1}{(q-\underline{k}_f)^2[(q-\underline{k}_i)^2+\alpha_j^2]^k}\right\} \qquad (3.49)$$

Substituting for cos α, cos β from (3.38) into (3.47) → (3.49) and using partial fractions one obtains integrals of the form

$$\lim_{\epsilon\to 0}\int \frac{1}{(q^2-k_{av}^2-i\epsilon)(q-\underline{k}_i)^2[(q-\underline{k}_f)^2+\mu^2]^m}\,dq$$

where if $\mu = 0$ then $m = 1$, and also only from (3.49) integrals of the form

$$\lim_{\epsilon\to 0}\int \frac{(q-\underline{k}_i)^2}{(q^2-k_{av}^2-i\epsilon)(q-\underline{k}_f)^2[(q-\underline{k}_f)^2+\mu^2]^m}\,dq$$

both of which are discussed in Chapter 4.

3.6 Further Investigation of the $\sum_{\mu=-1}^{1}$ $(1s)^2$ - (ns,mp_μ) - (ns,mp) transition

If (3.42) is used to calculate the real part of the second Born correction for the transition $(1s)^2$ - (ns,mp) due to the intermediate states $(ns,mp_{0,\pm1})$, then an infinite answer is obtained. This anomaly is investigated and is shown to be due to the method of evaluating the correction. In fact the correction is finite. The infinity is due to the second term of (3.42), namely

$$X=\sqrt{3}\,i\left[\cos\beta-3\cos\alpha\cos(\alpha+\beta)\right]|\underline{q}-\underline{k}_i|(\underline{q}-\underline{k}_f)^2\sum_{k_j'k'}\frac{Y(3;ns,mp;ns,mp;k)\,Y(j';ns,mp;1s,1s;k')}{\left[(\underline{q}-\underline{k}_f)^2+\alpha_3^2\right]^k\left[(\underline{q}-\underline{k}_i)^2+\alpha_{j'}^2\right]^{k'}} \qquad (3.49)$$

Using (3.38) to replace cos α , cos β and cos (α+β) and writing $t_i = |\underline{q} - \underline{k}_i|$, $t_f = |\underline{q} - \underline{k}_f|$ this expression may be written as

$$\frac{X}{t_i^2t_f^2}=\frac{\sqrt{3}\,i}{4K}\,\frac{\left[t_f^4+2t_i^2t_f^2+2K^2t_f^2-3t_i^4+6K^2t_i^2-3K^4\right]}{t_i^2t_f^2}\sum_{k_j'k'}\frac{Y(3;ns,mp;ns,mp;k)\,Y(j';ns,mp;1s,1s;k')}{\left[t_f^2+\alpha_3^2\right]^k\left[t_i^2+\alpha_{j'}^2\right]^{k'}}\,.$$

All factors of the form $\dfrac{t_i^4 - 2K^2t_i^2 + K^4}{t_i^2t_f^2\left[t_i^2+\alpha_{j'}^2\right]^{k'}\left[t_f^2+\alpha_3^2\right]^k}$ then give an infinite answer whilst all the other factors give finite results. To see why this infinite answer occurs the result (3.38) is employed in the form

$$\frac{1}{t^2(t^2+\alpha^2)^k}=\frac{\alpha^{-2k}}{t^2}-\sum_{l=1}^{k}\frac{\alpha^{-2(k+1-l)}}{(t^2+\alpha^2)^l}\,.$$

The infinite factor becomes a sum of terms of the form $\dfrac{t_i^4-2K^2t_i^2+K^4}{t_i^2(t_i^2+\alpha_{j'}^2)^{k'}(t_f^2+\alpha_3^2)^k}$

and $\dfrac{t_i^4-2K^2t_i^2+K^4}{t_i^2t_f^2(t_i^2+\alpha_{j'}^2)^{k'}}$, the former term being finite.

Using partial fractions the latter term reduces to a sum of terms of the form $\dfrac{1}{t_f^2\,(t_i^2+\alpha_{j'}^2)^{k'}}$ and $\dfrac{1}{t_i^2\,t_f^2}$.

Hence to find the real part of the second Born correction in this case, it is necessary to find the imaginary part of the integral

$$\lim_{\varepsilon\to 0}\int d\underline{q}\,\frac{1}{(q^2-k_f^2-i\varepsilon)(\underline{q}-\underline{k}_f)^2[(\underline{q}-\underline{k}_i)^2+\alpha_{j'}^2]^{k'}}\quad(\text{since } k_n=k_f,\ \underline{t}_i=\underline{k}_i-\underline{q},\ \underline{t}_f=\underline{q}-\underline{k}_f)$$

which is infinite. (note the multiplicative factor of i in (3.49)). However in section 4.4 it is shown that the integral

$$\operatorname{Im}\lim_{\varepsilon\to 0}\int d\underline{q}\,\frac{(\underline{q}-\underline{k}_i)^4-2K^2(\underline{q}-\underline{k}_i)^2+K^4}{(q^2-k_f^2-i\varepsilon)(\underline{q}-\underline{k}_i)^2(\underline{q}-\underline{k}_f)^2[(\underline{q}-\underline{k}_i)^2+\alpha_{j'}^2]^{k'}}$$

is in fact finite. Since we are unable to calculate the imaginary part of a $\underline{q}$ integral, it follows from the argument presented in section 3.1 that the ratio β cannot be computed. Consequently the third order differential cross section (2.40) can still be found but not the full second Born (2.39) or Padé (2.38) differential cross sections.

CHAPTER 4

EVALUATION OF THE KNOWN INTEGRALS.

4.1 Introduction

The method of evaluating the known integrals

$$J(n,m) = \lim_{\varepsilon\to 0}\int \frac{d\underline{q}}{(q^2-k^2-i\varepsilon)[(\underline{q}-\underline{k}_i)^2+\lambda^2]^n[(\underline{q}-\underline{k}_f)^2+\mu^2]^m} \tag{4.1}$$

and

$$I(n) = \lim_{\varepsilon\to 0}\int \frac{(\underline{q}-\underline{k}_i)^2}{(q^2-k^2-i\varepsilon)(\underline{q}-\underline{k}_f)^2[(\underline{q}-\underline{k}_f)^2+\mu^2]^n}\,d\underline{q} \tag{4.2}$$

is due to Dalitz (1951). The integrals

$$\lim_{\varepsilon\to 0}\int \frac{d\underline{q}}{(q^2-k^2-i\varepsilon)[(\underline{q}-\underline{P})^2+\Lambda^2]^n} \tag{4.3}$$

n = 1 and 2 may be evaluated analytically. Using the identity

$$\frac{1}{ab} = \int_{-1}^{1}\left[\frac{a(1+Z)+b(1-Z)}{2}\right]^{-2}\frac{dZ}{2} \tag{4.4}$$

J(1,1) may be transformed to an integral with respect to Z of a term of the form (4.3). Replacing (4.3) by its analytic expression and performing the Z integration gives an analytic expression for J(1,1). J(n,m) is obtained from J(1,1) by partial differentiation with respect to the variables λ^2 and μ^2. Recurrence relations and Leibnitz's theorem are used to compute the partial derivatives of J(1,1). A similar method is used to compute the integrals I(n).

4.2 Analytic Form of the Known Integrals.

To evaluate $\displaystyle \lim_{\varepsilon\to 0}\int \frac{d\underline{q}}{(q^2-k^2-i\varepsilon)[(\underline{q}-\underline{P})^2+\Lambda^2]}$,

$\underline{q}$ is written in spherical coordinates (q, θ, ϕ) with polar axis parallel to $\underline{P}$. As a consequence the integrand is independent of the angle ϕ and integration with respect to ϕ gives $\lim_{\epsilon\to 0} \pi \int_{-1}^{1} d\mu \int_{-\infty}^{\infty} \frac{q^2 dq}{(q^2-k^2-i\epsilon)(q^2-2qP\mu+P^2+\Lambda^2)}$.

The integral with respect to q is evaluated by closing the contour with a semicircle in the upper half plane and then using the calculus of residues to give

$$\pi^2 i \int_{-1}^{1} d\mu \left\{ \frac{k}{k^2-2kP\mu+P^2+\Lambda^2} + \frac{P\mu+i\gamma}{i\gamma\left[(P\mu+i\gamma)^2-k^2\right]} \right\},$$

where $\gamma^2 = P^2(1-\mu^2) + \Lambda^2$. Using partial fractions the last term of this expression reduces to a form that is immediately integrable giving

$$\lim_{\epsilon\to 0} \int \frac{d\underline{q}}{(q^2-k^2-i\epsilon)\left[(\underline{q}-\underline{P})^2+\Lambda^2\right]} = \frac{\pi^2 i}{P} \ln \frac{k+P+i\Lambda}{k-P+i\Lambda}. \tag{4.5}$$

Henceforth the limit process $\epsilon \to 0$ will be understood. Differentiating (4.5) with respect to Λ^2 gives

$$\int \frac{d\underline{q}}{(q^2-k^2-i\epsilon)\left[(\underline{q}-\underline{P})^2+\Lambda^2\right]^2} = \frac{\pi^2}{\Lambda\left[P^2+\Lambda^2-k^2-2ik\Lambda\right]}. \tag{4.6}$$

The identity (4.4) may be written as

$$\frac{1}{\left[(\underline{q}-\underline{k}_i)^2+\lambda^2\right]\left[(\underline{q}-\underline{k}_f)^2+\mu^2\right]} = \frac{1}{2}\int_{-1}^{1} \frac{dz}{\left[(\underline{q}-\underline{P})^2+\Lambda^2\right]^2} \tag{4.7}$$

where $\underline{P} = \frac{1}{2}\left[\underline{k}_i(1+z)+\underline{k}_f(1-z)\right]$

$$\Lambda^2 = \frac{K^2}{4}(1-z^2) + \frac{\lambda^2}{2}(1+z) + \frac{\mu^2}{2}(1-z) \tag{4.8}$$

and $\underline{K} = \underline{k}_i - \underline{k}_f$.

Consequently the integral $J(1,1)$ may be written as

$$\begin{aligned} J(1,1) &= \int \frac{d\underline{q}}{(q^2-k^2-i\epsilon)\left[(\underline{q}-\underline{k}_i)^2+\lambda^2\right]\left[(\underline{q}-\underline{k}_f)^2+\mu^2\right]} \\ &= \frac{1}{2}\int_{-1}^{1} dz \int \frac{d\underline{q}}{(q^2-k^2-i\epsilon)\left[(\underline{q}-\underline{P})^2+\Lambda^2\right]^2} \\ &= \frac{\pi^2}{2}\int_{-1}^{1} \frac{dz}{\Lambda(P^2+\Lambda^2-k^2-2ik\Lambda)} \end{aligned}$$

$$\therefore J(1,1) = \frac{\pi^2}{2}\int_{-1}^{1} \frac{dz}{\Lambda(\alpha+\beta z-2ik\Lambda)} \tag{4.9}$$

where

$$\alpha = \frac{1}{2}\left(k_i^2+k_f^2-2k^2+\lambda^2+\mu^2\right) \tag{4.10}$$

and $\quad \beta = \frac{1}{2}\left(k_i^2-k_f^2+\lambda^2-\mu^2\right).$

For convenience the integral J(1,1) is divided into its real and imaginary parts so that

$$J(1,1) = \frac{\pi^2}{2}\int_{-1}^{1} \frac{\alpha+\beta z+2ik\Lambda}{\Lambda[(\alpha+\beta z)^2+4k^2\Lambda^2]}\,dz. \tag{4.11}$$

Let $\quad A = -\frac{K^2}{4} \qquad B = \frac{\lambda^2-\mu^2}{4} \qquad C = \frac{K^2+2(\lambda^2+\mu^2)}{4}$

$$H = A\alpha - B\beta \qquad L = B\alpha - C\beta$$
$$G^2 = -A\alpha^2+2B\alpha\beta-C\beta^2+4k^2(B^2-AC). \tag{4.12}$$

Then $\quad \Lambda^2 = Az^2+2Bz+C \tag{4.13}$

and the real part of J(1,1) is

$$\mathrm{Re}\,J(1,1) = \frac{\pi^2}{2}\int_{-1}^{1} \frac{(\alpha+\beta z)}{\sqrt{Az^2+2Bz+C}\,[(\alpha+\beta z)^2+4k^2(Az^2+2Bz+C)]}\,dz$$

$$= \frac{\pi^2}{2G}\left[\tan^{-1}\left\{\frac{G(Az^2+2Bz+C)^{1/2}}{Hz+L}\right\}\right]_{-1}^{1} \tag{4.14}$$

where the principle value of the inverse tangent is chosen to lie between 0 and π.

$$\mathrm{Im}\,J(1,1) = \pi^2 k\int_{-1}^{1} \frac{dz}{(\alpha+\beta z)^2+4k^2(Az^2+2Bz+C)}$$

$$= \frac{\pi^2}{4G}\ln\left|\frac{\alpha^2-\beta^2-4k^2(A-C)+4kG}{\alpha^2-\beta^2-4k^2(A-C)-4kG}\right|. \tag{4.15}$$

It should be noted that the integrals of interest have $k_i \geq k_n$ and hence $\alpha + \beta Z \geq 0$ for $Z \in [-1,1]$. Since Λ^2 is also positive for $Z \in [-1,1]$ the integrand of (4.14) is always positive and hence so is the integral. This shows that the range of the principle value of the inverse tangents in the expression (4.14) must be chosen so as to ensure the positiveness of this expression.

To obtain $J(n,m)$ it is necessary to differentiate (4.14) and (4.15) with respect to λ^2 and μ^2 ,

$$J(n,m) = (-1)^{n+m} \left(\frac{\partial}{\partial\lambda^2}\right)^{n-1} \left(\frac{\partial}{\partial\mu^2}\right)^{m-1} J(1,1). \tag{4.16}$$

The method of calculating these derivatives is described in detail in section 4.6.

The behaviour of (4.14) in the limit $\lambda \to 0$ will now be examined. The left hand side reduces to the integral

$$J(\lambda=0,1) = \int \frac{d\underset{\sim}{q}}{(q^2-k^2-i\varepsilon)(\underset{\sim}{q}-\underset{\sim}{k}_i)^2[(\underset{\sim}{q}-\underset{\sim}{k}_f)^2+\mu^2]}.$$

When $Z = 1(-1)$ then $AZ^2 + 2BZ + C = \lambda^2(\mu^2)$ and so

$$J(\lambda=0,1) = \lim_{\lambda\to 0} \frac{\pi^2}{2G}\left[\tan^{-1}\frac{G\lambda}{H+L} - \tan^{-1}\frac{G\mu}{H-L}\right] \tag{4.17}$$

$$J(\lambda=0,n) = (-1)^{n-1}\left(\frac{\partial}{\partial\mu^2}\right)^{n-1} J(\lambda=0,1). \tag{4.18}$$

The term $\frac{\pi^2}{2G} \tan^{-1} \frac{G\mu}{H-L}$ is well behaved as $\lambda \to 0$ and may be evaluated using the techniques of section 4.6. To investigate the behaviour of the first term of (4.17) it is assumed that G and $H+L$ do not simultaneously vanish as $\lambda \to 0$. In practice this constraint has never been violated. From (4.12)

$$H+L = \frac{1}{4}(k^2-k_i^2)(K^2+\mu^2) + \frac{\lambda^2\mu^2}{4} + O(\lambda^2) \tag{4.19}$$

so that
$$\lim_{\lambda\to 0}\frac{G\lambda}{H+L} = \lim_{\lambda\to 0}\frac{G\lambda}{\frac{1}{4}(k^2-k_i^2)(k^2+\mu^2)+\lambda^2\mu^2/4+O(\lambda^2)}$$
$$= \begin{cases} 0^+ & k>k_i \\ \infty & k=k_i \\ 0^- & k<k_i \end{cases}$$

Consequently

$$\lim_{\lambda\to 0}\tan^{-1}\frac{G\lambda}{H+L} = \begin{cases} 0 & k>k_i \\ \pi/2 & k=k_i \\ \pi & k<k_i \end{cases} = \gamma^k_{k_i}\,.$$

$$\therefore J(\lambda=0,n) = (-1)^{n-1}\left(\frac{\partial}{\partial\mu^2}\right)^{n-1}\frac{\pi^2}{2G}\left[\gamma^k_{k_i} - \tan^{-1}\frac{G\mu}{H-L}\right]_{\lambda=0}\,. \tag{4.20}$$

Similarly

$$J(n,\mu=0) = (-1)^{n-1}\left(\frac{\partial}{\partial\lambda^2}\right)^{n-1}\frac{\pi^2}{2G}\left[\tan^{-1}\frac{G\lambda}{H+L} - \gamma^{k_f}_{k}\right]_{\mu=0} \tag{4.21}$$

where $\gamma^{k_f}_{k} = \begin{cases} 0 & k_f>k \\ \pi/2 & k_f=k \\ \pi & k_f<k\,. \end{cases}$

Combining the two limiting processes $\lambda\to 0$ and $\mu\to 0$ gives the value of the integral

$$J(\lambda=0,\mu=0) = \int\frac{d\underline{q}}{(q^2-k^2-i\varepsilon)(\underline{q}-\underline{k}_i)^2(\underline{q}-\underline{k}_f)^2} = \frac{\pi^2}{2G}\left[\gamma^k_{k_i} - \gamma^{k_f}_{k}\right]. \tag{4.22}$$

In the special case $\underline{k}_i = \underline{k}_f$ and $\lambda=\mu$ the simple expression (4.6) is used in preference to the more complex (4.14) and (4.15) .

$$\int\frac{d\underline{q}}{(q^2-k^2-i\varepsilon)[(\underline{q}-\underline{k}_i)^2+\lambda^2]^2} = \frac{\pi^2}{\lambda[k_i^2+\lambda^2-k^2-2ik\lambda]}$$
$$= \frac{\pi^2}{2k_i}\left[\frac{1}{\lambda}\left\{\frac{k_i+k}{\lambda^2+(k_i+k)^2} + \frac{k_i-k}{\lambda^2+(k_i-k)^2}\right\} + i\left\{\frac{1}{\lambda^2+(k_i-k)^2} - \frac{1}{\lambda^2+(k_i+k)^2}\right\}\right] \tag{4.23}$$

All the integrals that have been discussed above are finite except the following which are infinite:-

$$\mathrm{Im}\int\frac{d\underline{q}}{(q^2-k_i^2-i\varepsilon)(\underline{q}-\underline{k}_i)^2[(\underline{q}-\underline{k}_f)^2+\mu^2]^m}$$ for all k_f, μ and m.

$$\mathrm{Re}\int\frac{d\underline{q}}{(q^2-k^2-i\varepsilon)(\underline{q}-\underline{k}_i)^4}$$ for all k.

4.3 Analytic Form of an Integral Arising from Transitions to "d" Final States.

The integral considered in this section has the form

$$I(1)=\int\frac{(\underline{q}-\underline{k}_f)^2}{(q^2-k^2-i\varepsilon)(\underline{q}-\underline{k}_i)^2[(\underline{q}-\underline{k}_i)^2+\lambda^2]}\,d\underline{q}, \tag{4.24}$$

where it is assumed that $\lambda \neq 0$. Using the identity (4.4) together with a suitable change of the variable of integration gives

$$I(1)=2\int_0^1 t\,dt\int\frac{(\underline{q}-\underline{k}_f)^2}{(q^2-k^2-i\varepsilon)[(\underline{q}-\underline{k}_i)^2+\lambda^2t^2]^2}\,d\underline{q}. \tag{4.25}$$

Writing $\underline{q}$ in spherical coordinates (q,θ,ϕ) with polar axis parallel to $\underline{k}_i$ and the plane containing $\underline{k}_f$ as the $\phi = 0$ plane (so that $\underline{k}_f = (k_f,\gamma,0)$) then (4.25) becomes

$$I(1)=2\int_0^1 t\,dt\int\frac{q^2-2qk_f(\cos\theta\cos\gamma+\sin\theta\sin\gamma\cos\phi)+k_f^2}{(q^2-k^2-i\varepsilon)(q^2-2qk_i\cos\theta+k_i^2+\lambda^2t^2)^2}\,d\underline{q}. \tag{4.26}$$

The only ϕ dependence in the integrand is the cos ϕ term and since $\int_0^{2\pi}\cos\phi\,d\phi$ vanishes so does this term. Replacing cos θ by μ and using partial fractions (4.26) may be written as

$$I(1)=2\int_0^1 t\,dt\left\{\int\frac{d\underline{q}}{(q^2-k^2-i\varepsilon)(q^2-2qk_i\mu+k_i^2+\lambda^2t^2)}+\int\frac{(k_f^2-k_i^2-\lambda^2t^2)}{(q^2-k^2-i\varepsilon)(q^2-2qk_i\mu+k_i^2+\lambda^2t^2)^2}\,d\underline{q}\right. \tag{4.27}$$
$$\left.+\int\frac{2q\mu(k_i-k_f\cos\gamma)}{(q^2-k^2-i\varepsilon)(q^2-2qk_i\mu+k_i^2+\lambda^2t^2)^2}\,d\underline{q}\right\}.$$

The first two $\underline{q}$ integrals in this expression have been evaluated in section 4.2 (c.f. (4.5) and (4.6)). Partially differentiating both sides of (4.5) with respect to k_i gives

$$\int \frac{2q\mu - 2k_i}{(q^2-k^2-i\epsilon)(q^2-2qk_i\mu+k_i^2+\Lambda^2)^2}\,d\underline{q} = \frac{\pi^2 i}{k_i}\frac{2(k+i\Lambda)}{(k+i\Lambda)^2-k_i^2} + \frac{\pi^2 i}{k_i^2}\ln\frac{k+k_i+i\Lambda}{k-k_i+i\Lambda}\,. \tag{4.28}$$

Using (4.28) and (4.6) it is then a simple matter to obtain the third $\underline{q}$ integral of (4.27) and after some algebraic manipulation obtain the result,

$$I(1) = \frac{2\pi^2}{k_i\lambda}\int_0^1 \left\{(k_i - 2k_f\cos\gamma) + \frac{1}{2}\frac{k_f^2+2kk_f\cos\gamma+k^2}{k+k_i+i\lambda t} - \frac{1}{2}\frac{k_f^2-2kk_f\cos\gamma+k^2}{k-k_i+i\lambda t}\right\}dt + \frac{2i\pi^2 k_f\cos\gamma}{k_i^2}\int_0^1 t\ln\frac{k+k_i+i\lambda t}{k-k_i+i\lambda t}\,dt\,. \tag{4.29}$$

Only the real part of this integral is required. Care must be taken when evaluating the t integrals that the correct principle values are used for the various expressions arising. The final result is

$$\mathrm{Re}\, I(1) = \frac{\pi^2}{k_i^2\lambda^2}\left\{\tan^{-1}\frac{\lambda}{k-k_i} - \tan^{-1}\frac{\lambda}{k+k_i}\right\}\left\{k_f\cos\gamma(\lambda^2+k_i^2+k^2) - k_i(k_f^2+k^2)\right\} + \frac{2\pi^2}{k_i\lambda}(k_i - k_f\cos\gamma) + \frac{\pi^3 k_f\cos\gamma}{k_i^2}H(k_i-k), \tag{4.30}$$

where the principle value of the inverse tangents lies in the range $\left[-\pi/2, \pi/2\right]$ and $H(k_i-k)$ is the Heaviside distribution i.e.

$$H(k_i - k) = \begin{cases} 0 & k_i < k \\ 1 & k_i > k\,. \end{cases}$$

It is clear from (4.30) that there is a discontinuity in the integral $I(1)$ at the point $k_i=k$. To determine the value of $I(1)$ at this point it is necessary to set $k=k_i$ in (4.29) and then perform the t

integration to give

$$\mathrm{Re}\, I(1)\Big|_{k=k_i} = -\frac{\pi^2}{k_i^2\lambda^2}\tan^{-1}\frac{\lambda}{2k_i}\left\{k_f\cos\gamma\,(\lambda^2+2k_i^2)-k_i(k_i^2+k_f^2)\right\} + \frac{2\pi^2}{k_i\lambda}(k_i-k_f\cos\gamma)+\frac{\pi^3 k_f\cos\gamma}{2k_i^2}.$$

Finally cos γ is replaced by the variable $K^2 = (\underline{k}_i - \underline{k}_f)^2$ giving

$$\mathrm{Re}\, I(1) = \frac{\pi^2(k_i^2+k_f^2-K^2)}{2k_i^3}\,\gamma^k_{k_i} + \frac{\pi^2(K^2+k_i^2-k_f^2)}{k_i^2\lambda} + \frac{\pi^2}{2k_i^3\lambda^2}\left\{\tan^{-1}\frac{\lambda}{k-k_i}-\tan^{-1}\frac{\lambda}{k+k_i}\right\}\left\{\lambda^2(k_i^2+k_f^2-K^2)-K^2(k_i^2+k^2)+(k_i^2-k_f^2)(k_i^2-k^2)\right\}, \tag{4.31}$$

where $\tan^{-1}\dfrac{\lambda}{k-k_i} = 0$ if $k=k_i$ and $\gamma^k_{k_i} = \begin{cases} 0 & k>k_i \\ \pi/2 & k=k_i \\ \pi & k<k_i. \end{cases}$

4.4 The Pseudo-Infinite Integral of the $\sum_{\mu=-1}^{1} (1s)^2 - (ns,mp_\mu) - (ns,mp)$ Transition.

In section 3.6 an anomaly occurred because of the method used to evaluate the integral

$$X = \mathrm{Im}\int \frac{(\underline{q}-\underline{k}_i)^4 - 2K^2(\underline{q}-\underline{k}_i)^2 + K^4}{(q^2-k_f^2-i\epsilon)(\underline{q}-\underline{k}_i)^2(\underline{q}-\underline{k}_f)^2[(\underline{q}-\underline{k}_i)^2+\lambda^2]^2}\,d\underline{q}. \tag{4.32}$$

For convenience in this section $\underline{q} - \underline{k}_i$ and $\underline{q} - \underline{k}_f$ will be denoted by $\underline{t}_i$ and $\underline{t}_f$ respectively. Using partial fractions (4.32) may be written as

$$X = \mathrm{Im}\int\frac{d\underline{q}}{(q^2-k_f^2-i\epsilon)}\left\{\frac{K^4}{\lambda^2 t_i^2 t_f^2} + \frac{(\lambda^2-K^2)(\lambda^2+K^2)}{\lambda^4 t_f^2(t_i^2+\lambda^2)} - \frac{(\lambda^2+K^2)^2}{\lambda^2 t_f^2(t_i^2+\lambda^2)^2}\right\}. \tag{4.33}$$

However, as stated at the end of section 4.2, all three of these integrals are infinite. It is now shown that if these three terms are summed before performing the final integration, a finite result is obtained.

From (4.15)

$$\mathrm{Im}\, J(1,1) = \mathrm{Im} \int \frac{d\underset{\sim}{q}}{(q^2-k^2-i\varepsilon)[(\underset{\sim}{q}-\underset{\sim}{k}_i)^2+\lambda^2][(\underset{\sim}{q}-\underset{\sim}{k}_f)^2+\mu^2]}$$

$$= \pi^2 k \int_{-1}^{1} \frac{dZ}{(\alpha+\beta Z)^2 + 4k^2(AZ^2+2BZ+C)} .$$

Setting $k = k_f$ and $\mu = 0$ gives

$$I'(1) = \mathrm{Im} \int \frac{d\underset{\sim}{q}}{(q^2-k_f^2-i\varepsilon)(\underset{\sim}{q}-\underset{\sim}{k}_f)^2[(\underset{\sim}{q}-\underset{\sim}{k}_i)^2+\lambda^2]} \qquad (4.34)$$

$$= \pi^2 k_f \int_{-1}^{1} \frac{dZ}{(\alpha+\beta Z)^2 + 4k_f^2(AZ^2+2BZ+C)} ,$$

where

$$\alpha = \frac{1}{2}\left(k_i^2 - k_f^2 + \lambda^2\right) = \beta$$

$$A = -\frac{K^2}{4} \qquad B = \frac{\lambda^2}{4} \qquad C = \frac{K^2+2\lambda^2}{4} .$$

Applying the transformation $Z = \frac{1-y}{1+y}$ and substituting for α, β, A, B and C, (4.34) becomes

$$I'(1) = 2\pi^2 k_f \int_0^{\infty} \frac{dy}{4k_f^2(K^2+\lambda^2)y + X(\lambda^2)} , \qquad (4.35)$$

where $X(\lambda^2) = (k_i^2 - k_f^2 + \lambda^2)^2 + 4k_f^2\lambda^2$. Partial differentiation with respect to λ^2 gives

$$I'(2) = -\frac{\partial}{\partial\lambda^2} I'(1) = \mathrm{Im} \int \frac{d\underset{\sim}{q}}{(q^2-k_f^2-i\varepsilon)(\underset{\sim}{q}-\underset{\sim}{k}_f)^2[(\underset{\sim}{q}-\underset{\sim}{k}_i)^2+\lambda^2]^2}$$

$$= 2\pi^2 k_f \int_0^{\infty} \frac{4k_f^2 y + X'(\lambda^2)}{[4k_f^2(K^2+\lambda^2)y + X(\lambda^2)]^2}$$

$$= \frac{2\pi^2 k_f}{K^2+\lambda^2} \int_0^{\infty} \left\{ \frac{1}{4k_f^2(K^2+\lambda^2)y + X(\lambda^2)} + \frac{(K^2+\lambda^2)X'(\lambda^2) - X(\lambda^2)}{[4k_f^2(K^2+\lambda^2)y + X(\lambda^2)]^2} \right\} dy , \qquad (4.36)$$

where $X'(\lambda^2) = \frac{\partial}{\partial\lambda^2} X(\lambda^2)$. Setting $\lambda = 0$ in (4.35) gives

$$\mathrm{Im} \int \frac{d\underset{\sim}{q}}{(q^2-k_f^2-i\varepsilon)(\underset{\sim}{q}-\underset{\sim}{k}_i)^2(\underset{\sim}{q}-\underset{\sim}{k}_f)^2} = 2\pi^2 k_f \int_0^{\infty} \frac{dy}{4k_f^2K^2y + (k_f^2-k_i^2)^2} . \qquad (4.37)$$

Substituting (4.35), (4.36) and (4.37) into (4.33) gives

$$X = 2\pi^2 k_f \int_0^\infty \left\{ \frac{K^4}{\lambda^4[4k_f^2K^2y + (k_f^2-k_i^2)^2]} + \frac{(\lambda^2-K^2)(\lambda^2+K^2)}{\lambda^4[4k_f^2(K^2+\lambda^2)y + X(\lambda^2)]} - \frac{K^2+\lambda^2}{\lambda^2[4k_f^2(K^2+\lambda^2)y + X(\lambda^2)]} - \frac{(K^2+\lambda^2)[(K^2+\lambda^2)X'(\lambda^2) - X(\lambda^2)]}{\lambda^2[4k_f^2(K^2+\lambda^2)y + X(\lambda^2)]^2} \right\} dy \quad (4.38)$$

The last term of the integrand of (4.38) is of the form

$\int_0^\infty \frac{dy}{(ay+b)^2} = \frac{1}{ab}$ and is finite. The remaining three terms may be written as

$$2\pi^2 k_f \int_0^\infty \left\{ \frac{K^4}{\lambda^4[4k_f^2K^2y + (k_f^2-k_i^2)^2]} - \frac{K^2(K^2+\lambda^2)}{\lambda^4[4k_f^2(K^2+\lambda^2)y + X(\lambda^2)]} \right\} dy$$

$$= \frac{\pi^2 K^2}{2k_f\lambda^4}\left[\ln\left\{ \frac{4k_f^2K^2y + (k_f^2-k_i^2)^2}{4k_f^2(K^2+\lambda^2)y + X(\lambda^2)} \right\} \right]_0^\infty$$

$$= \frac{\pi^2 K^2}{2k_f\lambda^4} \ln\left\{ \frac{K^2}{K^2+\lambda^2} \, \frac{X(\lambda^2)}{(k_f^2-k_i^2)^2} \right\},$$

which is also finite. Consequently the integral (4.32) is finite with analytic form

$$\frac{\pi^2 K^2}{2k_f\lambda^4} \ln\left\{ \frac{K^2 X(\lambda^2)}{(K^2+\lambda^2)(k_f^2-k_i^2)^2} \right\} - \frac{\pi^2[(K^2+\lambda^2)X'(\lambda^2) - X(\lambda^2)]}{2k_f\lambda^2 X(\lambda^2)} \quad (4.39)$$

4.5 General Recurrence Relations

In the next two sections the following notation is used. Let $S(1,1)$ be a function of two variables say x and y. Then $S(n,m)$ is defined as

$$S(n,m) = \left(\frac{\partial}{\partial x}\right)^{n-1}\left(\frac{\partial}{\partial y}\right)^{m-1} S(1,1). \quad (4.40)$$

Consider the identity

$$S(n,m)P(1,1) = A(n,m) + \sum_{r=1}^{n}\sum_{s=1}^{m} B\begin{pmatrix} n & m \\ r & s \end{pmatrix} P(r,s)\, S(n+1-r, m+1-s) \quad (4.41)$$

where S, P and A are functions of the two variables x,y and $B\binom{n\ m}{r\ s}$ is a constant. To use this identity to compute higher derivatives of S knowing the lowest derivatives of S and all those of A and B, it is necessary to have a recurrence relation to determine the constants $B\binom{n\ m}{r\ s}$. The starting points are the two expressions

$$S(2,1)P(1,1) = A(2,1) + B\binom{2\ 1}{2\ 1}P(2,1)S(1,1) \tag{4.42}$$

and

$$S(1,2)P(1,1) = A(1,2) + B\binom{1\ 2}{1\ 2}P(1,2)S(1,1). \tag{4.43}$$

It should be noted that $B\binom{n\ m}{1\ 1} \equiv 0$ since otherwise there would occur terms containing S(n,m) on both sides of (4.41) and these would be combined and the constants $B\binom{n\ m}{r\ s}$ divided by $1- B\binom{n\ m}{1\ 1}$ to obtain a new set of constants with $B\binom{n\ m}{1\ 1} \equiv 0$. Using (4.41), S(n,1) and S(1,n) $n \geqq 3$ are calculated and then S(n,m) $n \geqq 2$, $m \geqq 2$. The recurrence relation is derived by differentiating (4.41) once with respect to x , obtaining

$$\begin{aligned} S(n+1,m)P(1,1) &= A(n+1,m) - S(n,m)P(2,1) \\ &\quad + \sum_{r=1}^{n}\sum_{s=1}^{m} B\binom{n\ m}{r\ s}\Big[P(r+1,s)S(n+1-r,m+1-s) + P(r,s)S(n+2-r,m+1-s)\Big] \\ &= A(n+1,m) + \sum_{r=2}^{n}\sum_{s=1}^{m}\Big[B\binom{n\ m}{r\ s} + B\binom{n\ \ m}{r-1\ s}\Big]P(r,s)S(n+2-r,m+1-s) - S(n,m)P(2,1) \\ &\quad + \sum_{s=1}^{m}\Big[B\binom{n\ m}{n\ s}P(n+1,s)S(1,m+1-s) + B\binom{n\ m}{1\ s}P(1,s)S(n+1,m+1-s)\Big]. \end{aligned} \tag{4.44}$$

The expression (4.44) is to be compared with

$$S(n+1,m)P(1,1) = A(n+1,m) + \sum_{r=1}^{n+1}\sum_{s=1}^{m} B\binom{n+1\ m}{r\ \ s}P(r,s)S(n+2-r,m+1-s)$$

to give the following recurrence relation, noting that due to the starting point (4.42), $n \geqq 2$.

From the coefficient of P(1,s) $$B\binom{n+1\ m}{1\ \ s} = B\binom{n\ m}{1\ s}. \tag{4.45}$$

From the coefficient of P(2,1) noting that $B\binom{n\ m}{1\ 1} \equiv 0$

$$B\binom{n+1\ m}{2\ \ s} = B\binom{n\ m}{2\ 1} - 1. \tag{4.46}$$

From the coefficient of P(n+1,s)

$$B\begin{pmatrix} n+1 & m \\ n+1 & s \end{pmatrix} = B\begin{pmatrix} n & m \\ n & s \end{pmatrix}, \quad n \geq 2. \tag{4.47}$$

Finally from the coefficient of P(r,s) in all cases not covered by the three previous relations

$$B\begin{pmatrix} n+1 & m \\ r & s \end{pmatrix} = B\begin{pmatrix} n & m \\ r & s \end{pmatrix} + B\begin{pmatrix} n & m \\ r-1 & s \end{pmatrix}. \tag{4.48}$$

Similar recurrence relations hold for derivatives with respect to y, which would increase m .

This method is used on two occasions when calculating the derivatives of Rℓ J(1,1) (c.f. (4.14)). An expression is given in (4.12) for the function $G^2(1,1)$ of the two variables λ^2 and μ^2 . In general it is not possible to find the square root of this function in closed form, so (4.41) is used to determine the derivatives of the square root and inverse square root of $G^2(1,1)$. Consider first

$$G(1,1) = \sqrt{G^2(1,1)}$$

$$G(2,1) = \frac{1}{2}\,\frac{G^2(2,1)\,G(1,1)}{G^2(1,1)} \qquad G(1,2) = \frac{1}{2}\,\frac{G^2(1,2)\,G(1,1)}{G^2(1,1)}.$$

The expressions for G(2,1) and G(1,2) are of the same form as (4.42) and (4.43) with $B\binom{2\ 1}{2\ 1} = B\binom{1\ 2}{1\ 2} = \tfrac{1}{2}$. The recurrence relations for $B\binom{n\ m}{r\ s}$ and (4.41) are used to determine G(n,m).

$$U(1,1) = [G^2(1,1)]^{-1/2}$$

$$U(2,1) = -\frac{1}{2}\,\frac{G^2(2,1)\,U(1,1)}{G^2(1,1)} \qquad U(1,2) = -\frac{1}{2}\,\frac{G^2(1,2)\,U(1,1)}{G^2(1,1)}.$$

and U(n,m) may be calculated in a like fashion to G(n,m).

Finally, it is necessary to determine the higher derivatives of the function V(n,m), which is defined in terms of the known derivatives Y(n,m) and Z(n,m) by the two relationships,

$$V(2,1) = \frac{Y(2,1)}{Z(1,1)} \quad \text{and} \quad V(1,2) = \frac{Y(1,2)}{Z(1,1)}.$$

By proceeding in a similar manner to the derivation of the recurrence relations for $B\binom{n\ m}{r\ s}$ one obtains the following three relations, from which all values of V(n,m) may be found.

$$V(n,1)Z(1,1) = Y(n,1) - \sum_{r=1}^{n-2} {}_{n-2}C_r \, Z(r+1,1)V(n-r,1) \tag{4.49}$$

$$V(1,m)Z(1,1) = Y(1,m) - \sum_{s=1}^{m-2} {}_{m-2}C_s \, Z(1,s+1)V(1,m-s) \tag{4.50}$$

$$V(n,m)Z(1,1) = Y(n,m) - \sum_{s=2}^{m} {}_{m-1}C_{s-1} \, Z(1,s)V(n,m+1-s)$$
$$- \sum_{r=1}^{n-2}\sum_{s=1}^{m} {}_{n-2}C_r \, {}_{m-1}C_{s-1} \, Z(r+1,s)V(n-r,m+1-s) \tag{4.51}$$

where ${}_nC_r$ is the binomial coefficient $\dfrac{n!}{r!(n-r)!}$.

4.6 Evaluation of the Higher Order Integrals.

To obtain the higher order integrals J(n,m) it is necessary to partially differentiate J(1,1) with respect to λ^2 and μ^2. The techniques used are those of section 4.5, together with Leibnitz's theorem.

From (4.14)

$$\mathcal{R}\ J(1,1) = \frac{\pi^2}{2G}\left[\tan^{-1}\left\{\frac{G(AZ^2+2BZ+C)}{HZ+L}\right\}\right]_{-1}^{1}$$

where the algebraic expressions of the functions G^2, H, L, A, B and C are given in (4.12). Replacing Z by ± 1,

$$\mathcal{R}\ J(1,1) = \frac{\pi^2}{2G}\left\{\tan^{-1}\frac{G\lambda}{L+H} - \tan^{-1}\frac{G\mu}{L-H}\right\}$$

$$= \frac{\pi^2}{2G}\tan^{-1}\frac{G\left[(L-H)\lambda-(L+H)\mu\right]}{L^2-H^2+G^2\lambda\mu}$$

$$= \frac{\pi^2}{2}U(1,1)\tan^{-1}\frac{Q(1,1)}{P(1,1)}$$

in the notation of section 4.5. From the algebraic form of the functions L, H, G^2, λ and μ, the values of their derivatives with respect to λ^2 μ^2 may be calculated. The recurrence relation (4.41) is used to determine the values of the derivatives of $U(1,1) = \left[G^2(1,1)\right]^{-\frac{1}{2}}$ and $G(1,1) = \left[G^2(1,1)\right]^{\frac{1}{2}}$. Leibnitz's theorem is used to calculate all the derivatives of $Q(1,1)$ and $P(1,1)$.

Consider now

$$V(1,1) = \tan^{-1}\frac{Q(1,1)}{P(1,1)}$$

$$V(2,1) = \frac{Q(2,1)P(1,1) - Q(1,1)P(2,1)}{P^2(1,1) + Q^2(1,1)} = \frac{Y(2,1)}{Z(1,1)}$$

$$V(1,2) = \frac{Q(1,2)P(1,1) - Q(1,1)P(1,2)}{P^2(1,1) + Q^2(1,1)} = \frac{Y(1,2)}{Z(1,1)}.$$

The higher derivatives of Y and Z may be calculated using Leibnitz's theorem and of V by using the recurrence relations (4.49), (4.50) and (4.51). Lastly

$$\mathrm{Re}\ J(1,1) = \frac{\pi^2}{2}U(1,1)V(1,1)$$

and Leibnitz's theorem is again used to compute the value of the higher derivatives of $J(1,1)$. In the cases where λ or μ vanishes the same steps are taken except that if say $\lambda = 0$ then $V(1,1) =$

$$Y_{k_1}^{k} - \tan^{-1}\frac{G\mu}{L-H}.$$

A similar method is used to compute the derivatives of

$$\mathrm{Im}\ J(1,1) = \frac{\pi^2}{4G}\ln\left|\frac{\alpha^2-\beta^2-4k^2(A-C)+4kG}{\alpha^2-\beta^2-4k^2(A-C)-4kG}\right|$$

$$= \frac{\pi^2}{4}U(1,1)\ln\frac{Q(1,1)}{P(1,1)}$$

(4.41) is used to compute the values of the higher derivatives of $U(1,1) = \left[G^2(1,1)\right]^{-\frac{1}{2}}$ and $G(1,1) = \left[G^2(1,1)\right]^{\frac{1}{2}}$, then Leibnitz's theorem gives the values of $Q(n,m)$ and $P(n,m)$.

$$V(2,1) = \frac{Q(2,1)P(1,1) - Q(1,1)P(2,1)}{P(1,1)Q(1,1)} = \frac{Y(2,1)}{Z(1,1)}$$

$$V(1,2) = \frac{Q(1,2)P(1,1) - Q(1,1)P(1,2)}{P(1,1)Q(1,1)} = \frac{Y(1,2)}{Z(1,1)}$$

Y(n,m) and Z(n,m) are calculated using Leibnitz's theorem, V(n,m) by the recurrence relations (4.49), (4.50) and (4.51) and J(n,m) by Leibnitz's theorem. In the cases where λ or μ vanish then the same method is used except that the form of Q(1,1) and P(1,1) will be less complex.

In the special case $\underline{k}_i = \underline{k}_f$ and $\lambda = \mu$ (c.f. (4.23)) then Leibnitz's theorem is sufficient to determine all the derivatives, as it also is to compute the derivatives of the integral arising when considering excitation to a d state (c.f. (4.31)).

In the case of forward scattering (i.e. $K = k_i - k_f$), G^2 becomes a perfect square $G = \frac{1}{2}\left| k_i \mu^2 - k_f \lambda^2 + (k^2 - k_i k_f)(k_i - k_f) \right|$.

4.7 The Computer Programs.

In this section, the computer programs used in this work are briefly described. Firstly the coefficients of the Cohen and McEachran wave functions are read in and the coefficients $A_{n\ell,i}$ (c.f. (3.11b)) computed. These coefficients are stored and are accessed when required. If a hydrogenic type wave function is to be used (i.e. for a ^1D state), then the nuclear charges α and β (c.f. (3.9)) must be input with the request for that type of wave function and the coefficients $A_{n\ell,i}$ will then be generated by the program.

Next the initial, intermediate and final states are input, together with the type of wave functions to be used. The program generates and stores the coefficients Y(j;f;n;k) and Y(j';n;i;k'). (See Appendix B.) Briefly these coefficients are defined by

$$I(n,m;\underline{t}) = F_{nm}(\hat{\underline{t}}) \sum_{jk} \frac{t^{\lambda_j}\, Y(j;n;m;k)}{(t^2 + \alpha_j^2)^k}$$

where $I(n,m;\underline{t})$ is defined by (3.13), $F_{nm}(\underline{t})$ is some function of the angle between $\underline{t}$ and $\underline{K}$ and λ_j is a non-zero integer.

Lastly the incident electron energies and the angles of scattering are read in and the program then computes the real or imaginary part of the second Born correction to the scattering amplitude. There are six different routines used to compute the known integrals (4.1) and (4.2). Three of them calculate the real part of these integrals and three the imaginary part. Of these three, one computes forward scattering amplitudes only making use of the simplification occurring to G^2 in this case. The logic of the other two routines are identical, the difference being that one uses single precision arithmetic (16 significant figures on the CDC 6600 used to perform these calculations) whilst the other uses double precision arithmetic. The latter routine is necessary because at small angles of scattering, rounding errors make the results of the recursion relations meaningless.

The known integrals required for the transition under investigation are computed, combined in the required manner to give the more complex integrals of (3.39) → (3.44) and (3.47) → (3.49) and then multiplied by the coefficients $Y(j;f;n;k)$ and $Y(j';n;i;k')$ to give the scattering amplitudes. The program then reads the data for the next transition to be considered.

To use these programs for collisions with atoms other than Helium, it is only necessary to replace the routines that compute the coefficients $Y(j;n;m;k)$ by ones that compute these coefficients for the atom in question. By computing second Born electron-Hydrogen atom cross sections, it was possible to check the computer program by comparing its results with those of other workers.

CHAPTER 5

RESULTS

5.1 Scattering Amplitudes

The scattering amplitudes are given in Appendix C. The first column of each Table is the angle of scattering, θ , the second column the first Born scattering amplitude and the third column the second Born correction to the scattering amplitude arising from closure, i.e. the term $\underset{n}{S}\ I_n(k_{av})$ of equation 3.45. The remaining columns are the corrections to the closure contribution due to the explicit inclusion of the named state, i.e. the term $I_n(k_n) - I_n(k_{av})$ of equation 3.45.
For elastic scattering and excitation of the 2^1S state, two rows of figures are given at every angle of scattering. The upper row is the real part and the lower row the imaginary part of the second Born correction. By examining these tables, the states not adequately represented by the simplifying approximation may be found.

For elastic scattering, the contribution from closure is of the same order of magnitude as the first Born scattering amplitude. The ground state is inadequately represented at all angles of scattering whilst the 2^1P and 3^1P states give significant corrections in the forward direction. The 2^1P state makes a large correction to closure for the excitation of the 2^1S state. The ground and 2^1S states make smaller, but still significant corrections, whilst the 3^1S and 3^1P states appear to be adequately represented by the simplifying approximation.

Considering the excitation of the 2^1P state, the contribution from closure is very small except at large angles of scattering. The correction from the 2^1P state is much larger than the closure contribution and the ground, 2^1S and 3^1P states are also inadequately represented.

For excitation of the 3^1D and 4^1D states, using values of Δ_{av} chosen to give the exact forward elastic scattering amplitudes, the contribution from closure is small, as are the corrections from the ^1S states. However the corrections from the ^1P states are much larger and will be discussed below. If the Holt and Moiseiwitsch (1968) choice of Δ_{av} is used, then the contribution from closure is of the same order of magnitude as the first Born scattering amplitude in the forward direction, but decreases faster than does first Born with increasing angle of scattering. The only states giving significant corrections at small angles of scattering are the 2^1P state, the 3^1P state for 3^1D excitation and the 4^1P state for 4^1D excitation.

The contribution from the closure approximation is small for the excitation of the model doubly excited states and of the same order of magnitude as the corrections from the intermediate states.

In Figure 5, the contributions to the second Born correction to the scattering amplitude from some ^1P intermediate states for the excitation of the 3^1D and 4^1D states of Helium by 200 eV electrons are plotted against the angle of scattering, i.e. (3.3) for certain intermediate states n is plotted against θ. For comparison the first Born scattering amplitudes are also shown. For excitation of the 3^1D state the largest contributions come from the 2^1P and 3^1P intermediate states. These contributions are of approximately the same magnitude but of opposite sign and nearly cancel. Similarly in the case of excitation of the 4^1D state the contributions from the 2^1P and 3^1P intermediate states nearly cancel that from the 4^1P intermediate state. It is clear that the contributions from the ^1P intermediate states converge rapidly. The contributions from the ^1S states and closure are small.

5.2 Elastic Scattering by Helium

Calculated elastic differential cross sections are shown in Figures 6 and 7, where they are compared with the SRB2 calculation of Holt et al (1971b) and experiment. The experimental data are those of Bromberg (1970) at 500 eV and at other energies those of Vriens et al (1968b) renormalised at 5^o to Chamberlain et al (1970). The SFB2 results lie well above all other values at all angles considered. The SRB2 results are in less good agreement with experiment than those of Holt et al (1971b) who choose $\Delta_{av} = E\ (3^1S)$. The SP1 results do not give the intense forward peak.

A comparison with the SRB2 calculation of Birman and Rosendorff (1969), who included only the 1^1S state explicitly, is possible at 200 eV. As is to be expected from Table 3, their results using $\Delta_{av} = 5.0$ are in agreement with the SRB2 results using $\Delta_{av} = 7.1$, but including more intermediate states explicitly.

Garibotti and Massaro (1971 b) do not quote the values of Δ_{av} they used for electron-Helium atom scattering but it would appear that they do not obtain the correct second Born forward elastic scattering amplitude. (c.f. section 2.5) Their differential cross sections are close to the SFB2 ones but approach the first Born values more quickly than do the SFB2.

Berrington et al (1972) give differential cross sections calculated using second order potentials. (c.f. section 2.7) Their results lie close to those of Holt et al (1971 b) for small angle scattering $(<5^o)$, although they are lower than Holt et al's in the forward direction. At larger angles of scattering, the second order potential results fall below the first Born results.

Finally, it is clear that the approximations adopted by Holt et al give a more satisfactory account of the elastic scattering of electrons by Helium atoms than do any of the other approximations discussed above.

In Table 7, the total elastic scattering cross sections are compared with the complete SRB2 calculation of Holt et al (1971 b), the second order potential calculation of Berrington et al (1972) and with experiment. They were obtained by numerical integration of the differential cross sections and agree in first Born with the very accurate values of Bell et al (1969) to better than 0.5%. The SRB2 results are much larger than the corresponding calculations of Holt et al and the SFB2 results are larger still. The SP1 results are in substantially better agreement with the experimental values than any of the second Born calculations. However owing to the shape of the Padé differential cross section, this good agreement is probably fortuitous. At all energies considered, the second order potential results lie slightly below the first Born results. The situation remains unsatisfactory.

5.3 Excitation of the 2^1S state of Helium

In Figures 8 and 9 various differential cross sections for the $1^1S \rightarrow 2^1S$ transition of Helium atoms by electron impact are compared with the experimental results of Vriens et al (1968 a) renormalised, at 5^o, to the absolute results of Chamberlain et al (1970). Also shown are the second order potential results of Berrington et al (1972). The experimental results in the forward direction are obtained from an analytic fit to the generalised oscillator strengths used by Vriens et al to extend the range of their results.

The imaginary part of the scattering amplitude is of the same order of magnitude as the first Born scattering amplitude in the forward direction which accounts for the very large SFB2 differential cross sections. The shape of the SP1 differential cross section is in disagreement with the shape of all the other differential cross sections. Near the forward direction, the SRB2 differential cross sections are larger than the first Born ones (in disagreement with experiment which is smaller) but at slightly larger angles ($\gtrsim 5^{o}$) become smaller than the first Born, in agreement with experiment. The shape of the second order potential differential cross sections is similar to that of the SRB2 ones. However they are sharply peaked in the forward direction and fall much further below the first Born differential cross sections than do the SRB2 ones.

Holt et al (1971 b) find that the SRB2 approximation with $\Delta_{av} = E(3^1S)$ is almost identical to the first Born approximation for this transition at the angles and energies considered here. Lassetre et al (1964) have measured relative differential cross sections at 511 eV for the transitions $1^1S \rightarrow 2^1S$ and $1^1S \rightarrow 2^1P$. They normalise their results to the first Born approximation for the $1^1S \rightarrow 2^1P$ differential cross section at four small angles. Their results agree closely with the first Born approximation and consequently with Holt et al's SRB2 results over the range of angles considered here.

Hidalgo and Geltman (1972) have calculated differential and total cross sections using the coulomb projected Born approximation. The wave functions they use give different first Born cross sections to those presented here, so an exact comparison is not possible. However their results have the following form. In the forward direction, their coulomb projected Born results lie approximately 5% below their first Born results, but at larger angles of scattering (>15^{o}) rise significantly above their first Born results.

In Table 8, the total $1^1S \rightarrow 2^1S$ cross sections are compared. As expected from consideration of the differential cross sections, the SFB2 result is much too large. Although the SP1 result appears to be in best agreement with experiment, owing to the shape of the differential cross section this is again probably fortuitous. The SRB2 results lie at least 5% lower than the first Born results at all energies considered, whereas Holt et al's SRB2 results lie within 1% of their first Born results. However none of the Born calculations lie within the error brackets (± 10%) of the experimental data, even at 400 eV.

The coulomb projected Born total cross sections lie below their respective first Born total cross sections at all energies considered, but the correction is only 60% of that to the first Born approximation by the SRB2 approximation. The second order potential results of Berrington et al (1972) give best agreement with the experimental data of Vriens et al, but are still outside the error brackets of the experiment.

5.4 Excitation of the 2^1P state of Helium

In this case, it proved difficult to calculate the imaginary part of the second Born correction arising from the 2^1P intermediate state (c.f. sections 3.6 and 4.4). Since this is expected to be an important contribution to the imaginary part, only SRB2 cross sections were calculated.

The differential cross sections are compared with experiment, Holt et al (1971 b) and Berrington et al (1972) in Table 9. The SRB2 results are a considerable improvement on the first Born results and are also superior to those of Holt et al, but still over estimate the forward peak. The second order potential differential cross sections lie slightly above the SRB2 ones at all angles, but below the first Born cross sections. The coulomb projected Born results lie below the first Born results in the forward direction but at angles of scattering greater than 15° lie well above the first Born differential cross sections. This is in contrast to the SRB2 differential cross sections which become negative, and hence meaningless, at angles greater than about 30°.

In Table 10 total cross sections are compared. The SRB2 total cross sections are lower than first Born, whereas Holt et al find their SRB2 total cross sections are larger than their first Born ones. Also the SRB2 results are in good agreement with the experimental results of Vriens et al down to 200 eV, whereas the first Born approximation is outside the quoted errors on this experiment up to 400 eV. Berrington et al's total cross sections lie slightly below their first Born total cross sections at the energies considered here. Hidalgo and Geltman's coulomb projected Born total cross sections lie well above their first Born total cross sections.

In figure 10, the first Born and SRB2 total cross sections are compared with the experimental results of Vriens et al renormalised and the results of three optical experiments (Moustafa Moussa (1967), de Jongh and van Eck (1971), Donaldson et al (1972)). The total cross sections of the optical experiments are made absolute by using the theoretical value for the optical oscillator strength. The SRB2 results are in good agreement with the recent results of Donaldson et al (1972) at all energies except the lowest considered (200 eV). The results of de Jongh and van Eck (1971) lie between the SRB2 and the first Born results while those of Moustafa Moussa (1967) lie below the SRB2 results.

5.5 Excitation of the 3 d and 4 d states of Hydrogen

Since to date, the excitation of d final states has not been investigated in the second Born approximation, results for collisions with both Hydrogen and Helium atoms are presented. The calculations have been performed using two different choices of Δ_{av} in the simplification approximation (c.f. section 2.4). Those results denoted by SRB2 used values of Δ_{av} chosen so that $R\ell\, f_{ii}^{SB2}(k_i^{\,2}, 0)$ takes the exact value given by Holt (1972). That is, Δ_{av} is chosen so that the simplified second Born scattering amplitude takes the correct value for forward elastic scattering. As in the case of Helium atom collisions, it is found that Δ_{av} is dependent on the incident electron energy and for incident electrons having wave numbers 2, 3, 4 and 5, Δ_{av} takes the values 1.07, 1.5, 2.0 and 2.45. The results denoted by HM were obtained by setting $\Delta_{av} = 0.5$ for all incident electron energies. This choice is very similar to that of Holt and Moiseiwitsch (1968) who choose Δ_{av} to be the eigen energy of the first state not explicitly included in the summation (i.e. if all states up to 5p are included explicitly then Δ_{av} = E(6p) = 0.46811). Also calculations have been performed in the truncation approximation. That is the infinite summation occurring in the expression for the second Born correction to the scattering amplitude is truncated, after having included those states whose contributions to the sum are expected to be large.

Total cross sections for the excitation of the 3d state of Hydrogen are presented in Table 11. Including s intermediate states in the truncation approximation has little effect on the total cross sections whereas the p intermediate states give significant contributions. In the truncation approximation the convergence is slow as the number of p states included increases and the total cross sections are decreased at all energies relative to the first Born results. The convergence in the SRB2 approximation is better and except at the highest incident electron energy the total cross sections lie below the first Born ones. The HM total cross sections lie above the first Born ones at all energies. The total cross sections for the excitation of the 4d state of Hydrogen are presented in Table 12 and the various approximations behave in a similar fashion to that for the excitation of the 3d state. The truncation and SRB2 approximations decrease the total cross section relative to first Born whilst the HM approximation increases it.

Bhadra and Ghosh (1971) have used the Glauber approximation to calculate total cross sections for the excitation of the 3d state of Hydrogen by electron impact. They find that the Glauber results lie below the first Born results at all incident electron energies, being 13% below at 50 eV ($k_i \simeq 2$). However the Glauber result tends rapidly to the first Born result with increasing incident electron energy, being indistinguishable from it at energies above 200 eV ($k_i \simeq 4$).

5.6 Excitation of the 3^1D and 4^1D states of Helium

Differential cross sections for the excitation of the 3^1D state are shown in Figures 11 and 12, both in the SRB2 approximation and the HM approximation with $\Delta_{av} = 0.88974$. The HM values always lie below the first Born results and tend to very small values in the forward direction. At the lowest energy the HM approximation fails in the forward direction giving a negative value of $I(0,k_i^2)$. The SRB2 approximation is not reliable at large scattering angles where $I(0,k_i^2)$ goes negative. At all the energies considered the SRB2 results lie below both the first Born and the HM approximation for $\theta > 5^o$ but are approximately 50% larger at small angles than the first Born values.

The total cross section results for 3^1D excitation are shown in Figure 13 where they are compared with the experimental results of Moustafa Moussa (1967), St. John et al (1964) and the first Born results. Numerical values are given in Table 13. All the theoretical predictions lie below Moustafa Moussa's experimental values (and consequently below those of St. John et al) and are outside their quoted experimental errors. The SRB2 cross sections lie below the first Born values and the HM results lie between the SRB2 and the first Born values. The second Born correction is small at impact energies above 400 eV but the SRB2 values lie 40% below the first Born values at 200 eV. Renormalising Moustafa Moussa's experimental values to the first Born approximation at 1 keV brings them into close agreement with the first Born values at all energies down to 200 eV but they remain inconsistent with either the SRB2 approximation or the HM approximation below 300 eV.

Differential cross sections for the 4^1D excitation are shown in Figures 14 and 15 and need not be discussed in detail, the pattern following that of the 3^1D case.

The corresponding total cross sections are shown in Figure 16 and the numerical values presented in Table 14. The second Born results differ little from the first Born results above 400 eV and are 10%

lower at 200 eV. All three theoretical results lie between the experimental values of Thomas and Bent (1967) and Hasselkamp et al (1971), which differ among themselves by 60% at 200 eV. The earlier results of Moustafa Moussa (1967) are in reasonable agreement with those of Hasselkamp et al (1971). The recent results of van Raan et al (1971) are a factor of two higher than the theoretical values at 200 eV but tend to Moustafa Moussa's values at higher energies. The measurements of St. John et al (1964) lie higher still and appear to have a different energy dependence. A renormalisation of the experimental results to the first Born values at 1 keV as suggested by Bell et al (1968) would appear to be plausible.

5.7 Excitation of Model Doubly Excited States of Helium

Helium is the simplest atom for which two electron processes may occur, i.e. double excitation, simultaneous excitation and ionization. Owing to the techniques used in evaluating the second Born correction it is necessary to use simple wave functions to represent the doubly excited states of Helium and consequently the results presented here are intended as a guide to the magnitude of the second Born correction rather than as absolute values.

The doubly excited states of Helium are embedded in the $(He^{+}(1s) + e)$ continuum and consequently one would expect there to be considerable interaction between the doubly excited states and the continuum. The wave functions used for the doubly excited states are suitably symmetrised products of hydrogenic orbitals and consequently the interaction with the continuum is not taken into account. The first Born results for the excitation of the $(2s,2p)^1P$, $(2s,3p)^1P$ and $(3s,2p)^1P$ states agree with those obtained by Gillespie (1971) using the same wave functions. He also reports first Born calculations using a Hylleraas ground state wave function for Helium which leads to significantly different absolute values.

The n=2 level of He^+ is degenerate (2s and 2p) so one would expect two separate series of excited Helium states to converge to this limit: namely (2s,np) and (2p,ns) . However experimentally Madden and Codling (1963) found only one such series. Cooper et al (1963) postulated that the series (2s,np) and (2p,ns) interact to form the new series (2n+) and (2n-) defined by

$$\Psi(2n\pm) = \frac{1}{\sqrt{2}}\left\{\Psi(2s,np) \pm \Psi(2p,ns)\right\}$$

where the $(1s)^2 \rightarrow (2n-)$ optical transition is quasi-forbidden. Using the wave functions described in section 3.2, the $(1s)^2 \rightarrow (2n+)$ optical transition is quasi-forbidden. This is in apparent contradiction with the calculations of Altick and Moore (1966) and Burke and McVicar (1965), who used more accurate wave functions which included the interaction with the continuum.

The calculated differential cross sections at 200 eV are shown in Figure 17. The intermediate states explicitly included are $(1s)^2$, (1s,2s,), (1s,3s), (1s,2p), (1s,3p), (2s,2p), (2s,3p) and (3s,2p). In each case the upper curve is the first Born result and the lower one the SRB2 result. The SRB2 differential cross sections become negative at scattering angles greater than 25^o.

Total cross sections are presented in Table 15. In no case is the second Born correction negligible at any energy below 1 keV while at the lowest energy considered (200 eV) the SRB2 results are almost a factor of two smaller than those obtained in the first Born correction. This is not unexpected since the first Born result is identically zero for double excitation if uncorrelated seperable wave functions are used, whereas the second Born correction would be non-zero. The total cross sections are very slowly varying with incident electron energy in the range considered.

The process of simultaneous excitation and ionization is of interest in auroral physics and has only been treated in the first Born approximation. The correction at 200 eV for the sequence (2s,np) seems only weakly dependent on n and it could be argued that it is likely to remain significant for large n and hence for (2s,kp) $k^2 > 0$, which is a typical example of a simultaneous excitation and ionization process.

5.8 Conclusions

It has been shown in section 2.3 that the second Born scattering amplitude satisfies certain identities and these identities may be used to calculate the "average energy" parameter Δ_{av} to be used in a simplifying approximation similar to that of Holt and Moiseiwitsch (1968). With this choice of Δ_{av} the simplified second Born elastic scattering amplitude takes the correct value in the forward direction. For elastic scattering and excitation of the 2^1S state of Helium various approximations were used to obtain differential cross sections from the second Born scattering amplitudes. Of these, the third order approximation of Kingston et al (1960)gave best agreement with the experimental data of Vriens et al (1968a, b) renormalised to Chamberlain et al (1970).

The results using this choice of Δ_{av} and the third order approximation (to be denoted by SRB2) give poor agreement with experimental elastic scattering data but improved agreement for the $1^1S \rightarrow 2^1S$ and $1^1S \rightarrow 2^1P$ inelastic transitions. For these cases, the second Born correction decreases the first Born total cross section by at least 3% at impact energies of forty times the threshold, and consequently experimental relative cross sections normalised to first Born at or below this energy may be less reliable than is usually supposed.

For excitation of the ^1D states of Helium it is not possible to draw unambiguous conclusions either on the reliability of the second Born correction from experiment, or on the accuracy of a particular experiment from the second Born calculations. However it would appear to be plausible to normalise relative experimental measurements to the first Born values at energies above 1 keV.

For excitation of the model doubly excited states large corrections occur in the SRB2 approximation and it is suggested that similar corrections to the first Born approximation will arise in considering simultaneous excitation and ionization at low and intermediate energies.

TABLE 1

A comparison between the truncated, Holt and Moiseiwitsch and exact forward elastic scattering amplitudes for electron-hydrogen atom collisions.

$k_i(a_o^{-1})$	$R\ell\, f_{ii}(0,k_i^2)$			$Im\, f_{ii}(0,k_i^2)$		
	Truncated*	HM+	Exact	Truncated*	HM+	Exact
3	0.664	1.149	1.207	0.815	1.571	1.456
4	0.501	0.843	0.878	0.708	1.264	1.334
5	0.401	0.665	0.689	0.625	1.160	1.111

*All states up to and including those with principle quantum number three summed.

+All states up to and including those with principle quantum number 100 explicitly included.

TABLE 2

A comparison between the Holt and Moiseiwitsch and dispersion relation real forward elastic scattering amplitudes for electron-helium atom collisions.

Electron energy (eV)	HM+	Dispersion Relation
200	2.245	1.706
300	1.956	1.482
400	1.788	1.360
500	1.676	1.286

+ $f_{ii}^{B1}(0,k_i^2) + f_{ii}^{B2}(0,k_i^2)$

TABLE 3

Values of Δ_{av} (in atomic units) used in this thesis.

Electron energy (eV)	Rℓ f			Im f		
	N=1	N=2	N=3	N=1	N=2	N=3
200	5.65	6.7	7.1	3.15	3.6	3.8
300	7.4	8.9	9.55	3.2	3.7	3.9
400	8.9	10.9	11.65	3.5	4.1	4.35
500	10.2	12.5	13.45	3.6	4.45	4.7
1000	15.05	18.75	20.3	4.55	5.5	5.9

TABLE 4

Coefficients of the Cohen-McEachran and modified $(1s,2s)2^1S$ wave function.

	a_1	a_2	a_3	a_4	$\langle 1s \| 2s \rangle$
Cohen-McEachran	1.0	2.0986	-3.8653(-1)	4.6059(-2)	-1.504(-2)
Modified	1.0	2.2786	-3.6640(-1)	5.1369(-2)	-1.035(-6)

The figures in parenthesis indicate the power of ten by which that entry should be multiplied.

TABLE 5

First Born cross sections for the $1^1S \rightarrow 2^1S$ process in units of $10^{-3}\ \pi\ a_o^2$.

Energy (eV)	200	300	400	500
Bell et al.	11.7	8.0	6.0	4.8
Cohen-McEachran function	9.7	6.6	5.0	4.1
Modified function	11.6	7.9	6.0	4.8

TABLE 6

Nuclear charges and eigen energies used in the wave functions of the singly excited d states and the model doubly excited states.

State	α	β	$\varepsilon_n^{\dagger}$ (a.u.)
(1s,3d)	2.0	1.02	-0.05564
(1s,4d)	2.0	1.02	-0.03128
(2s,2p)	2.0	1.58	1.3121
(2s,3p)	2.0	1.0	1.421
(3s,2p)	1.2	2.0	1.421

†Energies are relative to the single ionization threshold.

TABLE 7

Total cross sections for the elastic scattering of electrons by Helium atoms (in πa_o^2) .

Energy (eV)	200	300	400	500	1000
First Born	0.221	0.151	0.115	0.0922	0.0468
SFB2	0.633	0.353	0.234	0.172	---
SRB2	0.430	0.252	0.175	0.133	0.0593
SP1	0.240	0.158	0.121	0.0977	---
Holt et al.	0.352	0.211	0.149	0.115	---
Berrington et al.	0.215	0.147	0.107	0.088	0.045
Experiment†	0.251	0.171	0.128	0.108	---

† 200, 300 and 400 eV, Vriens et al (1968b) renormalised to Chamberlain et al (1970) at 5^o. 500 eV Bromberg (1969).

TABLE 8

Total cross section for the excitation of the 2^1S state of Helium by electron impact (in $10^{-3}\ \pi a_o^2$).

Energy (eV)	200	300	400	500	1000
First Born	11.6	7.87	5.97	4.81	2.40
SFB2	20.6	12.0	8.34	6.35	---
SRB2	10.4	7.28	5.62	4.57	2.33
SP1	7.02	5.40	4.42	3.73	---
Holt et al.	11.63	7.84	5.92	4.75	---
Berrington et al.	9.26	6.64	5.20	4.24	2.23
Experiment†	6.7	5.3	4.7	---	---

† Vriens et al (1968a) renormalised.

TABLE 9

Differential cross sections for the excitation of the 2^1P state of Helium by electron impact (in a_o^2 / sr).

Energy (eV)	200			400		
Angle	0^o	5^o	10^o	0^o	5^o	10^o
First Born	15.13	3.47	0.69	32.96	1.90	0.21
SRB2	13.43	3.00	0.56	30.86	1.73	0.17
Holt et al.	14.74	3.47	0.67	31.83	1.93	0.21
Berrington et al.	13.8	3.3	0.65	---	1.83	0.18
Experiment†	11.90	2.91	0.56	28.94	1.78	0.18

†Vriens et al (1968a) normalised.

TABLE 10

Total cross sections for the excitation of the 2^1P state of Helium by electron impact (in 10^{-2} π a_o^2).

Energy (eV)	200	300	400	500	1000
First Born	10.6	8.3	7.0	6.1	3.56
SRB2	8.8	7.4	6.4	5.6	3.39
Holt et al†	10.91	8.55	7.11	6.14	3.72
Berrington et al.	10.5	8.22	6.81	5.81	3.62
Vriens et al. ‡	8.6	7.4	6.2	--	--

†The values quoted are those given by Hunt (1971). Columns 3 and 4 of Table 3 of the paper by Holt et al (1971b) appear to have been interchanged.

‡Vriens et al (1968a) renormalised.

TABLE 11

Total cross section for the excitation of the 3d state of Hydrogen by electron impact (10^{-3} π a_o^2).

Approximation	States explicitly included.	Wave number (a_o^{-1}) 2	3	4	5
First Born	---	13.65	6.882	4.032	2.628
Truncation	1s→3s.	13.85	6.873	4.018	2.619
Truncation	1s→3s,2p→3p.	9.466	5.723	3.653	2.479
Truncation	1s→3s,2p→5p.	11.45	6.212	3.811	2.541
SRB2	1s→3s,2p→3p.	9.184	6.138	3.899	2.637
SRB2	1s→3s,2p→5p.	10.50	6.367	3.950	2.641
HM	1s→3s,2p→5p.	14.42	7.181	4.181	2.707

TABLE 12

Total cross section for excitation of the 4d state of Hydrogen by electron impact (10^{-3} π a_o^2) .

Approximation	States explicitly included.	Wave number (a_o^{-1}) 2	3	4	5
First Born	---	6.463	3.261	1.910	1.245
Truncation	1s→3s,2p→5p.	4.614	2.660	1.700	1.160
SRB2	1s→3s,2p→5p.	4.275	2.784	1.782	1.215
HM	1s→3s,2p→5p.	7.127	3.440	1.988	1.284

TABLE 13

Total cross section for the excitation of the 3^1D state of Helium by electron impact (10^{-20} cm^2) .

Approximation	States explicitly included.	Energy (eV) 200	300	400	500	1000
First Born	---	5.133	3.579	2.732	2.200	1.102
SRB2	1s→3s,2p→6p.	3.287	2.760	2.266	1.888	1.032
HM	1s→3s,2p→5p.	3.878	3.040	2.443	2.025	1.069

TABLE 14

Total cross section for the excitation of the 4^1D state of Helium by electron impact (10^{-20} cm^2).

Approximation	States explicitly included.	Energy (eV) 200	300	400	500	1000
First Born	---	2.729	1.905	1.455	1.172	0.587
SRB2	1s→4s,2p→6p.	2.366	1.779	1.427	1.178	0.597
HM	1s→4s,2p→5p.	2.008	1.591	1.285	1.068	0.566

TABLE 15

Total cross section for the excitation of the model doubly excited states of Helium by electron impact (πa_o^2).

Final State	Approximation	200 eV	300 eV	400 eV	500 eV	1000 eV
(2s,2p)	B1	2.00(-3)	1.96(-3)	1.82(-3)	1.68(-3)	1.23(-3)
	SRB2	1.18(-3)	1.43(-3)	1.43(-3)	1.38(-3)	1.10(-3)
(2s,3p)	B1	1.58(-4)	1.64(-4)	1.56(-4)	1.46(-4)	1.10(-4)
	SRB2	0.84(-4)	1.10(-4)	1.14(-4)	1.11(-4)	0.92(-4)
(3s,2p)	B1	4.08(-4)	3.96(-4)	3.66(-4)	3.36(-4)	2.42(-4)
	SRB2	2.11(-4)	2.73(-4)	2.80(-4)	2.70(-4)	2.14(-4)
(23 -)	B1	5.35(-4)	5.34(-4)	4.99(-4)	4.61(-4)	3.38(-4)
	SRB2	2.78(-4)	3.62(-4)	3.73(-4)	3.63(-4)	2.94(-4)
(23 +)	B1	3.03(-5)	2.63(-5)	2.30(-5)	2.04(-5)	1.34(-5)
	SRB2	1.73(-5)	2.03(-5)	1.97(-5)	1.84(-5)	1.32(-5)

The figure in brackets after each entry indicates the power of ten by which that entry is to be multiplied.

FIGURE 1

Variation of $Re\, f_{ii}^{SB2}(0,k_i^2)$ with Δ_{av} for 400 eV electrons incident on atomic Helium.

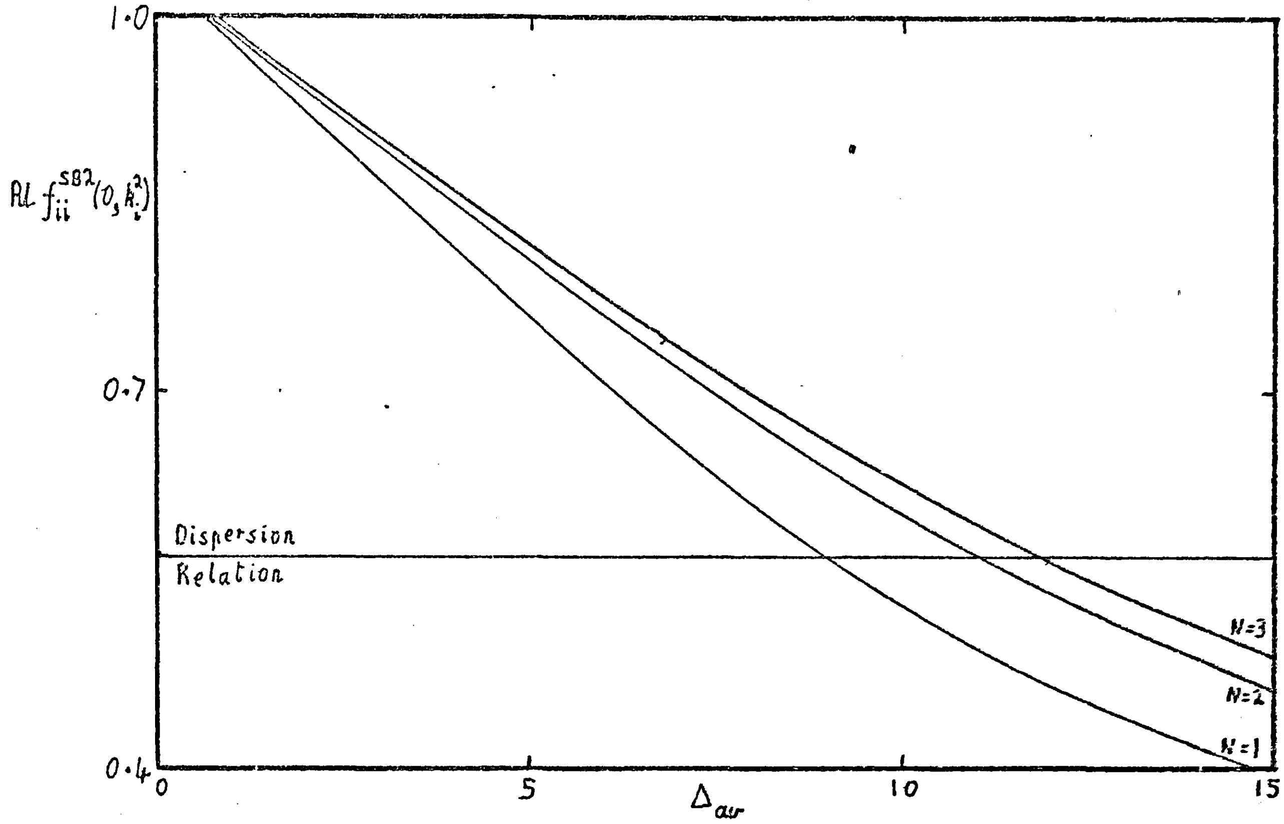


FIGURE 2

Variation of $Im\, f_{ii}^{SB2}(0,k_i^2)$ with Δ_{av} for 400 eV electrons on atomic Helium.

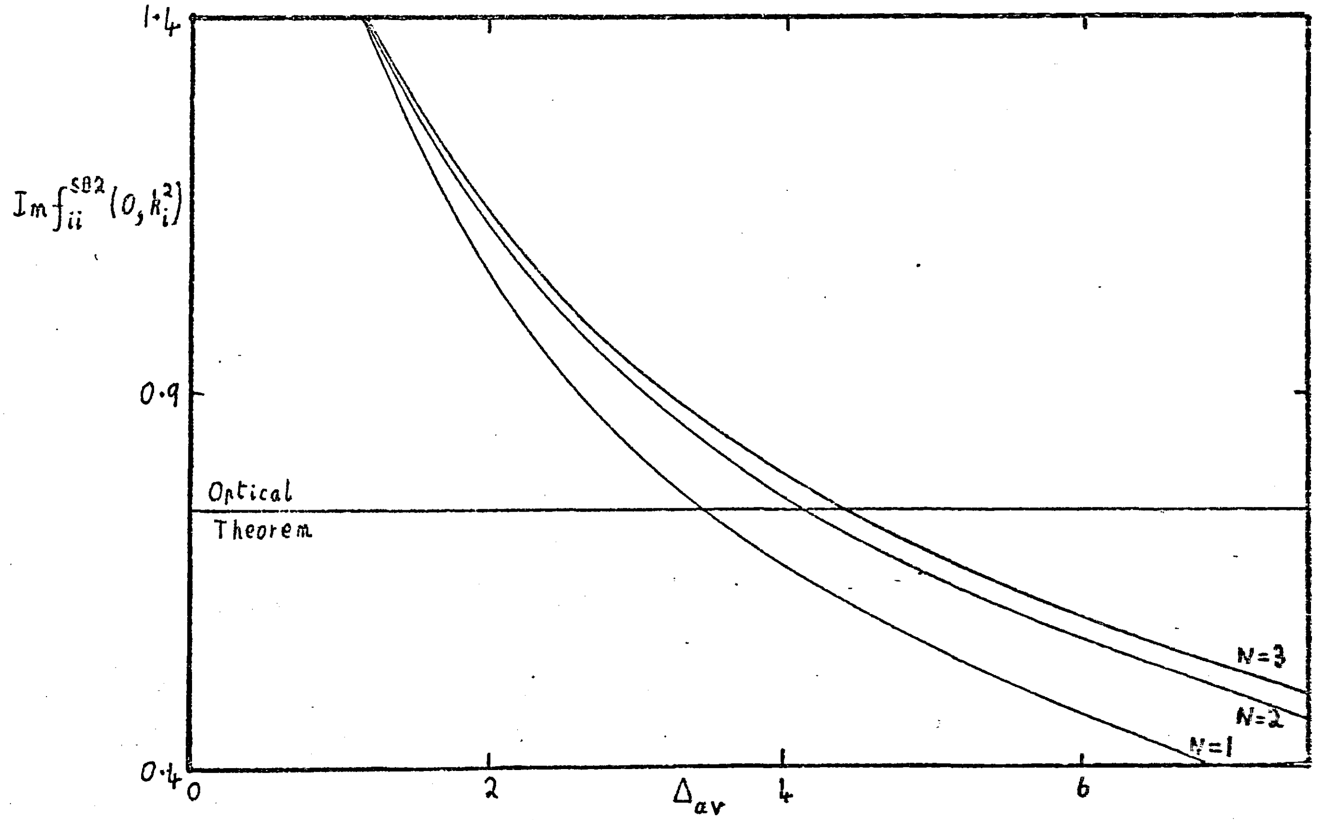


FIGURE 3

Variation of Rl and Im $f_{ii}^{SB2}(0,k_i^2)$ with Δ_{av} for electrons ($k_i = 4 a_0^{-1}$) incident on atomic Hydrogen. ($1^1S, 2^1S, 2^1P$ states explicitly included)

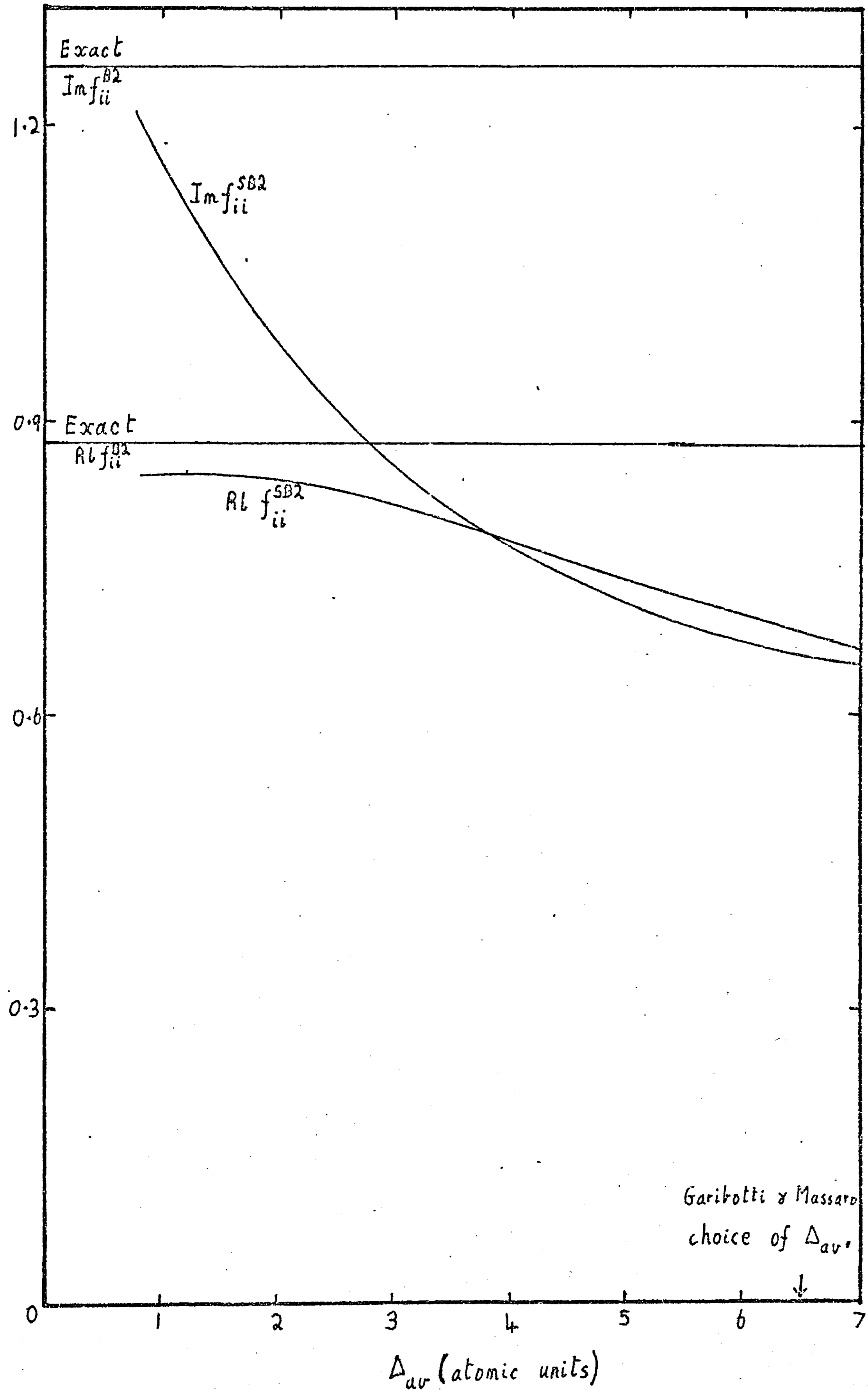


FIGURE 4

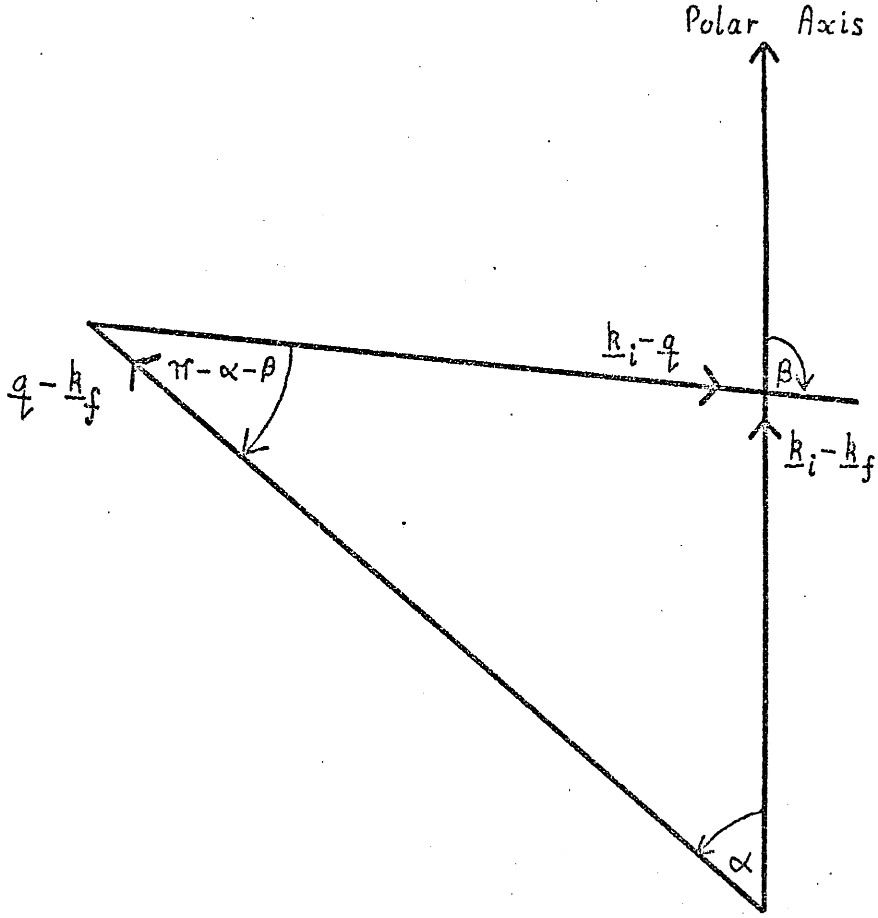

Polar Axis
$\underline{k}_i - \underline{q}$
β
$\pi - \alpha - \beta$
$\underline{q} - \underline{k}_f$
$\underline{k}_i - \underline{k}_f$
α

FIGURE CAPTION

Figure 5. Variation of $R\ell\ f_{if}^{SB2(n)}(\theta,k_i^2)$ with angle of scattering θ for excitation of the 3^1D and 4^1D states of Helium by the impact of 200 eV electrons.

——— First Born scattering amplitude.

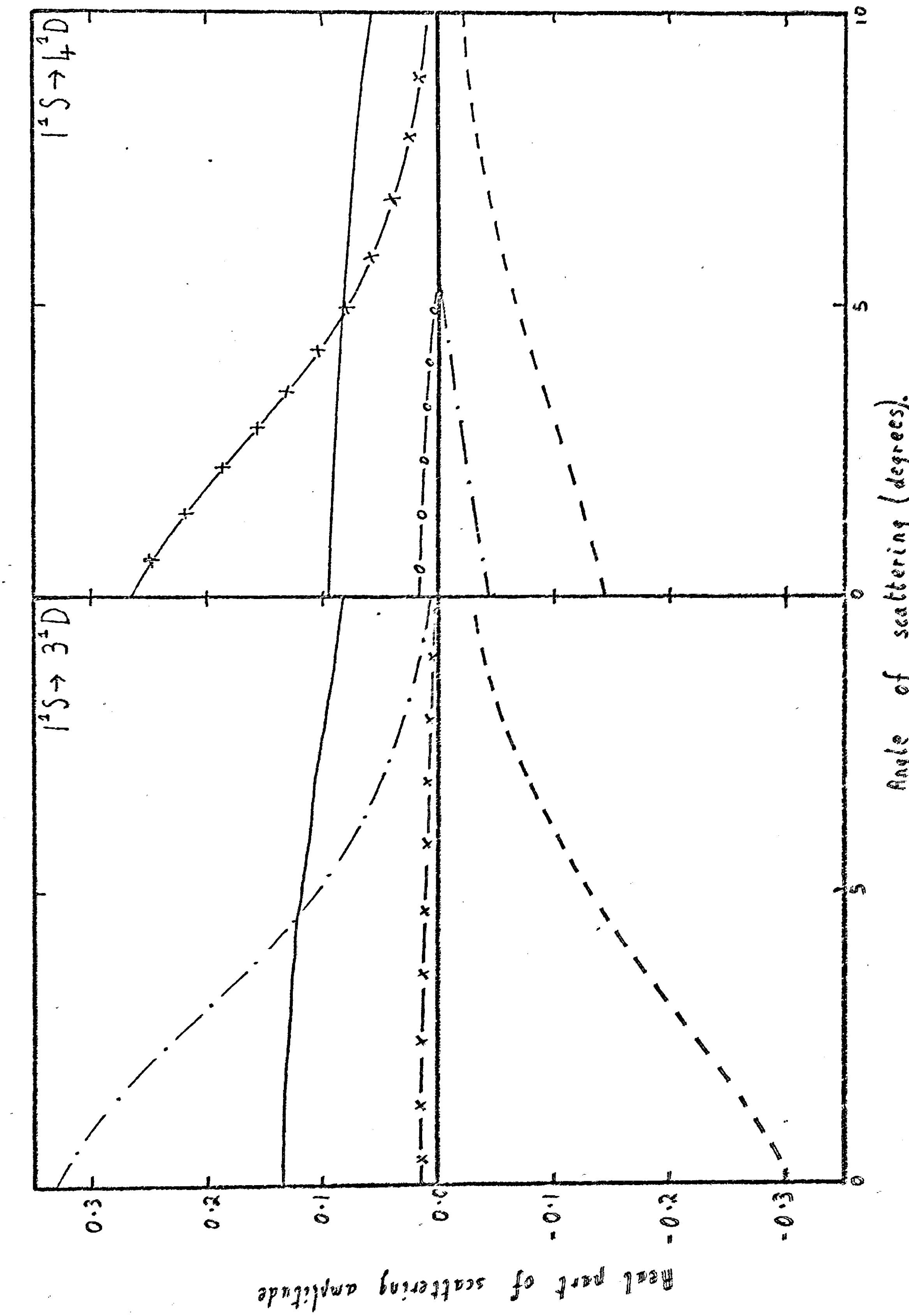
— — — Contribution from 2^1P intermediate state.

—·—·— Contribution from 3^1P intermediate state.

—X—X— Contribution from 4^1P intermediate state.

—O—O— Contribution from 5^1P intermediate state.

FIGURE 5



1¹S → 3¹D
1¹S → 4¹D
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
5
10
0
5
10
Real part of scattering amplitude
Angle of scattering (degrees).

FIGURE CAPTION

Figures 6 and 7. Differential cross sections for the elastic scattering of 500, 400, 300 and 200 eV electrons by Helium (in a_o^2 / sr).

——————— First Born Approximation

— — — SRB2 Approximation

—X—X— SP1 Approximation

—0—0— SFB2 Approximation

—•—•— Holt et al (1971b)

△ Experimental results of Bromberg (1969) at 500 eV, Vriens et al (1968b) renormalised to Chamberlain et al (1970), at 400, 300 and 200 eV.

FIGURE 6

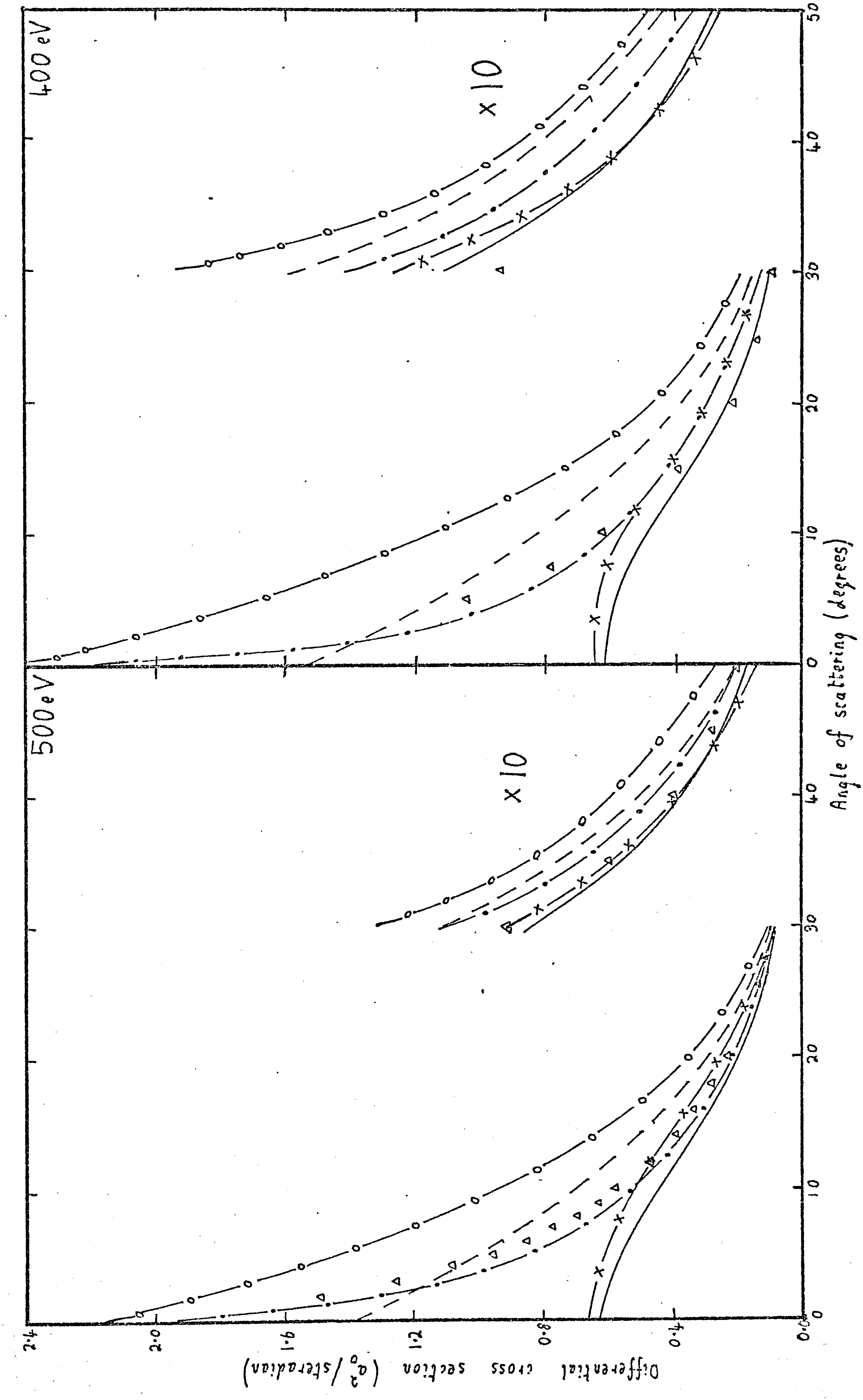

400 eV
500 eV
x 10
x 10
Angle of scattering (degrees)
Differential cross section (a_0^2/steradian)
0
10
20
30
40
50
0·0
0·4
0·8
1·2
1·6
2·0
2·4

FIGURE 7

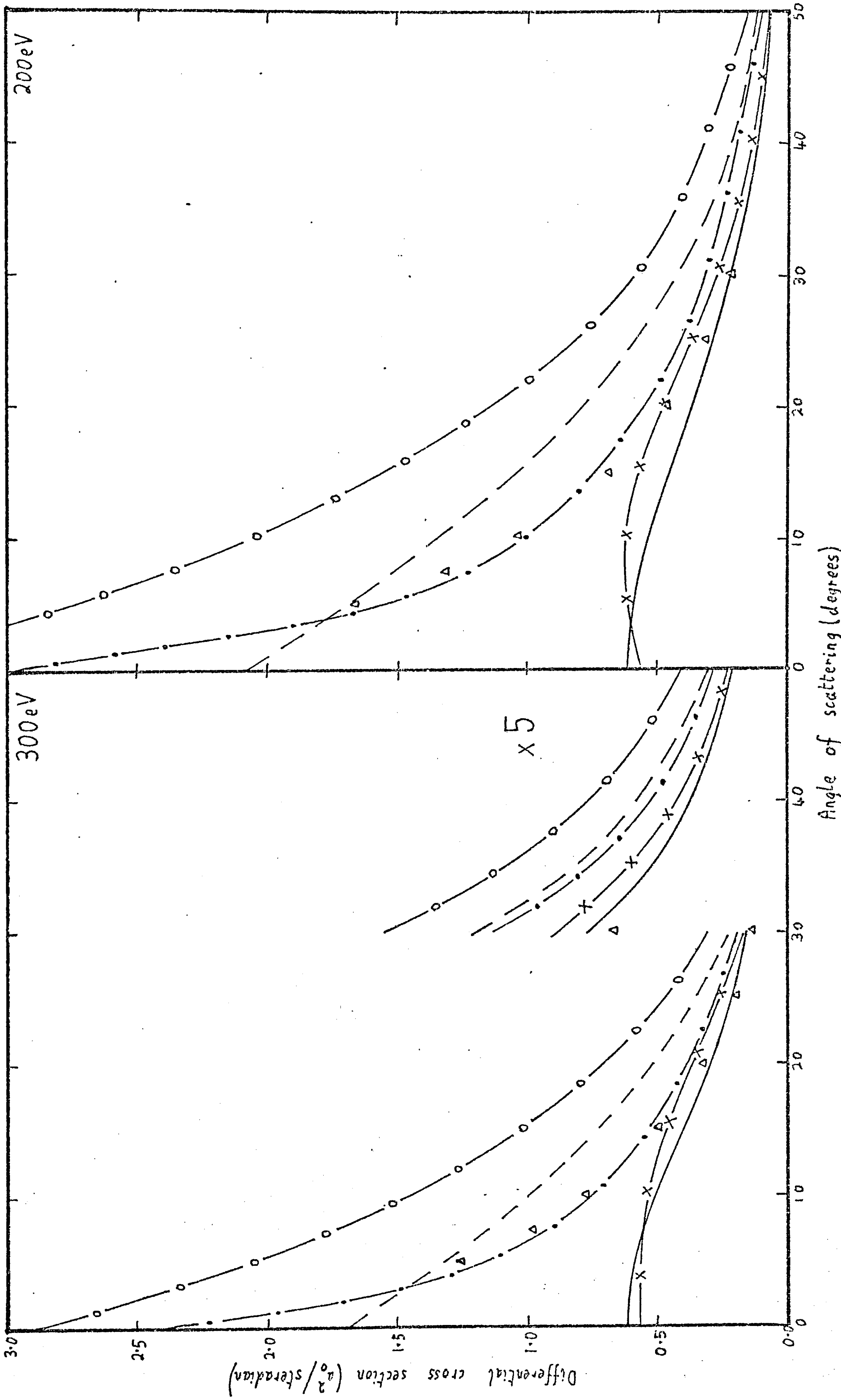

200eV
300eV
×5
Angle of scattering (degrees)
Differential cross section (a_0^2/steradian)
0
10
20
30
40
50
0·0
0·5
1·0
1·5
2·0
2·5
3·0

FIGURE CAPTION

Figures 8 and 9. Differential cross sections for the excitation of the 2^1S state of Helium by the impact of 500, 400, 300 and 200 eV electrons (in a_o^2 / sr).

——— First Born Approximation

— — — SRB2 Approximation

—X—X— SP1 Approximation

—O—O— SFB2 Approximation

—•—•— Berrington et al (1972)

△ Experimental results of Vriens et al (1968a) renormalised to Chamberlain et al (1970).

FIGURE 8

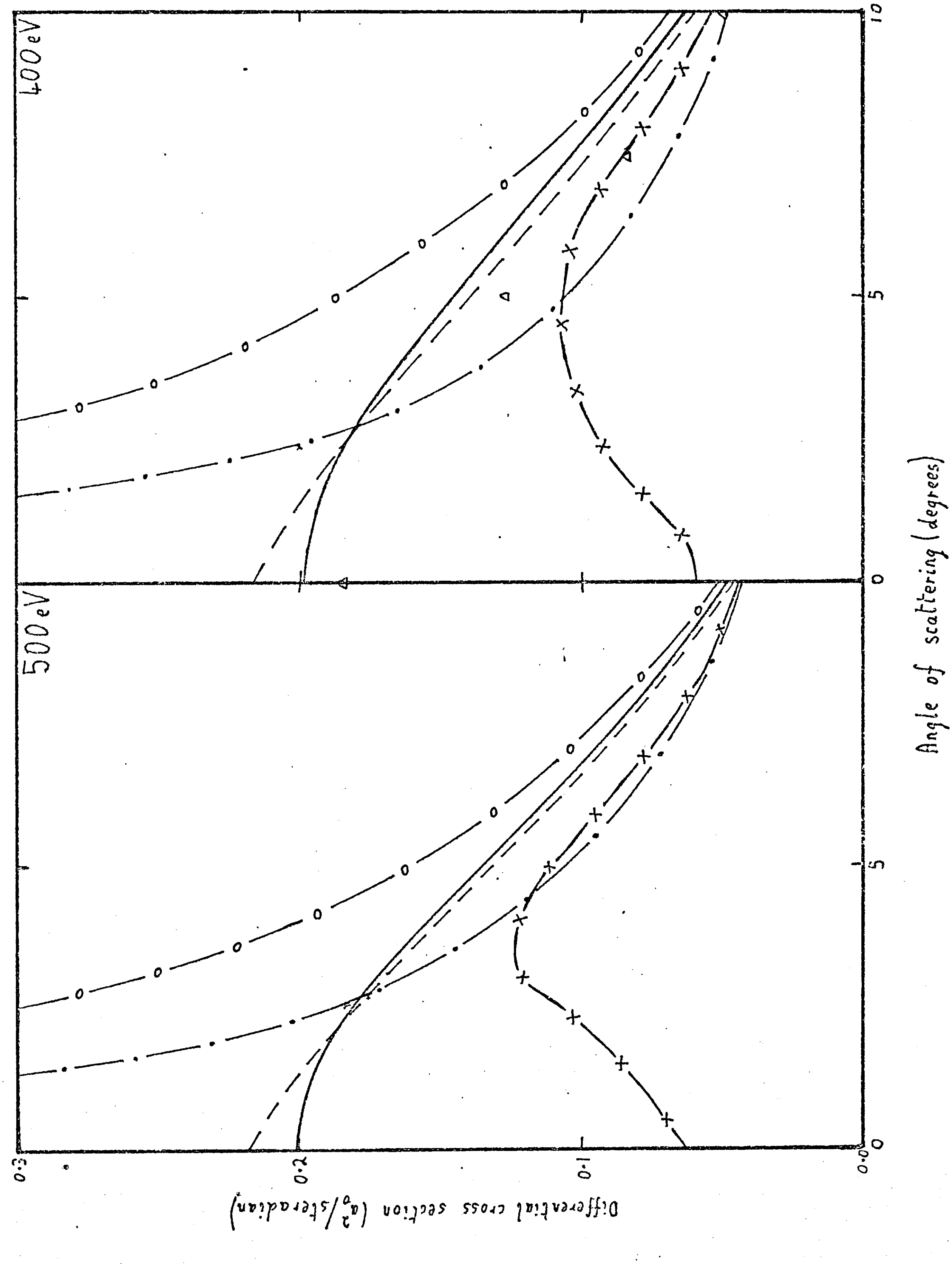

400eV
500eV
Angle of scattering (degrees)
Differential cross section (a_0^2/steradian)
0
5
10
0·0
0·1
0·2
0·3

FIGURE 9

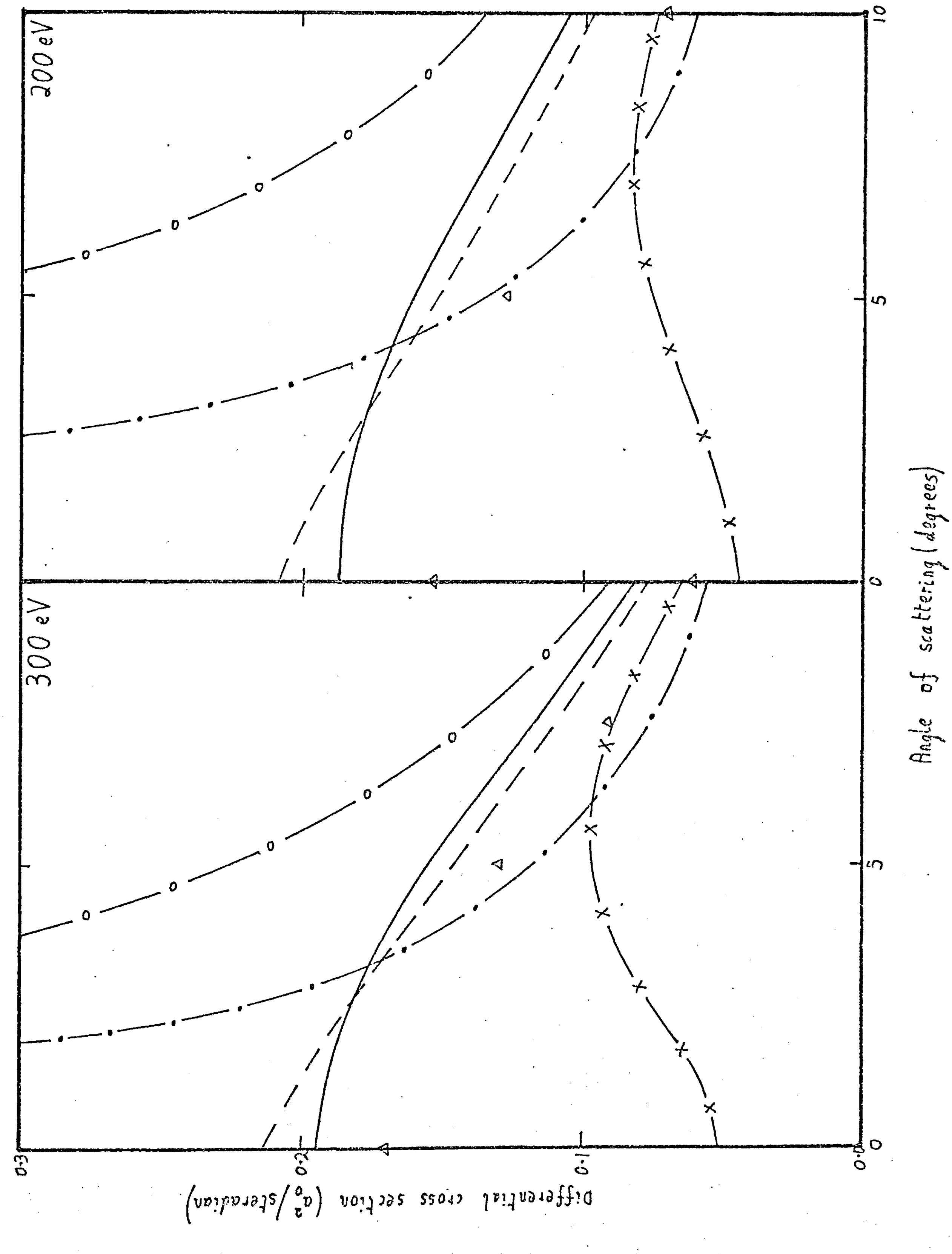

300 eV
200 eV
0
5
10
0
5
0·0
0·1
0·2
0·3
Angle of scattering (degrees)
Differential cross section (a_0^2/steradian)

FIGURE CAPTION

Figure 10. Total cross sections for the excitation of the 2^1P state of Helium by electron impact. (in πa_o^2).

——— First Born Approximation

— — — SRB2 Approximation

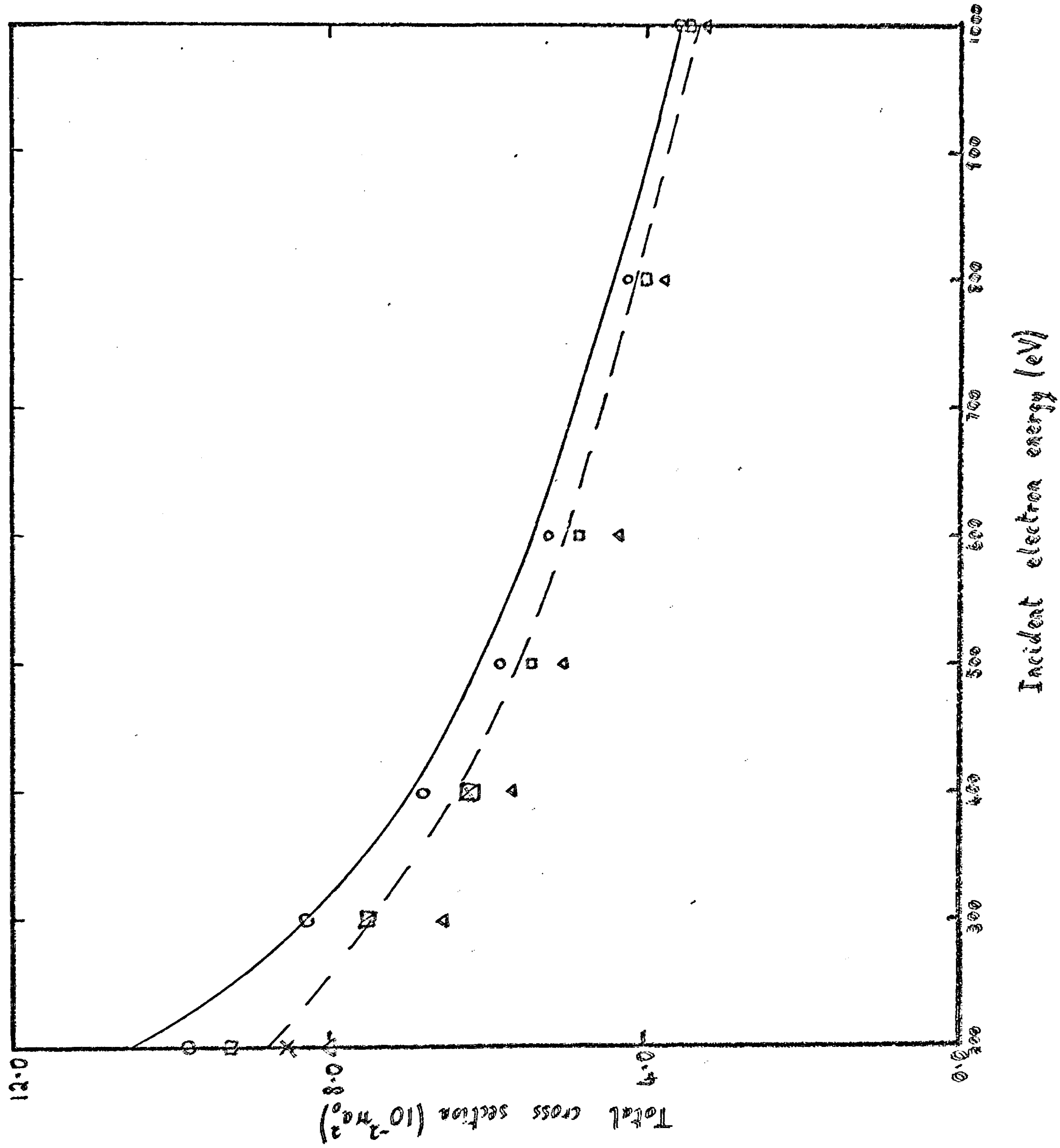
× Vriens et al (1968a) renormalised to Chamberlain et al (1970)

o de Jongh and van Eck (1971)

□ Donaldson et al (1972)

△ Moustafa Moussa (1967)

FIGURE 10



Total cross section ($10^{-2}\pi a_0^2$)
Incident electron energy (eV)
0.0
4.0
8.0
12.0
300
400
500
600
700
800
900
1000

FIGURE CAPTION

Figures 11 and 12. Differential cross sections for the excitation of the 3^1D state of Helium by the impact of 500, 400, 300 and 200 eV electrons (in a_o^2 / sr).

——— First Born Approximation

— — — SRB2 Approximation

—•—•— HM Approximation

FIGURE 11

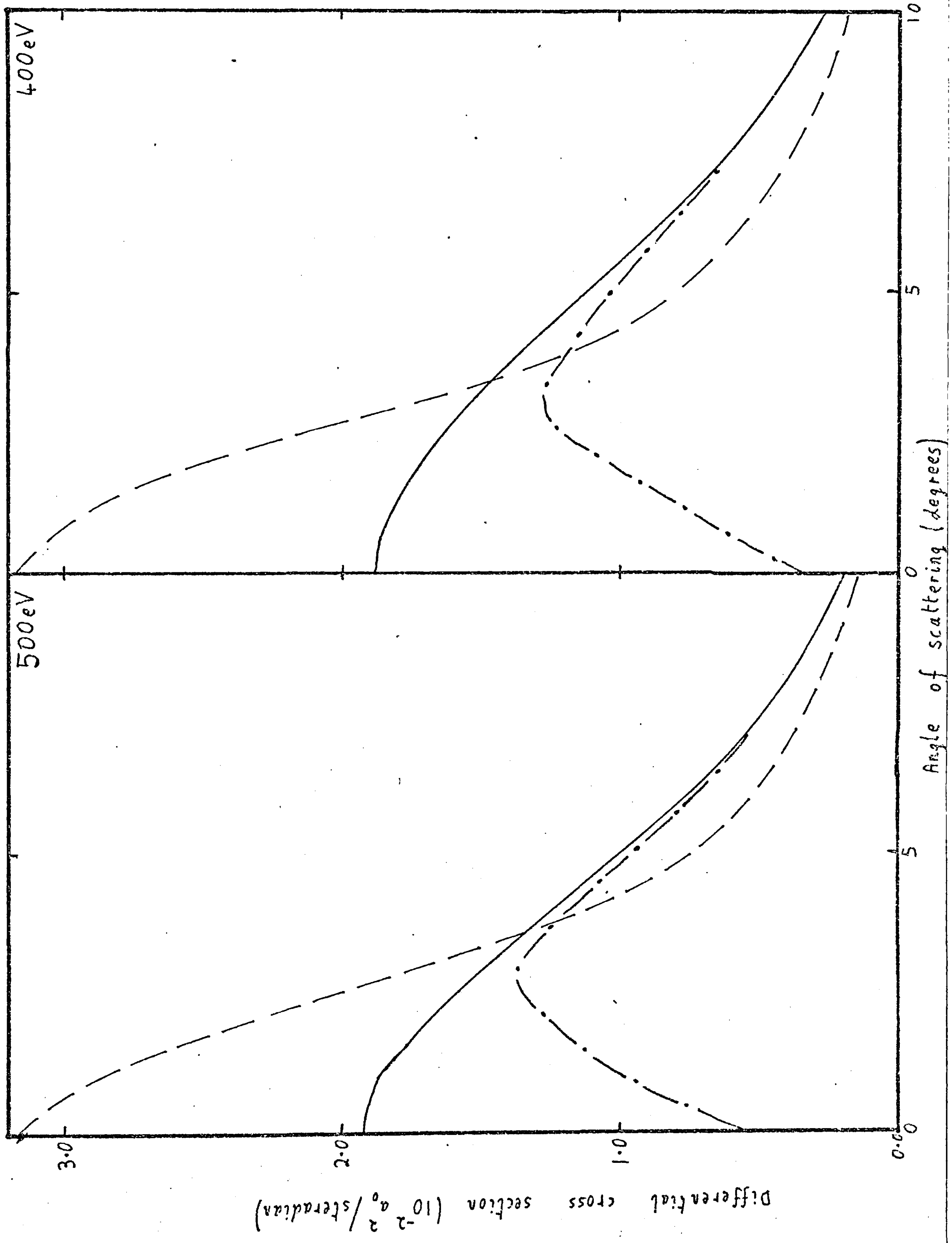

400eV
500eV
0
5
10
0·0
1·0
2·0
3·0
Angle of scattering (degrees)
Differential cross section ($10^{-2} a_0^2$/steradian)

FIGURE 12

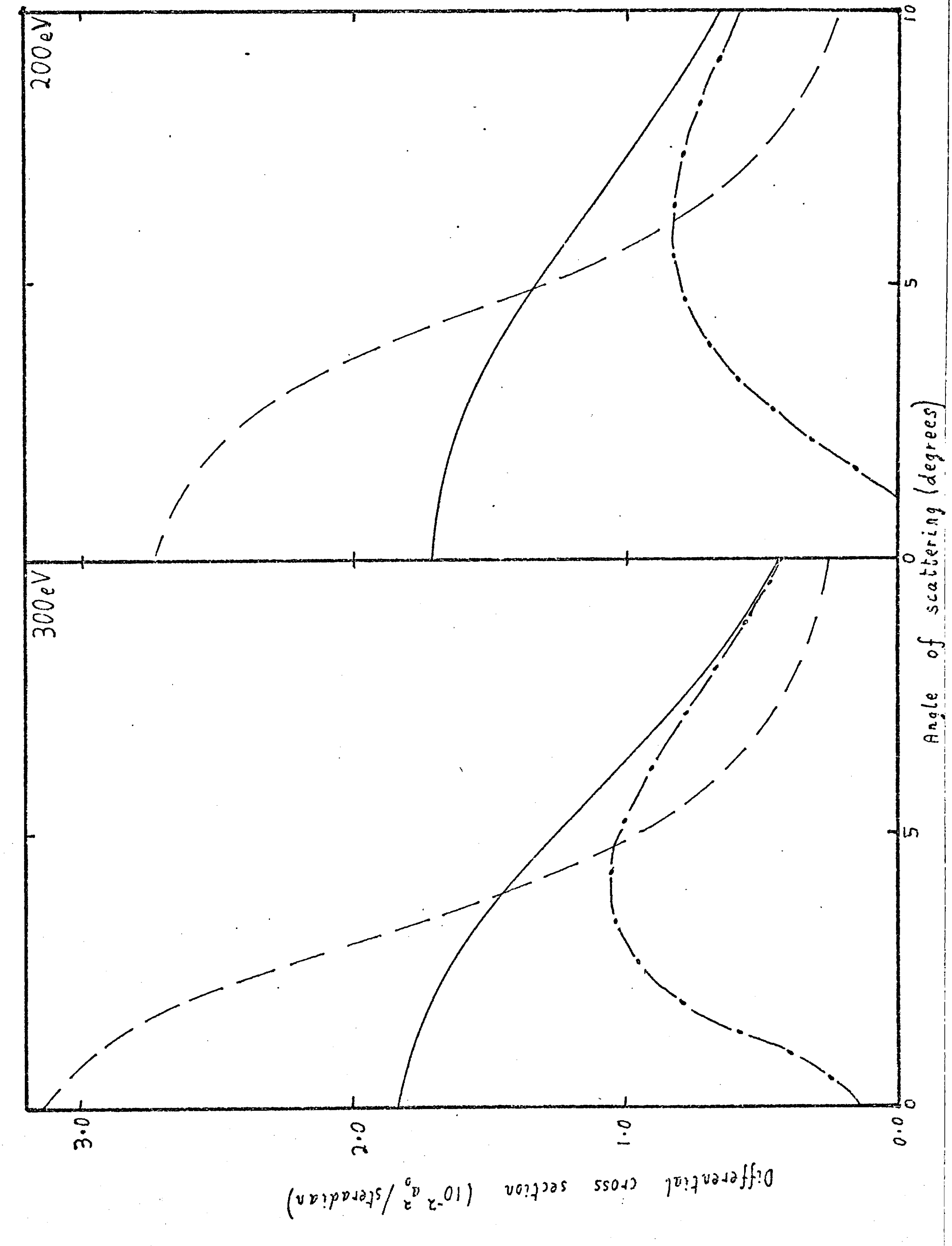

200eV
300eV
0
5
10
0
5
0.0
1.0
2.0
3.0
Angle of scattering (degrees)
Differential cross section ($10^{-2} a_0^2$/steradian)

FIGURE CAPTION

Figure 13. Total cross sections for the excitation of the 3^1D state of Helium by electron impact (in πa_o^2).

——·—— First Born Approximation

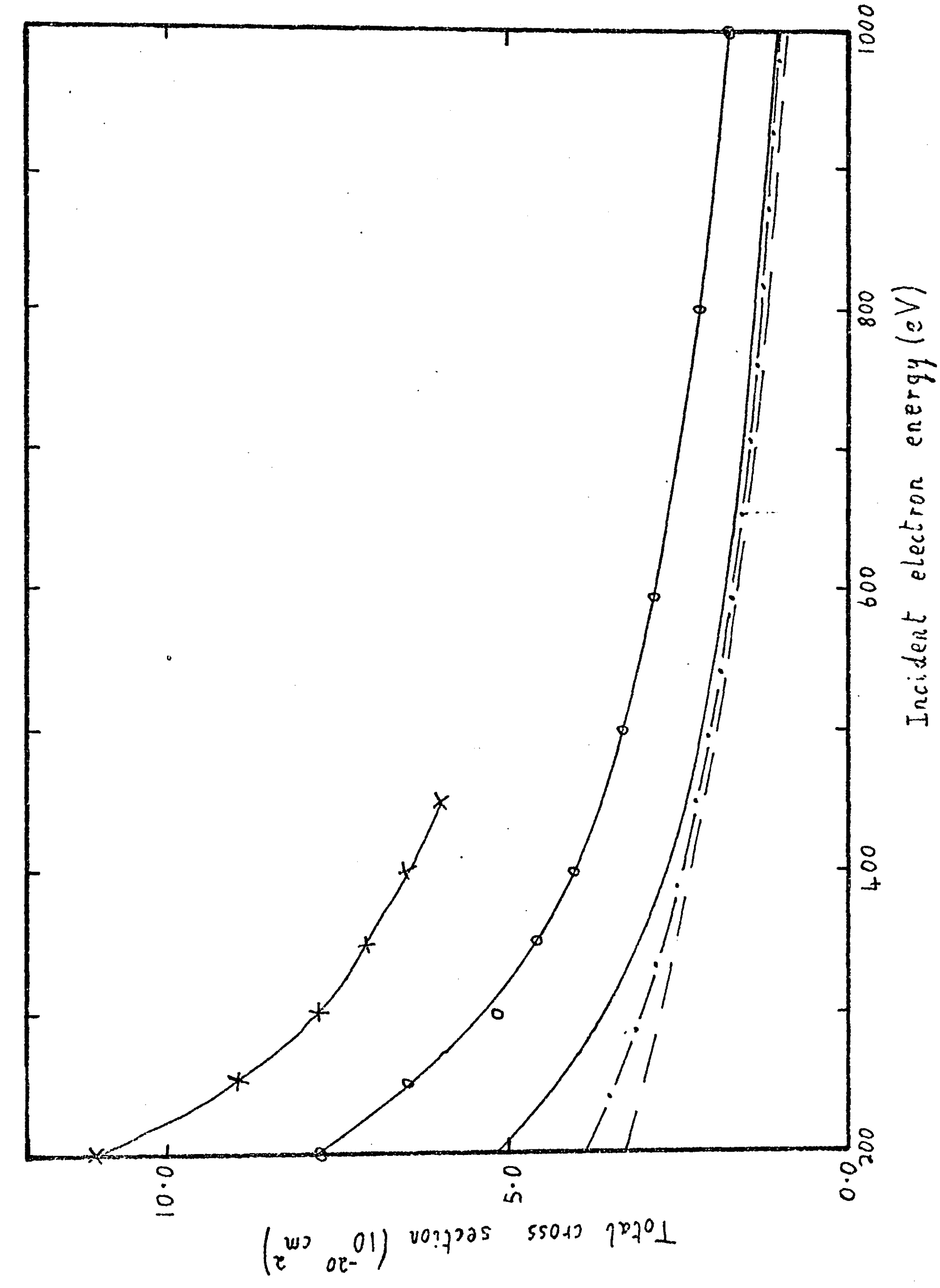
— — — SRB2 Approximation

—•—•— HM Approximation

——×—— St. John et al (1964)

——θ—— Moustafa Moussa (1967).

FIGURE 13



Total cross section (10^-20 cm^2)
0·0
5·0
10·0
200
400
600
800
1000
Incident electron energy (eV)

FIGURE CAPTION

Figures 14 and 15. Differential cross sections for the excitation of the 4^1D state of Helium by the impact of 500, 400, 300 and 200 eV electrons (in a_o^2 / sr).

——— First Born Approximation

— — — SRB2 Approximation

—•—•— HM Approximation.

FIGURE 14

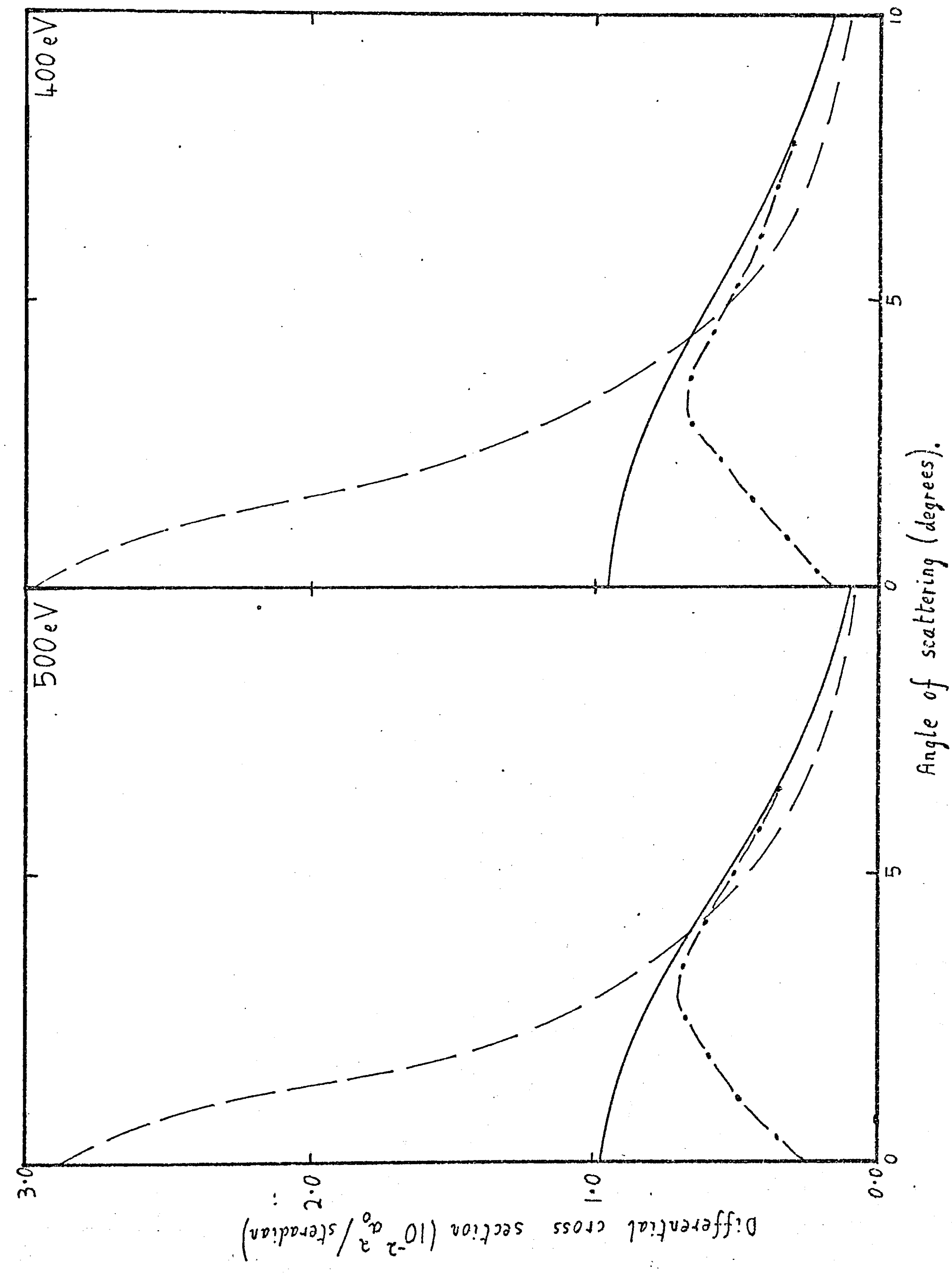

400 eV
500 eV
0
5
10
0.0
1.0
2.0
3.0
Angle of scattering (degrees).
Differential cross section ($10^{-2} a_0^2$/steradian)

FIGURE 15

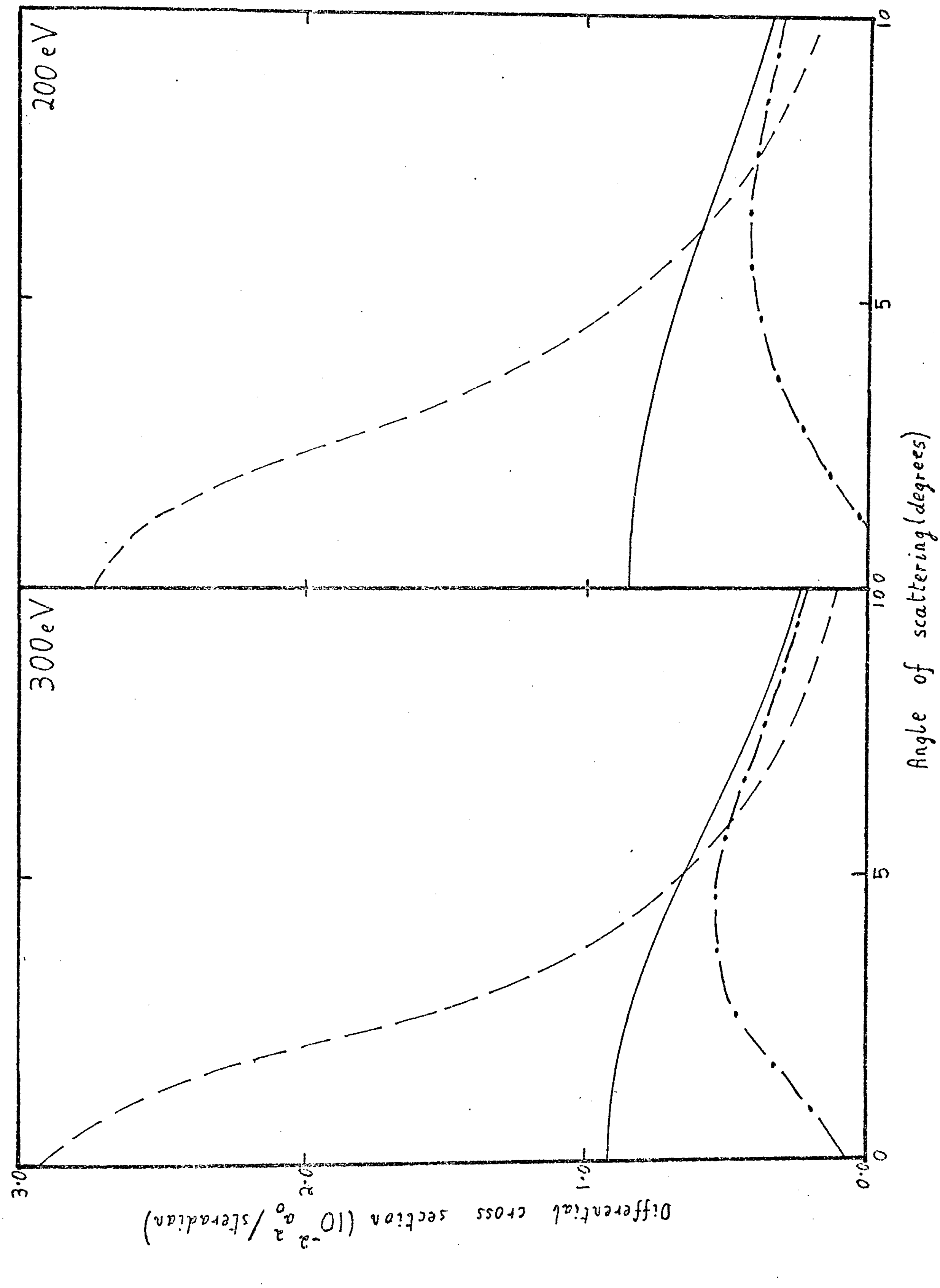

200 eV
300 eV
Angle of scattering (degrees)
Differential cross section ($10^{-2}\ a_0^2$/steradian)
0
5
10
0
5
10
0·0
1·0
2·0
3·0

FIGURE CAPTION

Figure 16. Total cross section for the excitation of the 4^1D state of Helium by electron impact (in πa_o^2).

— — — SRB2 Approximation

—×— van Raan et al (1971)

—o— Moustafa Moussa (1967)

—•— Hasselkamp et al (1971)

—△— Thomas and Bent (1967).

FIGURE 16

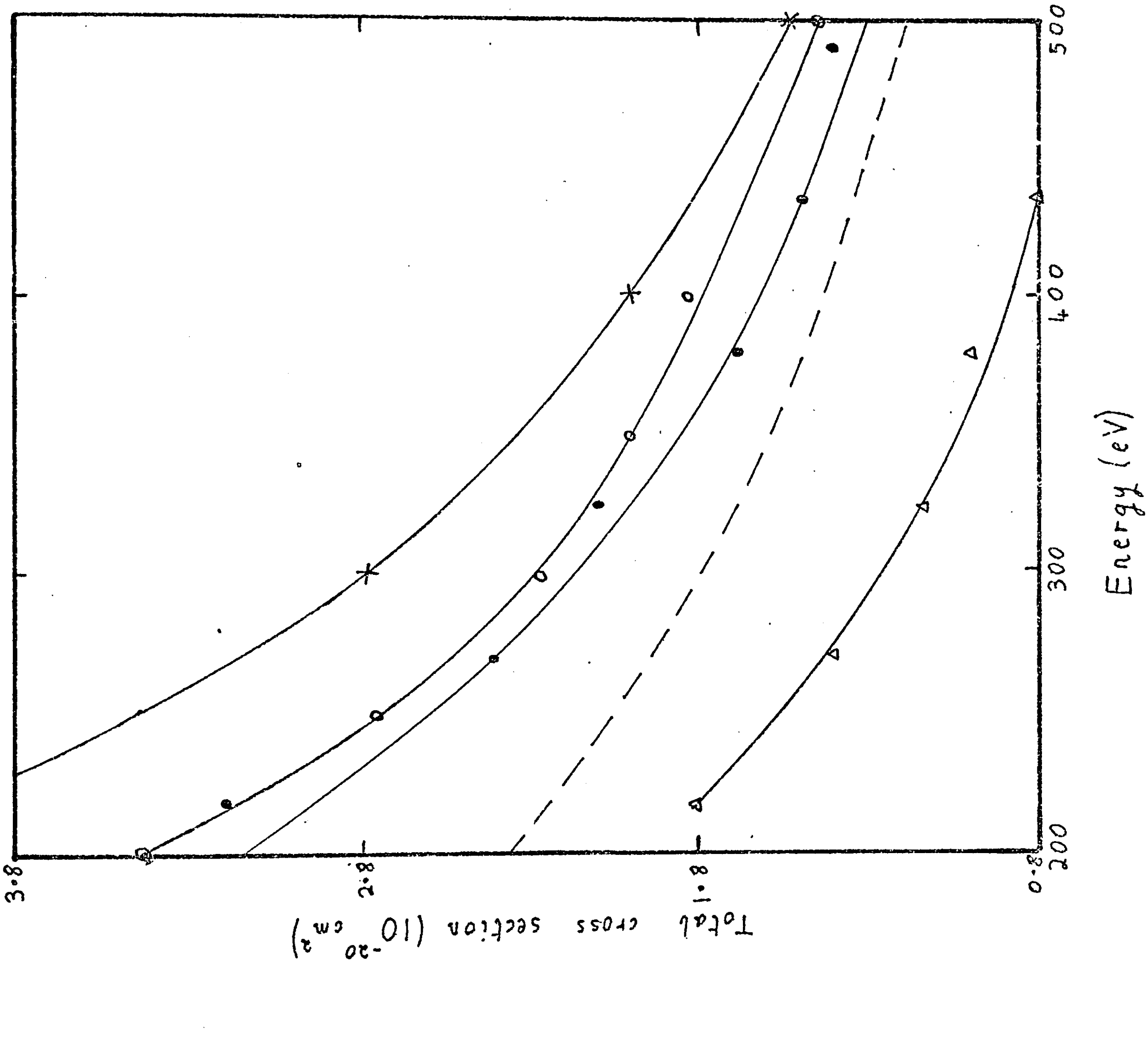

3·8
2·8
1·8
0·8
200
300
400
500
Energy (eV)
Total cross section (10^{-20} cm^2)

FIGURE CAPTION

Figure 17. Differential cross sections for the excitation of the model doubly excited states of Helium by 200 eV electrons (in a_o^2 / sr).

——————— (2s2p)

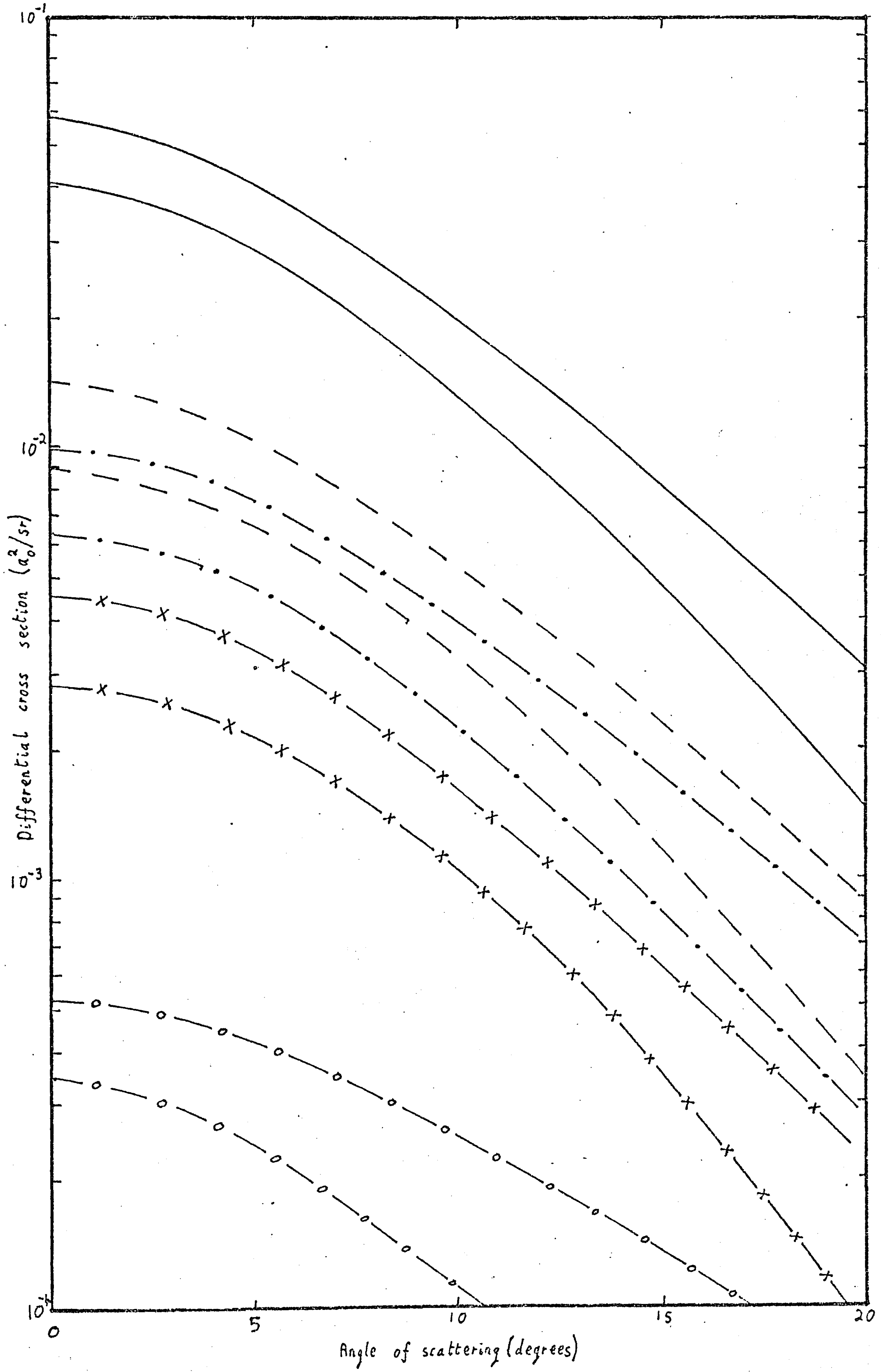
— — — (23 -)

—•—•— (3s,2p)

—X—X— (2s,3p)

—0—0— (23 +)

FIGURE 17



10^{-1}
10^{-2}
10^{-3}
10^{-4}
Differential cross section (a_0^2/sr)
0
5
10
15
20
Angle of scattering (degrees)

APPENDIX A

$$\gamma_{0,l\mu,l'\mu'}(\hat{\underline{t}}) = \delta^{l}_{l'}\,\delta^{\mu}_{\mu'} \quad \text{where } \delta^{x}_{y} \text{ is the kronecker delta.} \tag{A1}$$

$$\gamma_{1,00,1\mu}(\hat{\underline{t}}) = (-1)^{\mu}\sqrt{4\pi}\, i\, Y^{*}_{1-\mu}(\hat{\underline{t}}) = \sqrt{4\pi}\, i\, Y_{1\mu}(\hat{\underline{t}}) \tag{A2}$$

$$\gamma_{1,1\mu,00}(\hat{\underline{t}}) = \sqrt{4\pi}\, i\, Y^{*}_{1\mu}(\hat{\underline{t}}) \tag{A3}$$

$$\gamma_{1,20,1\mu}(\hat{\underline{t}}) = \sqrt{24\pi}\, i \begin{pmatrix} 2 & 1 & 1 \\ 0 & -\mu & \mu \end{pmatrix} Y^{*}_{1-\mu}(\hat{\underline{t}}) \tag{A4}$$

$$\gamma_{2,10,1\mu}(\hat{\underline{t}}) = -\sqrt{24\pi} \begin{pmatrix} 1 & 2 & 1 \\ 0 & -\mu & \mu \end{pmatrix} Y^{*}_{2-\mu}(\hat{\underline{t}}) \tag{A5}$$

$$\gamma_{2,2\mu,00}(\hat{\underline{t}}) = -\sqrt{4\pi}\, Y^{*}_{2\mu}(\hat{\underline{t}}) \tag{A6}$$

$$\gamma_{3,20,1\mu}(\hat{\underline{t}}) = 3\sqrt{4\pi}\, i \begin{pmatrix} 2 & 3 & 1 \\ 0 & -\mu & \mu \end{pmatrix} Y^{*}_{3-\mu}(\hat{\underline{t}}) \tag{A7}$$

APPENDIX B

$$I(ns,ms;n's,m's;\underline{t}) = t^2 \sum_{jk} \frac{Y(j;ns,ms;n's,m's;k)}{(t^2+\alpha_j^2)^k} \tag{B1}$$

The $\underline{t}$ dependence is contained in terms of the form E(ns,n's;$\underline{t}$) - E(ns,n's;0) (c.f. (3.16)). Using the transformation (3.36), (B1) is obtained.

$$I(ns,ms;n's,m'p_\mu;\underline{t}) = \sqrt{4\pi}\, i\, t\, Y_{1\mu}(\hat{\underline{t}}) \sum_{jk} \frac{Y(j;ns,ms;n's,m'p;k)}{(t^2+\alpha_j^2)^k} \tag{B2}$$

$$I(ns,mp_\mu;n's,m's;\underline{t}) = \sqrt{4\pi}\, i\, t\, Y^*_{1\mu}(\hat{\underline{t}}) \sum_{jk} \frac{Y(j;n's,m's;ns,mp;k)}{(t^2+\alpha_j^2)^k} \tag{B3}$$

The $\underline{t}$ dependence is contained in terms of the form E(ms,m'p_μ;$\underline{t}$) [E(mp_μ,m's;$\underline{t}$)] which may be written as $\mathcal{Y}_{1,00,1\mu}(\hat{\underline{t}})\, R_{1,m0,m'1}(t)$ $\left[\mathcal{Y}_{1,1\mu,00}(\hat{\underline{t}})\, R_{1,m'0,m1}(t)\right]$ Using (A2) [(A3)] and (3.23) together with (3.31) gives (B2) [(B3)] respectively.

$$I(ns,md_\mu;n's,m's;\underline{t}) = -\sqrt{4\pi}\, t^2\, Y^*_{2\mu}(\hat{\underline{t}}) \sum_{jk} \frac{Y(j;n's,m's;ns,md;k)}{(t^2+\alpha_j^2)^k} \tag{B4}$$

The $\underline{t}$ dependence is contained in terms of the form E(md_μ,m's;$\underline{t}$) which may be written as $\mathcal{Y}_{2,2\mu,00}(\hat{\underline{t}})\quad R_{2,m2,m'0}(t)$. Using (A6) and (3.23) together with (3.31) gives (B4).

$$I(ns,mp;n's,m'p_\mu;\underline{t}) = \delta_0^\mu t^2 \sum_{j=1,2,k} \frac{Y(j;ns,mp;n's,m'p;k)}{(t^2+\alpha_j^2)^k}$$
$$-\sqrt{24\pi}\, t^2\, Y^*_{2-\mu}(\hat{\underline{t}}) \begin{pmatrix} 1 & 2 & 1 \\ 0 & -\mu & \mu \end{pmatrix} \sum_{k} \frac{Y(3,ns,mp;n's,m'p;k)}{(t^2+\alpha_3^2)^k} \tag{B5}$$

$$= 2E(mp,m'p_\mu;0)\left[E(ns,n's;\underline{t}) - E(ns,n's;0)\right]$$

$$+2E(ns,n's;0)\left[E(mp,m'p_\mu;\underline{t}) - E(mp,m'p_\mu;0)\right] \quad \text{(from (3.16))}$$

APPENDIX B continued.

$$= 2\,E(mp, m'p_\mu; 0)\left[E(ns, n's; \underline{t}) - E(ns, n's; 0)\right]$$

$$+ 2E(ns, n's; 0)\left[\sum_{\lambda=0,2} \Upsilon_{\lambda,10,1\mu}(\hat{\underline{t}})\; R_{\lambda,m1,m'1}(t) - E(mp, m'p_\mu; 0)\right]$$

The last term of (B5) is obtained from the $\lambda=2$ term of this expression by using (A5) and (3.23) together with (3.31). The first term is obtained from the remainder of this expression by applying the transformation (3.36) twice.

$$I(ns, md; n's, m'p_\mu; \underline{t}) = \sqrt{24\pi}\, i t\, Y^*_{1-\mu}(\hat{\underline{t}}) \begin{pmatrix} 2 & 1 & 1 \\ 0 & -\mu & \mu \end{pmatrix} \sum_k \frac{Y(1; n's, m'p; ns, md; k)}{(t^2+\alpha_1^2)^k}$$

$$+ 3\sqrt{4\pi}\, i t^3\, Y^*_{3-\mu}(\hat{\underline{t}}) \begin{pmatrix} 2 & 3 & 1 \\ 0 & -\mu & \mu \end{pmatrix} \sum_k \frac{Y(2; n's, m'p; ns, md; k)}{(t^2+\alpha_2^2)^k} \qquad \text{(B6)}$$

Only one term contains the $\underline{t}$ dependence of this matrix element, namely $E(md, m'p_\mu; \underline{t})$ and using (3.33), (A6) and (A7) one obtains (B6).

APPENDIX C

The following tables of scattering amplitudes are contained in this Appendix:-

Table C1. Elastic scattering of electrons by Helium atoms.

Table C2. Excitation of the 2^1S state of Helium by electron impact.

Table C3. Excitation of the 2^1P state of Helium by electron impact.

Table C4. Excitation of the 3^1D state of Helium by electron impact (SRB2 Approximation).

Table C5. Excitation of the 3^1D state of Helium by electron impact (HM Approximation).

Table C6. Excitation of the 4^1D state of Helium by electron impact (SRB2 Approximation).

Table C7. Excitation of the 4^1D state of Helium by electron impact (HM Approximation).

Table C8. Excitation of the (2s,2p) state of Helium by electron impact.

Table C9) Excitation of the (3s,2p) state of Helium by electron impact.

Table C10. Excitation of the (2s,3p) state of Helium by electron impact.

TABLE C1

THE SCATTERING AMPLITUDE FOR ELASTIC SCATTERING OF ELECTRONS BY HELIUM ATOMS
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	7.878E-01	9.377E-01	-1.430E-01	-5.661E-05	1.044E-04	9.539E-02	3.149E-02
		6.294E-01	5.159E-02	7.206E-03	3.741E-03	9.898E-02	3.293E-02
1.0	7.873E-01	9.141E-01	-1.430E-01	-5.859E-05	1.033E-04	7.453E-02	2.512E-02
		6.286E-01	5.157E-02	7.197E-03	3.736E-03	9.679E-02	3.231E-02
2.5	7.843E-01	8.780E-01	-1.430E-01	-6.892E-05	9.804E-05	4.689E-02	1.649E-02
		6.248E-01	5.149E-02	7.149E-03	3.711E-03	8.652E-02	2.936E-02
5.0	7.738E-01	8.169E-01	-1.429E-01	-1.047E-04	7.957E-05	1.627E-02	6.378E-03
		6.114E-01	5.121E-02	6.981E-03	3.624E-03	6.117E-02	2.171E-02
7.5	7.568E-01	7.558E-01	-1.429E-01	-1.606E-04	5.045E-05	1.641E-03	1.100E-03
		5.902E-01	5.074E-02	6.711E-03	3.483E-03	3.782E-02	1.417E-02
10.0	7.343E-01	6.964E-01	-1.428E-01	-2.316E-04	1.294E-05	-4.218E-03	-1.249E-03
		5.626E-01	5.009E-02	6.353E-03	3.294E-03	2.079E-02	8.347E-03
15.0	6.766E-01	5.866E-01	-1.425E-01	-3.941E-04	-7.559E-05	-5.741E-03	-2.172E-03
		4.953E-01	4.829E-02	5.448E-03	2.808E-03	2.529E-03	1.636E-03
20.0	6.093E-01	4.933E-01	-1.420E-01	-5.447E-04	-1.618E-04	-3.759E-03	-1.577E-03
		4.231E-01	4.591E-02	4.423E-03	2.242E-03	-3.590E-03	-9.206E-04
25.0	5.400E-01	4.181E-01	-1.414E-01	-6.495E-04	-2.257E-04	-1.875E-03	-8.809E-04
		3.562E-01	4.308E-02	3.417E-03	1.671E-03	-4.581E-03	-1.556E-03
30.0	4.741E-01	3.593E-01	-1.406E-01	-6.962E-04	-2.571E-04	-6.883E-04	-3.931E-04
		2.999E-01	3.992E-02	2.528E-03	1.154E-03	-3.830E-03	-1.458E-03
45.0	3.173E-01	2.527E-01	-1.373E-01	-5.801E-04	-1.878E-04	2.735E-04	1.174E-04
		1.956E-01	2.973E-02	8.247E-04	1.714E-04	-1.128E-03	-5.978E-04
60.0	2.198E-01	2.020E-01	-1.326E-01	-3.610E-04	-5.593E-05	1.842E-04	1.434E-04
		1.512E-01	2.033E-02	1.864E-04	-1.243E-04	-2.119E-04	-1.761E-04
90.0	1.245E-01	1.563E-01	-1.219E-01	-1.050E-04	4.408E-05	3.310E-05	7.120E-05
		1.171E-01	7.435E-03	-5.201E-05	-1.229E-04	6.166E-06	1.529E-07
135.0	7.653E-02	1.292E-01	-1.093E-01	-3.356E-06	3.612E-05	3.191E-06	2.971E-05
		9.632E-02	-4.979E-04	-4.683E-05	-6.686E-05	1.975E-06	7.122E-06
180.0	6.590E-02	1.218E-01	-1.049E-01	1.223E-05	2.763E-05	8.624E-07	2.200E-05
		9.021E-02	-2.235E-03	-3.907E-05	-5.588E-05	7.018E-07	5.069E-06

THE SCATTERING AMPLITUDE FOR ELASTIC SCATTERING OF ELECTRONS BY HELIUM ATOMS
E= 300E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	7.878E-01	6.948E-01	-1.096E-01	-2.855E-04	-5.578E-05	8.222E-02	2.720E-02
		6.660E-01	2.672E-02	4.778E-03	2.427E-03	9.116E-02	3.008E-02
1.0	7.870E-01	6.733E-01	-1.096E-01	-2.872E-04	-5.660E-05	5.639E-02	1.925E-02
		6.645E-01	2.671E-02	4.768E-03	2.422E-03	8.703E-02	2.891E-02
2.5	7.825E-01	6.402E-01	-1.095E-01	-2.957E-04	-6.083E-05	2.649E-02	9.695E-03
		6.567E-01	2.664E-02	4.717E-03	2.397E-03	6.999E-02	2.392E-02
5.0	7.669E-01	5.843E-01	-1.094E-01	-3.245E-04	-7.527E-05	2.873E-03	1.461E-03
		6.300E-01	2.638E-02	4.538E-03	2.307E-03	3.840E-02	1.396E-02
7.5	7.422E-01	5.292E-01	-1.093E-01	-3.675E-04	-9.710E-05	-3.996E-03	-1.264E-03
		5.896E-01	2.596E-02	4.257E-03	2.166E-03	1.719E-02	6.759E-03
10.0	7.100E-01	4.767E-01	-1.090E-01	-4.181E-04	-1.234E-04	-5.136E-03	-1.881E-03
		5.401E-01	2.539E-02	3.898E-03	1.981E-03	5.192E-03	2.439E-03
15.0	6.314E-01	3.842E-01	-1.083E-01	-5.144E-04	-1.759E-04	-3.318E-03	-1.381E-03
		4.332E-01	2.383E-02	3.049E-03	1.534E-03	-3.631E-03	-1.039E-03
20.0	5.461E-01	3.117E-01	-1.073E-01	-5.693E-04	-2.089E-04	-1.399E-03	-6.660E-04
		3.381E-01	2.184E-02	2.194E-03	1.065E-03	-4.258E-03	-1.505E-03
25.0	4.648E-01	2.577E-01	-1.060E-01	-5.677E-04	-2.108E-04	-3.643E-04	-2.293E-04
		2.659E-01	1.957E-02	1.469E-03	6.548E-04	-2.989E-03	-1.170E-03
30.0	3.931E-01	2.185E-01	-1.044E-01	-5.200E-04	-1.843E-04	6.024E-05	-1.739E-05
		2.154E-01	1.716E-02	9.249E-04	3.432E-04	-1.760E-03	-7.702E-04
45.0	2.417E-01	1.534E-01	-9.826E-02	-2.892E-04	-5.132E-05	1.490E-04	1.016E-04
		1.421E-01	1.024E-02	1.612E-04	-5.395E-05	-2.235E-04	-1.589E-04
60.0	1.593E-01	1.230E-01	-9.095E-02	-1.241E-04	2.094E-05	5.094E-05	6.729E-05
		1.138E-01	4.968E-03	-5.556E-06	-8.537E-05	-1.361E-05	-2.255E-05
90.0	8.634E-02	9.269E-02	-7.686E-02	-4.236E-06	2.894E-05	3.705E-06	2.180E-05
		8.572E-02	-5.789E-04	-2.776E-05	-4.485E-05	1.652E-06	3.627E-06
135.0	5.201E-02	7.235E-02	-6.310E-02	2.246E-05	8.683E-06	-5.171E-07	5.786E-06
		6.585E-02	-2.896E-03	-1.574E-05	-2.569E-05	4.809E-09	1.396E-06
180.0	4.461E-02	6.667E-02	-5.877E-02	2.394E-05	3.386E-06	-5.671E-07	3.400E-06
		6.032E-02	-3.246E-03	-1.285E-05	-2.222E-05	-7.030E-08	8.778E-07

THE SCATTERING AMPLITUDE FOR ELASTIC SCATTERING OF ELECTRONS BY HELIUM ATOMS
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	7.878E-01	5.675E-01	-9.139E-02	-3.794E-04	-1.237E-04	7.336E-02	2.429E-02
		6.101E-01	2.048E-02	3.937E-03	1.991E-03	8.665E-02	2.859E-02
1.0	7.867E-01	5.471E-01	-9.138E-02	-3.807E-04	-1.243E-04	4.382E-02	1.515E-02
		6.081E-01	2.047E-02	3.926E-03	1.985E-03	8.034E-02	2.679E-02
2.5	7.808E-01	5.157E-01	-9.134E-02	-3.875E-04	-1.276E-04	1.474E-02	5.634E-03
		5.981E-01	2.039E-02	3.869E-03	1.957E-03	5.780E-02	2.004E-02
5.0	7.602E-01	4.627E-01	-9.118E-02	-4.098E-04	-1.382E-04	-1.797E-03	-4.044E-04
		5.643E-01	2.013E-02	3.672E-03	1.859E-03	2.536E-02	9.470E-03
7.5	7.280E-01	4.112E-01	-9.092E-02	-4.408E-04	-1.533E-04	-4.574E-03	-1.646E-03
		5.150E-01	1.970E-02	3.369E-03	1.707E-03	7.975E-03	3.374E-03
10.0	6.872E-01	3.632E-01	-9.055E-02	-4.734E-04	-1.697E-04	-4.057E-03	-1.587E-03
		4.578E-01	1.911E-02	2.992E-03	1.514E-03	-1.379E-04	3.395E-04
15.0	5.916E-01	2.826E-01	-8.951E-02	-5.168E-04	-1.929E-04	-1.822E-03	-8.167E-04
		3.454E-01	1.755E-02	2.157E-03	1.070E-03	-4.079E-03	-1.369E-03
20.0	4.942E-01	2.235E-01	-8.806E-02	-5.077E-04	-1.892E-04	-4.942E-04	-2.788E-04
		2.579E-01	1.562E-02	1.401E-03	6.496E-04	-3.053E-03	-1.156E-03
25.0	4.073E-01	1.824E-01	-8.624E-02	-4.507E-04	-1.576E-04	1.517E-05	-3.515E-05
		1.993E-01	1.350E-02	8.340E-04	3.264E-04	-1.686E-03	-7.187E-04
30.0	3.350E-01	1.540E-01	-8.409E-02	-3.691E-04	-1.108E-04	1.449E-04	5.440E-05
		1.621E-01	1.135E-02	4.614E-04	1.178E-04	-8.154E-04	-4.044E-04
45.0	1.946E-01	1.083E-01	-7.634E-02	-1.497E-04	2.482E-06	6.939E-05	6.564E-05
		1.111E-01	5.717E-03	4.232E-05	-6.739E-05	-5.926E-05	-5.685E-05
60.0	1.245E-01	8.627E-02	-6.795E-02	-4.261E-05	3.025E-05	1.583E-05	3.367E-05
		8.907E-02	2.001E-03	-1.861E-05	-5.582E-05	-4.973E-07	-4.142E-06
90.0	6.590E-02	6.257E-02	-5.359E-02	1.354E-05	1.599E-05	1.604E-07	8.327E-06
		6.461E-02	-1.248E-03	-1.664E-05	-2.748E-05	2.960E-07	1.541E-06
135.0	3.932E-02	4.646E-02	-4.126E-02	1.841E-05	1.898E-06	-3.501E-07	1.751E-06
		4.789E-02	-2.261E-03	-9.214E-06	-1.653E-05	-5.008E-08	4.407E-07
180.0	3.367E-02	4.209E-02	-3.767E-02	1.719E-05	-7.667E-07	-2.707E-07	9.771E-07
		4.345E-02	-2.359E-03	-7.691E-06	-1.438E-05	-4.868E-08	2.740E-07

THE SCATTERING AMPLITUDE FOR ELASTIC SCATTERING OF ELECTRONS BY HELIUM ATOMS
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	7.878E-01	4.893E-01	-7.984E-02	-4.330E-04	-1.624E-04	6.684E-02	2.213E-02
		5.704E-01	1.660E-02	3.353E-03	1.688E-03	8.268E-02	2.727E-02
1.0	7.864E-01	4.695E-01	-7.983E-02	-4.340E-04	-1.629E-04	3.449E-02	1.208E-02
		5.680E-01	1.659E-02	3.342E-03	1.682E-03	7.407E-02	2.480E-02
2.5	7.790E-01	4.390E-01	-7.977E-02	-4.390E-04	-1.650E-04	7.778E-03	3.136E-03
		5.556E-01	1.651E-02	3.280E-03	1.652E-03	4.767E-02	1.673E-02
5.0	7.535E-01	3.877E-01	-7.957E-02	-4.548E-04	-1.719E-04	-3.385E-03	-1.110E-03
		5.152E-01	1.624E-02	3.071E-03	1.549E-03	1.687E-02	6.459E-03
7.5	7.144E-01	3.386E-01	-7.923E-02	-4.744E-04	-1.806E-04	-4.105E-03	-1.557E-03
		4.583E-01	1.581E-02	2.755E-03	1.391E-03	3.022E-03	1.490E-03
10.0	6.657E-01	2.940E-01	-7.875E-02	-4.907E-04	-1.879E-04	-3.022E-03	-1.237E-03
		3.958E-01	1.522E-02	2.375E-03	1.195E-03	-2.343E-03	-5.873E-04
15.0	5.562E-01	2.220E-01	-7.742E-02	-4.908E-04	-1.868E-04	-1.006E-03	-4.883E-04
		2.835E-01	1.369E-02	1.582E-03	7.709E-04	-3.574E-03	-1.272E-03
20.0	4.510E-01	1.725E-01	-7.561E-02	-4.385E-04	-1.577E-04	-1.369E-04	-1.102E-04
		2.059E-01	1.185E-02	9.320E-04	4.061E-04	-2.080E-03	-8.317E-04
25.0	3.620E-01	1.397E-01	-7.338E-02	-3.537E-04	-1.096E-04	1.028E-04	2.568E-05
		1.588E-01	9.898E-03	4.977E-04	1.598E-04	-9.604E-04	-4.457E-04
30.0	2.914E-01	1.178E-01	-7.081E-02	-2.638E-04	-5.948E-05	1.229E-04	6.205E-05
		1.306E-01	7.995E-03	2.438E-04	2.460E-05	-3.965E-04	-2.235E-04
45.0	1.625E-01	8.279E-02	-6.205E-02	-7.969E-05	2.146E-05	3.326E-05	4.226E-05
		9.152E-02	3.414E-03	5.787E-06	-5.624E-05	-1.740E-05	-2.262E-05
60.0	1.020E-01	6.509E-02	-5.331E-02	-1.076E-05	2.679E-05	5.351E-06	1.836E-05
		7.262E-02	7.472E-04	-1.710E-05	-3.795E-05	5.535E-07	-3.702E-07
90.0	5.322E-02	4.544E-02	-3.970E-02	1.570E-05	9.480E-06	-2.741E-07	3.721E-06
		5.103E-02	-1.251E-03	-1.105E-05	-1.880E-05	3.753E-08	7.166E-07
135.0	3.159E-02	3.242E-02	-2.912E-02	1.392E-05	3.182E-07	-1.892E-07	6.928E-07
		3.694E-02	-1.711E-03	-6.207E-06	-1.151E-05	-3.504E-08	1.809E-07
180.0	2.702E-02	2.901E-02	-2.622E-02	1.237E-05	-1.121E-06	-1.339E-07	3.886E-07
		3.331E-02	-1.723E-03	-5.249E-06	-1.002E-05	-2.729E-08	1.128E-07

THE SCATTERING AMPLITUDE FOR ELASTIC SCATTERING OF ELECTRONS BY HELIUM ATOMS
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	7.878E-01	3.191E-01	-5.390E-02	-6.832E-04	-2.032E-04	4.900E-02	1.624E-02
1.0	7.850E-01	3.004E-01	-5.388E-02	-6.827E-04	-2.028E-04	1.203E-02	4.457E-03
2.5	7.703E-01	2.715E-01	-5.377E-02	-6.798E-04	-2.010E-04	-1.824E-03	-5.629E-04
5.0	7.219E-01	2.244E-01	-5.338E-02	-6.674E-04	-1.936E-04	-2.992E-03	-1.165E-03
7.5	6.528E-01	1.827E-01	-5.275E-02	-6.408E-04	-1.794E-04	-1.706E-03	-7.339E-04
10.0	5.749E-01	1.487E-01	-5.188E-02	-5.974E-04	-1.577E-04	-7.490E-04	-3.656E-04
15.0	4.265E-01	1.027E-01	-4.954E-02	-4.719E-04	-9.784E-05	-2.091E-05	-4.064E-05
20.0	3.116E-01	7.706E-02	-4.658E-02	-3.365E-04	-3.839E-05	7.017E-05	2.838E-05
25.0	2.307E-01	6.209E-02	-4.328E-02	-2.264E-04	8.205E-07	4.651E-05	3.273E-05
30.0	1.748E-01	5.236E-02	-3.984E-02	-1.485E-04	1.878E-05	2.244E-05	2.473E-05
45.0	8.827E-02	3.546E-02	-3.026E-02	-4.178E-05	1.828E-05	1.323E-06	6.966E-06
60.0	5.322E-02	2.590E-02	-2.298E-02	-1.304E-05	8.641E-06	-1.341E-07	1.935E-06
90.0	2.702E-02	1.587E-02	-1.448E-02	-2.004E-06	1.650E-06	-6.854E-08	2.424E-07
135.0	1.589E-02	1.026E-02	-9.476E-03	-3.792E-07	-6.438E-08	-1.783E-08	4.164E-08
180.0	1.357E-02	8.950E-03	-8.284E-03	-2.272E-07	-2.618E-07	-1.159E-08	2.491E-08

TABLE C2

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2S) STATE OF HELIUM BY ELECTRON IMPACT
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	4.452E-01	1.820E-01	-2.139E-02	-2.145E-02	-1.078E-03	-1.246E-01	7.983E-03
		1.104E-01	1.648E-02	1.126E-01	-7.511E-03	5.422E-01	2.791E-02
1.0	4.439E-01	1.796E-01	-2.139E-02	-2.146E-02	-1.076E-03	-1.269E-01	8.091E-03
		1.099E-01	1.647E-02	1.124E-01	-7.487E-03	5.095E-01	2.829E-02
2.5	4.369E-01	1.684E-01	-2.137E-02	-2.151E-02	-1.068E-03	-1.298E-01	7.715E-03
		1.075E-01	1.642E-02	1.115E-01	-7.360E-03	3.844E-01	2.900E-02
5.0	4.132E-01	1.408E-01	-2.132E-02	-2.167E-02	-1.040E-03	-1.125E-01	3.520E-03
		9.952E-02	1.626E-02	1.084E-01	-6.919E-03	1.813E-01	2.490E-02
7.5	3.775E-01	1.118E-01	-2.122E-02	-2.189E-02	-9.956E-04	-8.178E-02	-1.679E-03
		8.811E-02	1.598E-02	1.033E-01	-6.226E-03	6.247E-02	1.542E-02
10.0	3.342E-01	8.644E-02	-2.109E-02	-2.213E-02	-9.398E-04	-5.243E-02	-4.430E-03
		7.543E-02	1.560E-02	9.684E-02	-5.341E-03	5.787E-03	6.130E-03
15.0	2.427E-01	5.211E-02	-2.072E-02	-2.240E-02	-8.188E-04	-1.473E-02	-3.215E-03
		5.335E-02	1.459E-02	8.115E-02	-3.301E-03	-2.148E-02	-2.669E-03
20.0	1.647E-01	3.636E-02	-2.022E-02	-2.215E-02	-7.219E-04	-8.633E-04	-7.247E-04
		4.057E-02	1.332E-02	6.468E-02	-1.398E-03	-1.602E-02	-2.898E-03
25.0	1.084E-01	3.079E-02	-1.961E-02	-2.133E-02	-6.600E-04	2.339E-03	1.771E-04
		3.558E-02	1.190E-02	4.983E-02	-2.252E-05	-9.005E-03	-1.648E-03
30.0	7.114E-02	2.909E-02	-1.890E-02	-2.008E-02	-6.106E-04	2.357E-03	3.093E-04
		3.435E-02	1.042E-02	3.771E-02	7.578E-04	-4.719E-03	-8.375E-04
45.0	2.251E-02	2.654E-02	-1.649E-02	-1.580E-02	-3.849E-04	7.557E-04	1.461E-04
		3.266E-02	6.402E-03	1.643E-02	1.086E-03	-6.727E-04	-1.405E-04
60.0	8.704E-03	2.302E-02	-1.408E-02	-1.245E-02	-1.486E-04	1.874E-04	5.530E-05
		2.792E-02	3.607E-03	7.980E-03	7.764E-04	-1.195E-04	-3.752E-05
90.0	2.081E-03	1.699E-02	-1.037E-02	-8.535E-03	1.007E-04	9.818E-06	7.431E-06
		1.873E-02	1.027E-03	2.716E-03	4.286E-04	-9.281E-06	-8.345E-06
135.0	5.943E-04	1.231E-02	-7.536E-03	-6.050E-03	2.193E-04	-2.589E-06	-2.511E-07
		1.210E-02	1.164E-04	1.108E-03	2.722E-04	-3.447E-07	-2.323E-06
180.0	4.011E-04	1.104E-02	-6.769E-03	-5.411E-03	2.421E-04	-2.525E-06	-7.227E-07
		1.046E-02	-9.283E-06	8.519E-04	2.403E-04	6.624E-07	-1.368E-06

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2S) STATE OF HELIUM BY ELECTRON IMPACT
E= 300E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	4.493E-01	1.395E-01	-1.546E-02	-1.778E-02	-8.285E-04	-9.048E-02	6.281E-03
		1.425E-01	9.385E-03	7.582E-02	-7.573E-03	5.146E-01	2.116E-02
1.0	4.472E-01	1.362E-01	-1.546E-02	-1.778E-02	-8.270E-04	-9.521E-02	6.696E-03
		1.415E-01	9.377E-03	7.562E-02	-7.540E-03	4.559E-01	2.224E-02
2.5	4.365E-01	1.230E-01	-1.544E-02	-1.780E-02	-8.194E-04	-9.684E-02	6.175E-03
		1.363E-01	9.332E-03	7.461E-02	-7.372E-03	2.828E-01	2.364E-02
5.0	4.011E-01	9.500E-02	-1.536E-02	-1.785E-02	-7.931E-04	-7.242E-02	9.781E-04
		1.199E-01	9.176E-03	7.115E-02	-6.796E-03	8.825E-02	1.654E-02
7.5	3.502E-01	6.943E-02	-1.524E-02	-1.789E-02	-7.528E-04	-4.338E-02	-2.910E-03
		9.870E-02	8.923E-03	6.585E-02	-5.917E-03	8.289E-03	5.766E-03
10.0	2.928E-01	4.990E-02	-1.508E-02	-1.785E-02	-7.044E-04	-2.170E-02	-3.332E-03
		7.812E-02	8.584E-03	5.930E-02	-4.842E-03	-1.644E-02	-9.990E-04
15.0	1.862E-01	2.885E-02	-1.461E-02	-1.736E-02	-6.104E-04	-2.206E-03	-8.704E-04
		5.051E-02	7.705E-03	4.487E-02	-2.608E-03	-1.566E-02	-3.056E-03
20.0	1.110E-01	2.234E-02	-1.401E-02	-1.626E-02	-5.403E-04	1.523E-03	1.143E-04
		3.998E-02	6.664E-03	3.182E-02	-8.889E-04	-7.640E-03	-1.502E-03
25.0	6.563E-02	2.069E-02	-1.329E-02	-1.478E-02	-4.767E-04	1.474E-03	2.002E-04
		3.671E-02	5.583E-03	2.187E-02	7.847E-05	-3.369E-03	-6.371E-04
30.0	3.973E-02	1.984E-02	-1.249E-02	-1.321E-02	-3.986E-04	9.563E-04	1.494E-04
		3.468E-02	4.555E-03	1.498E-02	4.855E-04	-1.476E-03	-2.961E-04
45.0	1.075E-02	1.639E-02	-1.003E-02	-9.293E-03	-1.435E-04	1.703E-04	4.417E-05
		2.659E-02	2.222E-03	5.392E-03	5.385E-04	-1.454E-04	-3.805E-05
60.0	3.737E-03	1.283E-02	-7.897E-03	-6.791E-03	1.411E-05	2.627E-05	1.179E-05
		1.912E-02	1.000E-03	2.426E-03	3.794E-04	-2.018E-05	-7.680E-06
90.0	7.801E-04	8.073E-03	-5.092E-03	-4.127E-03	1.375E-04	-1.194E-06	8.154E-07
		1.075E-02	1.912E-04	8.339E-04	2.158E-04	-3.574E-07	-7.251E-07
135.0	2.039E-04	5.060E-03	-3.283E-03	-2.592E-03	1.711E-04	-1.105E-06	1.446E-07
		6.393E-03	6.079E-06	3.904E-04	1.392E-04	2.142E-06	6.102E-08
180.0	1.346E-04	4.334E-03	-2.838E-03	-2.226E-03	1.722E-04	-8.433E-07	1.900E-07
		5.445E-03	-1.130E-05	3.189E-04	1.225E-04	4.266E-06	1.003E-07

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2S) STATE OF HELIUM BY ELECTRON IMPACT
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	4.513E-01	1.165E-01	-1.253E-02	-1.572E-02	-7.001E-04	-7.147E-02	4.980E-03
		1.333E-01	7.637E-03	6.290E-02	-6.708E-03	4.936E-01	1.758E-02
1.0	4.485E-01	1.125E-01	-1.252E-02	-1.572E-02	-6.987E-04	-7.829E-02	5.701E-03
		1.320E-01	7.628E-03	6.268E-02	-6.670E-03	4.087E-01	1.945E-02
2.5	4.341E-01	9.818E-02	-1.250E-02	-1.572E-02	-6.915E-04	-7.679E-02	4.798E-03
		1.254E-01	7.581E-03	6.155E-02	-6.474E-03	2.126E-01	2.079E-02
5.0	3.877E-01	7.103E-02	-1.241E-02	-1.567E-02	-6.665E-04	-4.985E-02	-4.983E-04
		1.053E-01	7.414E-03	5.771E-02	-5.810E-03	4.334E-02	1.122E-02
7.5	3.242E-01	4.862E-02	-1.227E-02	-1.555E-02	-6.291E-04	-2.488E-02	-2.795E-03
		8.153E-02	7.148E-03	5.200E-02	-4.827E-03	-8.856E-03	1.207E-03
10.0	2.571E-01	3.342E-02	-1.208E-02	-1.529E-02	-5.859E-04	-9.624E-03	-2.043E-03
		6.120E-02	6.796E-03	4.523E-02	-3.684E-03	-1.778E-02	-2.742E-03
15.0	1.460E-01	2.007E-02	-1.155E-02	-1.428E-02	-5.060E-04	4.121E-04	-1.359E-04
		3.942E-02	5.912E-03	3.151E-02	-1.557E-03	-9.870E-03	-2.010E-03
20.0	7.905E-02	1.708E-02	-1.087E-02	-1.277E-02	-4.369E-04	1.215E-03	1.551E-04
		3.327E-02	4.918E-03	2.057E-02	-2.296E-04	-3.887E-03	-7.497E-04
25.0	4.357E-02	1.613E-02	-1.010E-02	-1.112E-02	-3.561E-04	7.825E-04	1.190E-04
		3.072E-02	3.948E-03	1.321E-02	3.351E-04	-1.482E-03	-2.891E-04
30.0	2.505E-02	1.513E-02	-9.281E-03	-9.593E-03	-2.637E-04	4.176E-04	7.553E-05
		2.799E-02	3.086E-03	8.619E-03	4.891E-04	-5.771E-04	-1.234E-04
45.0	6.091E-03	1.142E-02	-6.946E-03	-6.286E-03	-4.583E-05	4.912E-05	1.643E-05
		1.908E-02	1.361E-03	2.937E-03	3.818E-04	-4.375E-05	-1.416E-05
60.0	1.965E-03	8.323E-03	-5.140E-03	-4.362E-03	5.699E-05	4.162E-06	3.262E-06
		1.280E-02	5.997E-04	1.335E-03	2.543E-04	-4.649E-06	-2.608E-06
90.0	3.757E-04	4.756E-03	-3.039E-03	-2.453E-03	1.188E-04	-1.083E-06	1.540E-07
		6.788E-03	1.586E-04	4.988E-04	1.418E-04	9.371E-07	-1.435E-07
135.0	9.319E-05	2.801E-03	-1.844E-03	-1.452E-03	1.210E-04	-4.887E-07	9.881E-08
		3.962E-03	5.743E-05	2.572E-04	9.052E-05	8.346E-06	3.095E-08
180.0	6.073E-05	2.366E-03	-1.571E-03	-1.229E-03	1.165E-04	-3.542E-07	1.030E-07
		3.367E-03	4.470E-05	2.153E-04	7.929E-05	6.615E-06	2.770E-08

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2S) STATE OF HELIUM BY ELECTRON IMPACT
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	4.525E-01	1.023E-01	-1.079E-02	-1.446E-02	-6.261E-04	-5.945E-02	4.000E-03
		1.276E-01	6.435E-03	5.399E-02	-6.107E-03	4.754E-01	1.487E-02
1.0	4.489E-01	9.770E-02	-1.078E-02	-1.446E-02	-6.247E-04	-6.776E-02	4.977E-03
		1.260E-01	6.425E-03	5.375E-02	-6.064E-03	3.663E-01	1.751E-02
2.5	4.310E-01	8.268E-02	-1.075E-02	-1.443E-02	-6.178E-04	-6.301E-02	3.652E-03
		1.179E-01	6.375E-03	5.252E-02	-5.844E-03	1.628E-01	1.833E-02
5.0	3.743E-01	5.650E-02	-1.066E-02	-1.430E-02	-5.939E-04	-3.585E-02	-1.257E-03
		9.455E-02	6.203E-03	4.842E-02	-5.109E-03	1.911E-02	7.243E-03
7.5	3.003E-01	3.671E-02	-1.049E-02	-1.403E-02	-5.586E-04	-1.495E-02	-2.286E-03
		6.922E-02	5.929E-03	4.250E-02	-4.052E-03	-1.435E-02	-1.114E-03
10.0	2.268E-01	2.471E-02	-1.028E-02	-1.358E-02	-5.189E-04	-4.314E-03	-1.151E-03
		5.014E-02	5.572E-03	3.575E-02	-2.883E-03	-1.538E-02	-2.910E-03
15.0	1.170E-01	1.597E-02	-9.691E-03	-1.220E-02	-4.440E-04	8.856E-04	6.111E-05
		3.341E-02	4.703E-03	2.310E-02	-9.347E-04	-6.364E-03	-1.286E-03
20.0	5.885E-02	1.433E-02	-8.963E-03	-1.048E-02	-3.656E-04	8.198E-04	1.164E-04
		2.920E-02	3.774E-03	1.408E-02	6.005E-05	-2.161E-03	-4.184E-04
25.0	3.084E-02	1.338E-02	-8.160E-03	-8.835E-03	-2.720E-04	4.295E-04	7.232E-05
		2.634E-02	2.918E-03	8.611E-03	3.798E-04	-7.375E-04	-1.531E-04
30.0	1.708E-02	1.221E-02	-7.343E-03	-7.433E-03	-1.797E-04	1.997E-04	4.175E-05
		2.309E-02	2.202E-03	5.458E-03	4.220E-04	-2.618E-04	-6.229E-05
45.0	3.822E-03	8.553E-03	-5.172E-03	-4.616E-03	-3.318E-06	1.617E-05	6.896E-06
		1.444E-02	9.068E-04	1.818E-03	2.823E-04	-1.607E-05	-6.252E-06
60.0	1.165E-03	5.902E-03	-3.646E-03	-3.077E-03	6.614E-05	1.307E-07	1.032E-06
		9.272E-03	4.033E-04	8.434E-04	1.828E-04	-1.800E-06	-1.008E-06
90.0	2.095E-04	3.169E-03	-2.031E-03	-1.641E-03	9.623E-05	-6.644E-07	5.679E-08
		4.764E-03	1.304E-04	3.367E-04	1.008E-04	-2.663E-07	-3.186E-09
135.0	5.018E-05	1.816E-03	-1.194E-03	-9.432E-04	8.730E-05	-2.428E-07	5.579E-08
		2.760E-03	6.245E-05	1.824E-04	6.373E-05	-1.740E-07	1.193E-08
180.0	3.244E-05	1.529E-03	-1.012E-03	-7.946E-04	8.180E-05	-1.706E-07	4.996E-08
		2.344E-03	5.197E-05	1.542E-04	5.562E-05	-3.044E-06	2.052E-08

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2S) STATE OF HELIUM BY ELECTRON IMPACT
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	-4.545E-01	-6.993E-02	7.132E-03	1.110E-02	4.530E-04	3.452E-02	-1.817E-03
1.0	-4.474E-01	-6.345E-02	7.124E-03	1.108E-02	4.518E-04	4.374E-02	-3.115E-03
2.5	-4.123E-01	-4.773E-02	7.081E-03	1.095E-02	4.457E-04	3.106E-02	-6.840E-04
5.0	-3.135E-01	-2.633E-02	6.932E-03	1.049E-02	4.240E-04	1.003E-02	1.425E-03
7.5	-2.096E-01	-1.522E-02	6.696E-03	9.743E-03	3.919E-04	1.583E-03	4.717E-04
10.0	-1.305E-01	-1.116E-02	6.388E-03	8.806E-03	3.536E-04	-3.366E-04	-6.751E-06
15.0	-4.908E-02	-9.397E-03	5.630E-03	6.829E-03	2.575E-04	-3.533E-04	-5.424E-05
20.0	-2.014E-02	-8.263E-03	4.805E-03	5.199E-03	1.538E-04	-1.250E-04	-2.479E-05
25.0	-9.148E-03	-6.955E-03	4.022E-03	4.004E-03	7.496E-05	-3.841E-05	-1.041E-05
30.0	-4.512E-03	-5.736E-03	3.336E-03	3.137E-03	2.400E-05	-1.098E-05	-4.113E-06
45.0	-7.821E-04	-3.201E-03	1.921E-03	1.660E-03	-3.797E-05	3.458E-07	-1.797E-07
60.0	-2.044E-04	-1.933E-03	1.190E-03	9.948E-04	-4.785E-05	3.243E-07	-5.415E-09
90.0	-3.160E-05	-9.411E-04	5.922E-04	4.827E-04	-3.945E-05	8.212E-08	-1.158E-08
135.0	-6.997E-06	-5.341E-04	3.386E-04	2.728E-04	-2.832E-05	2.199E-08	-5.209E-09
180.0	-4.443E-06	-4.522E-04	2.870E-04	2.307E-04	-2.520E-05	1.451E-08	-3.640E-09

TABLE C3

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	3.995E+00	-4.961E-02	4.018E-02	9.040E-02	3.525E-03	-3.209E-01	1.585E-02
1.0	3.782E+00	-5.139E-02	3.797E-02	8.538E-02	3.335E-03	-2.988E-01	1.286E-02
2.5	3.021E+00	-5.876E-02	3.005E-02	6.736E-02	2.656E-03	-2.189E-01	1.851E-03
5.0	1.912E+00	-7.211E-02	1.829E-02	4.065E-02	1.657E-03	-1.017E-01	-1.238E-02
7.5	1.253E+00	-7.809E-02	1.098E-02	2.435E-02	1.045E-03	-3.949E-02	-1.398E-02
10.0	8.513E-01	-7.583E-02	6.166E-03	1.416E-02	6.468E-04	-1.328E-02	-8.932E-03
15.0	4.133E-01	-5.664E-02	4.493E-05	3.091E-03	1.462E-04	-7.170E-03	-6.007E-04
20.0	2.070E-01	-3.366E-02	-3.755E-03	-1.515E-03	-1.411E-04	-1.337E-02	3.773E-04
25.0	1.061E-01	-1.588E-02	-6.248E-03	-3.000E-03	-2.677E-04	-1.621E-02	-2.903E-04
30.0	5.562E-02	-4.424E-03	-7.858E-03	-3.102E-03	-2.674E-04	-1.621E-02	-6.816E-04
45.0	9.157E-03	7.426E-03	-9.570E-03	-1.791E-03	-3.614E-05	-1.244E-02	-8.263E-04
60.0	1.801E-03	7.746E-03	-9.178E-03	-9.294E-04	5.424E-05	-9.606E-03	-7.185E-04
90.0	9.816E-05	5.359E-03	-7.423E-03	-3.153E-04	5.127E-05	-6.899E-03	-5.506E-04
135.0	-6.922E-06	3.709E-03	-5.834E-03	-1.174E-04	3.913E-05	-5.341E-03	-4.363E-04
180.0	-8.684E-06	3.322E-03	-5.393E-03	-8.577E-05	3.629E-05	-4.940E-03	-4.057E-04

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 300E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	4.989E+00	-3.066E-02	3.434E-02	7.362E-02	2.643E-03	-2.985E-01	2.080E-02
1.0	4.434E+00	-3.334E-02	3.043E-02	6.520E-02	2.345E-03	-2.585E-01	1.533E-02
2.5	3.009E+00	-4.276E-02	2.034E-02	4.338E-02	1.576E-03	-1.535E-01	3.709E-04
5.0	1.629E+00	-5.499E-02	1.028E-02	2.174E-02	8.168E-04	-5.199E-02	-1.067E-02
7.5	9.757E-01	-5.664E-02	5.140E-03	1.111E-02	4.346E-04	-1.493E-02	-7.728E-03
10.0	6.126E-01	-5.033E-02	1.924E-03	5.112E-03	1.986E-04	-5.978E-03	-2.568E-03
15.0	2.548E-01	-2.960E-02	-2.045E-03	-4.437E-04	-7.799E-05	-9.584E-03	3.905E-04
20.0	1.100E-01	-1.246E-02	-4.342E-03	-1.965E-03	-1.850E-04	-1.212E-02	-2.065E-04
25.0	4.914E-02	-2.432E-03	-5.657E-03	-1.993E-03	-1.691E-04	-1.167E-02	-5.224E-04
30.0	2.266E-02	2.516E-03	-6.327E-03	-1.622E-03	-9.967E-05	-1.037E-02	-5.922E-04
45.0	2.616E-03	5.100E-03	-6.376E-03	-7.012E-04	2.822E-05	-7.094E-03	-5.109E-04
60.0	3.597E-04	3.862E-03	-5.512E-03	-3.216E-04	3.528E-05	-5.388E-03	-4.096E-04
90.0	-2.231E-06	2.175E-03	-4.041E-03	-9.581E-05	2.541E-05	-3.784E-03	-2.896E-04
135.0	-6.722E-06	1.360E-03	-3.032E-03	-3.322E-05	2.004E-05	-2.856E-03	-2.144E-04
180.0	-5.005E-06	1.190E-03	-2.777E-03	-2.388E-05	1.868E-05	-2.624E-03	-1.953E-04

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	5.815E+00	-2.202E-02	3.069E-02	6.372E-02	2.172E-03	-2.787E-01	2.274E-02
1.0	4.783E+00	-2.550E-02	2.515E-02	5.215E-02	1.780E-03	-2.213E-01	1.483E-02
2.5	2.837E+00	-3.583E-02	1.459E-02	3.009E-02	1.036E-03	-1.096E-01	-1.399E-03
5.0	1.397E+00	-4.592E-02	6.460E-03	1.326E-02	4.659E-04	-2.912E-02	-8.423E-03
7.5	7.858E-01	-4.430E-02	2.619E-03	5.829E-03	2.004E-04	-7.575E-03	-3.855E-03
10.0	4.635E-01	-3.600E-02	2.430E-04	1.925E-03	4.017E-05	-5.668E-03	-3.657E-04
15.0	1.700E-01	-1.674E-02	-2.617E-03	-1.131E-03	-1.245E-04	-9.465E-03	9.857E-05
20.0	6.527E-02	-4.438E-03	-4.131E-03	-1.547E-03	-1.422E-04	-9.677E-03	-3.630E-04
25.0	2.615E-02	1.319E-03	-4.855E-03	-1.265E-03	-8.470E-05	-8.443E-03	-4.642E-04
30.0	1.088E-02	3.472E-03	-5.091E-03	-9.226E-04	-2.515E-05	-7.159E-03	-4.565E-04
45.0	9.332E-04	3.379E-03	-4.572E-03	-3.435E-04	2.639E-05	-4.730E-03	-3.508E-04
60.0	8.462E-05	2.229E-03	-3.728E-03	-1.458E-04	2.200E-05	-3.573E-03	-2.702E-04
90.0	-8.655E-06	1.163E-03	-2.620E-03	-4.001E-05	1.591E-05	-2.480E-03	-1.838E-04
135.0	-3.692E-06	7.227E-04	-1.948E-03	-1.317E-05	1.256E-05	-1.863E-03	-1.336E-04
180.0	-2.508E-06	6.351E-04	-1.785E-03	-9.327E-06	1.167E-05	-1.712E-03	-1.214E-04

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	6.539E+00	-1.721E-02	2.813E-02	5.699E-02	1.874E-03	-2.622E-01	2.350E-02
1.0	4.938E+00	-2.138E-02	2.114E-02	4.276E-02	1.407E-03	-1.894E-01	1.340E-02
2.5	2.639E+00	-3.204E-02	1.097E-02	2.206E-02	7.280E-04	-8.074E-02	-2.566E-03
5.0	1.218E+00	-4.000E-02	4.341E-03	8.766E-03	2.860E-04	-1.771E-02	-6.340E-03
7.5	6.501E-01	-3.604E-02	1.304E-03	3.258E-03	8.670E-05	-5.388E-03	-1.709E-03
10.0	3.630E-01	-2.684E-02	-5.667E-04	5.621E-04	-2.970E-05	-6.137E-03	2.429E-04
15.0	1.195E-01	-9.783E-03	-2.748E-03	-1.173E-03	-1.242E-04	-8.530E-03	-1.118E-04
20.0	4.157E-02	-9.222E-04	-3.784E-03	-1.153E-03	-9.631E-05	-7.721E-03	-3.669E-04
25.0	1.518E-02	2.426E-03	-4.165E-03	-8.355E-04	-3.611E-05	-6.393E-03	-3.878E-04
30.0	5.782E-03	3.269E-03	-4.176E-03	-5.710E-04	2.625E-06	-5.299E-03	-3.591E-04
45.0	3.759E-04	2.353E-03	-3.461E-03	-1.925E-04	1.976E-05	-3.455E-03	-2.589E-04
60.0	1.583E-05	1.441E-03	-2.723E-03	-7.734E-05	1.519E-05	-2.598E-03	-1.945E-04
90.0	-6.831E-06	7.317E-04	-1.875E-03	-2.001E-05	1.115E-05	-1.791E-03	-1.295E-04
135.0	-2.092E-06	4.597E-04	-1.393E-03	-6.339E-06	8.743E-06	-1.345E-03	-9.378E-05
180.0	-1.363E-06	4.061E-04	-1.278E-03	-4.444E-06	8.112E-06	-1.236E-03	-8.536E-05

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,2P)	SB2(1S,3P)
0.0	-8.856E+00	8.664E-03	-2.102E-02	-4.031E-02	-1.209E-03	2.063E-01	-2.208E-02
1.0	-4.555E+00	1.479E-02	-1.069E-02	-2.046E-02	-6.081E-04	9.680E-02	-6.408E-03
2.5	-1.900E+00	2.412E-02	-4.149E-03	-7.897E-03	-2.205E-04	2.602E-02	3.595E-03
5.0	-7.219E-01	2.438E-02	-8.909E-04	-2.032E-03	-2.471E-05	4.305E-03	1.098E-03
7.5	-3.140E-01	1.587E-02	5.857E-04	-6.201E-05	5.351E-05	4.683E-03	-2.727E-04
10.0	-1.420E-01	7.774E-03	1.445E-03	5.538E-04	7.844E-05	5.381E-03	5.372E-06
15.0	-3.143E-02	-2.037E-04	2.217E-03	5.365E-04	4.272E-05	4.355E-03	2.212E-04
20.0	-7.601E-03	-1.950E-03	2.327E-03	3.163E-04	2.035E-06	3.196E-03	2.105E-04
25.0	-1.962E-03	-1.872E-03	2.167E-03	1.815E-04	-9.561E-06	2.454E-03	1.781E-04
30.0	-5.234E-04	-1.469E-03	1.930E-03	1.076E-04	-9.793E-06	1.985E-03	1.490E-04
45.0	2.205E-06	-6.598E-04	1.325E-03	2.799E-05	-6.471E-06	1.271E-03	9.423E-05
60.0	6.725E-06	-3.697E-04	9.799E-04	9.694E-06	-5.120E-06	9.424E-04	6.738E-05
90.0	1.327E-06	-1.926E-04	6.633E-04	2.145E-06	-3.732E-06	6.457E-04	4.405E-05
135.0	2.721E-07	-1.267E-04	4.960E-04	6.215E-07	-2.882E-06	4.865E-04	3.238E-05
180.0	1.657E-07	-1.131E-04	4.559E-04	4.271E-07	-2.667E-06	4.480E-04	2.966E-05

TABLE C4

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) SB2(1S,1S)	SB2(1S,3P) SB2(1S,2S)	SB2(1S,4P) SB2(1S,3S)	SB2(1S,5P)	SB2(1S,6P)
0.0	1.347E-01	-4.195E-03	-2.940E-01	3.234E-01	1.425E-02	4.427E-03	2.011E-03
			8.778E-04	1.958E-03	7.237E-04		
1.0	1.341E-01	-4.633E-03	-2.766E-01	3.036E-01	1.451E-02	4.512E-03	2.043E-03
			7.904E-04	1.993E-03	5.497E-04		
2.5	1.305E-01	-6.469E-03	-2.130E-01	2.275E-01	1.546E-02	5.052E-03	2.320E-03
			4.854E-04	2.086E-03	-4.571E-05		
5.0	1.188E-01	-1.030E-02	-1.153E-01	1.031E-01	1.238E-02	4.943E-03	2.510E-03
			9.232E-05	2.016E-03	-7.309E-04		
7.5	1.022E-01	-1.330E-02	-5.488E-02	3.415E-02	4.715E-03	2.224E-03	1.240E-03
			-9.253E-05	1.673E-03	-9.069E-04		
10.0	8.347E-02	-1.474E-02	-2.126E-02	8.418E-03	5.628E-04	3.010E-04	1.867E-04
			-1.871E-04	1.200E-03	-8.374E-04		
15.0	4.947E-02	-1.332E-02	1.136E-03	7.836E-04	-2.584E-04	-1.501E-04	-9.017E-05
			-2.863E-04	2.486E-04	-4.530E-04		
20.0	2.646E-02	-9.329E-03	2.923E-03	4.345E-05	-2.610E-05	-1.570E-05	-1.004E-05
			-3.441E-04	-3.840E-04	-1.007E-04		
25.0	1.352E-02	-5.662E-03	1.934E-03	-4.434E-04	-1.187E-06	4.020E-06	3.140E-06
			-3.817E-04	-6.352E-04	7.427E-05		
30.0	6.866E-03	-3.172E-03	1.139E-03	-5.758E-04	-1.261E-05	-1.065E-06	4.907E-07
			-4.025E-04	-6.364E-04	1.123E-04		
45.0	1.053E-03	-3.967E-04	2.542E-04	-3.788E-04	-1.638E-05	-5.980E-06	-2.943E-06
			-3.849E-04	-3.051E-04	4.142E-05		
60.0	2.263E-04	8.160E-05	7.167E-05	-2.014E-04	-7.713E-06	-2.990E-06	-1.526E-06
			-3.058E-04	-1.205E-04	7.168E-06		
90.0	2.456E-05	2.037E-04	7.650E-06	-6.852E-05	-1.954E-06	-7.545E-07	-3.846E-07
			-1.582E-04	-2.016E-05	-6.070E-06		
135.0	3.857E-06	2.234E-04	-8.625E-07	-2.613E-05	-6.331E-07	-2.440E-07	-1.242E-07
			-5.708E-05	-1.354E-07	-5.429E-06		
180.0	2.188E-06	2.263E-04	-1.337E-06	-1.937E-05	-4.530E-07	-1.746E-07	-8.889E-08
			-3.278E-05	1.672E-06	-4.788E-06		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E= 300E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) SB2(1S,1S)	SB2(1S,3P) SB2(1S,2S)	SB2(1S,4P) SB2(1S,3S)	SB2(1S,5P)	SB2(1S,6P)
0.0	1.377E-01	-2.319E-03	-2.611E-01	2.981E-01	8.868E-03	2.454E-03	1.066E-03
			7.145E-04	1.340E-03	4.639E-04		
1.0	1.367E-01	-2.899E-03	-2.287E-01	2.613E-01	9.498E-03	2.579E-03	1.092E-03
			5.613E-04	1.378E-03	2.315E-04		
2.5	1.312E-01	-4.952E-03	-1.443E-01	1.576E-01	1.196E-02	3.767E-03	1.678E-03
			1.931E-04	1.423E-03	-3.093E-04		
5.0	1.137E-01	-8.463E-03	-5.979E-02	4.561E-02	6.804E-03	3.027E-03	1.622E-03
			-8.395E-05	1.231E-03	-6.247E-04		
7.5	9.060E-02	-1.051E-02	-1.982E-02	7.628E-03	6.991E-04	3.986E-04	2.489E-04
			-1.785E-04	8.619E-04	-5.905E-04		
10.0	6.736E-02	-1.069E-02	-3.466E-03	9.840E-04	-3.400E-04	-1.809E-04	-1.029E-04
			-2.250E-04	4.532E-04	-4.445E-04		
15.0	3.224E-02	-7.667E-03	2.163E-03	2.243E-04	-3.175E-05	-2.114E-05	-1.368E-05
			-2.781E-04	-1.778E-04	-1.240E-04		
20.0	1.401E-02	-4.193E-03	1.349E-03	-2.632E-04	3.762E-06	4.783E-06	3.260E-06
			-3.089E-04	-4.227E-04	4.149E-05		
25.0	6.000E-03	-2.006E-03	6.883E-04	-3.643E-04	-7.903E-06	-7.437E-07	2.548E-07
			-3.230E-04	-4.121E-04	6.972E-05		
30.0	2.649E-03	-8.881E-04	3.535E-04	-3.176E-04	-1.171E-05	-3.573E-06	-1.549E-06
			-3.217E-04	-3.173E-04	5.154E-05		
45.0	3.144E-04	-1.709E-06	5.832E-05	-1.407E-04	-5.557E-06	-2.145E-06	-1.092E-06
			-2.600E-04	-1.057E-04	8.843E-06		
60.0	5.937E-05	8.361E-05	1.190E-05	-6.198E-05	-1.968E-06	-7.654E-07	-3.915E-07
			-1.786E-04	-3.597E-05	-1.942E-06		
90.0	5.663E-06	8.074E-05	-1.244E-07	-1.726E-05	-4.257E-07	-1.635E-07	-8.320E-08
			-6.965E-05	-5.039E-06	-3.668E-06		
135.0	8.232E-07	6.745E-05	-7.171E-07	-5.842E-06	-1.275E-07	-4.880E-08	-2.460E-08
			-5.314E-06	-4.640E-08	-2.402E-06		
180.0	4.581E-07	6.366E-05	-6.330E-07	-4.202E-06	-8.937E-08	-3.400E-08	-1.745E-08
			9.753E-06	3.397E-07	-2.023E-06		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P)	SB2(1S,6P)
0.0	1.392E-01	-1.527E-03	-2.374E-01	2.775E-01	5.492E-03	1.343E-03	5.582E-04
			5.929E-04	1.021E-03	3.412E-04		
1.0	1.377E-01	-2.223E-03	-1.898E-01	2.232E-01	6.478E-03	1.487E-03	5.666E-04
			3.882E-04	1.059E-03	6.992E-05		
2.5	1.303E-01	-4.340E-03	-1.011E-01	1.105E-01	9.949E-03	3.198E-03	1.428E-03
			4.654E-05	1.062E-03	-3.611E-04		
5.0	1.077E-01	-7.494E-03	-3.367E-02	2.068E-02	3.303E-03	1.629E-03	9.225E-04
			-1.295E-04	8.291E-04	-4.895E-04		
7.5	7.986E-02	-8.759E-03	-7.238E-03	1.622E-03	-2.184E-04	-9.371E-05	-4.412E-05
			-1.832E-04	4.817E-04	-3.963E-04		
10.0	5.449E-02	-8.139E-03	5.429E-04	3.850E-04	-1.955E-04	-1.127E-04	-6.769E-05
			-2.116E-04	1.453E-04	-2.496E-04		
15.0	2.178E-02	-4.747E-03	1.429E-03	-2.993E-05	6.202E-06	3.484E-06	1.895E-06
			-2.465E-04	-2.613E-04	-1.789E-05		
20.0	8.093E-03	-2.118E-03	6.701E-04	-2.602E-04	-2.171E-06	1.412E-06	1.326E-06
			-2.639E-04	-3.262E-04	4.959E-05		
25.0	3.071E-03	-8.331E-04	3.028E-04	-2.401E-04	-8.076E-06	-2.254E-06	-9.037E-07
			-2.652E-04	-2.531E-04	4.078E-05		
30.0	1.240E-03	-2.930E-04	1.416E-04	-1.800E-04	-7.404E-06	-2.629E-06	-1.272E-06
			-2.532E-04	-1.702E-04	2.356E-05		
45.0	1.266E-04	4.129E-05	1.805E-05	-6.444E-05	-2.260E-06	-8.811E-07	-4.513E-07
			-1.814E-04	-4.728E-05	1.289E-06		
60.0	2.216E-05	5.388E-05	2.439E-06	-2.525E-05	-7.092E-07	-2.746E-07	-1.402E-07
			-1.135E-04	-1.463E-05	-2.611E-06		
90.0	1.955E-06	4.013E-05	-4.787E-07	-6.250E-06	-1.424E-07	-5.454E-08	-2.763E-08
			-3.732E-05	-1.840E-06	-2.235E-06		
135.0	2.719E-07	3.137E-05	-3.433E-07	-1.979E-06	-4.122E-08	-1.565E-08	-8.091E-09
			2.617E-06	-1.875E-08	-1.329E-06		
180.0	1.497E-07	2.954E-05	-2.751E-07	-1.402E-06	-2.879E-08	-1.092E-08	-5.352E-09
			1.143E-05	1.132E-07	-1.105E-06		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P)	SB2(1S,6P)
0.0	1.401E-01	-1.104E-03	-2.193E-01	2.606E-01	3.258E-03	6.579E-04	2.522E-04
			5.041E-04	8.264E-04	2.696E-04		
1.0	1.382E-01	-1.898E-03	-1.581E-01	1.904E-01	4.616E-03	8.344E-04	2.550E-04
			2.642E-04	8.608E-04	-2.329E-05		
2.5	1.290E-01	-4.031E-03	-7.377E-02	7.936E-02	8.478E-03	2.865E-03	1.308E-03
			-2.262E-05	8.349E-04	-3.498E-04		
5.0	1.016E-01	-6.858E-03	-1.990E-02	9.423E-03	1.384E-03	7.583E-04	4.544E-04
			-1.367E-04	5.907E-04	-3.861E-04		
7.5	7.041E-02	-7.497E-03	-2.242E-03	0.	-2.928E-04	-1.529E-04	-8.671E-05
			-1.714E-04	2.732E-04	3.860E-04		
10.0	4.442E-02	-6.387E-03	1.324E-03	3.012E-04	-7.436E-05	-4.620E-05	-2.878E-05
			-1.916E-04	-4.881E-07	-1.437E-04		
15.0	1.524E-02	-3.096E-03	9.097E-04	-1.276E-04	6.397E-06	4.792E-06	3.016E-06
			-2.171E-04	-2.550E-04	1.670E-05		
20.0	5.008E-03	-1.160E-03	3.695E-04	-2.065E-04	-4.587E-06	-5.506E-07	5.799E-08
			-2.260E-04	-2.387E-04	3.806E-05		
25.0	1.741E-03	-3.795E-04	1.526E-04	-1.592E-04	-6.175E-06	-2.061E-06	-9.565E-07
			-2.196E-04	-1.607E-04	2.343E-05		
30.0	6.615E-04	-9.835E-05	6.590E-05	-1.093E-04	-4.491E-06	-1.681E-06	-8.407E-07
			-2.024E-04	-9.967E-05	1.162E-05		
45.0	6.098E-05	3.986E-05	6.533E-06	-3.377E-05	-1.072E-06	-4.179E-07	-2.141E-07
			-1.322E-04	-2.463E-05	-8.317E-07		
60.0	1.012E-05	3.535E-05	3.860E-07	-1.220E-05	-3.153E-07	-1.216E-07	-6.196E-08
			-7.750E-05	-7.108E-06	-2.199E-06		
90.0	8.472E-07	2.333E-05	-3.423E-07	-2.789E-06	-6.061E-08	-2.311E-08	-1.184E-08
			-2.273E-05	-8.315E-07	-1.472E-06		
135.0	1.145E-07	1.816E-05	-1.747E-07	-8.462E-07	-1.743E-08	-6.570E-09	-3.521E-09
			3.793E-06	-8.661E-09	-8.400E-07		
180.0	6.261E-08	1.725E-05	-1.340E-07	-5.932E-07	-1.233E-08	-4.501E-09	-2.070E-09
			9.446E-06	4.851E-08	-6.944E-07		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P)	SB2(1S,6P)
0.0	1.418E-01	-3.877E-04	-1.678E-01	2.082E-01	-1.463E-03	-6.596E-04	-3.202E-04
			2.867E-04	4.246E-04	1.312E-04		
1.0	1.380E-01	-1.457E-03	-7.107E-02	9.241E-02	1.979E-03	-3.116E-06	-1.406E-04
			1.912E-05	4.393E-04	-1.432E-04		
2.5	1.201E-01	-3.475E-03	-2.291E-02	1.968E-02	3.455E-03	1.530E-03	8.066E-04
			-7.667E-05	3.658E-04	-2.146E-04		
5.0	7.582E-02	-4.920E-03	-1.383E-03	6.000E-05	-2.163E-04	-1.091E-04	-6.079E-05
			-1.026E-04	1.465E-04	-1.490E-04		
7.5	3.910E-02	-3.877E-03	9.359E-04	1.651E-04	-1.345E-05	-1.013E-05	-6.771E-06
			-1.150E-04	-4.197E-05	-5.735E-05		
10.0	1.815E-02	-2.296E-03	5.893E-04	-2.950E-05	7.419E-06	4.328E-06	2.561E-06
			-1.239E-04	-1.288E-04	-6.367E-07		
15.0	3.710E-03	-5.505E-04	1.561E-04	-9.470E-05	-2.293E-06	-3.965E-07	-6.482E-08
			-1.294E-04	-1.118E-04	1.603E-05		
20.0	8.610E-04	-9.544E-05	4.342E-05	-5.825E-05	-2.318E-06	-8.415E-07	-4.131E-07
			-1.197E-04	-5.867E-05	7.106E-06		
25.0	2.394E-04	1.986E-06	1.303E-05	-3.223E-05	-1.202E-06	-4.655E-07	-2.376E-07
			-1.022E-04	-2.957E-05	2.174E-06		
30.0	7.834E-05	1.902E-05	4.085E-06	-1.786E-05	-5.810E-07	-2.269E-07	-1.164E-07
			-8.334E-05	-1.545E-05	-8.676E-09		
45.0	5.588E-06	1.268E-05	-5.633E-08	-3.732E-06	-9.095E-08	-3.503E-08	-1.777E-08
			-4.179E-05	-2.781E-06	-1.030E-06		
60.0	8.111E-07	7.363E-06	-1.414E-07	-1.103E-06	-2.382E-08	-9.036E-09	-4.574E-09
			-2.102E-05	-6.662E-07	-7.372E-07		
90.0	5.983E-08	4.579E-06	-4.821E-08	-2.092E-07	-4.590E-09	-1.695E-09	-6.531E-10
			-4.789E-06	-6.531E-08	-3.755E-07		
135.0	7.584E-09	4.010E-06	-1.654E-08	-5.666E-08	-1.932E-09	-6.975E-10	-3.058E-10
			2.001E-06	-5.875E-10	-2.040E-07		
180.0	4.089E-09	3.948E-06	-1.202E-08	-3.839E-08	-1.704E-09	-3.001E-10	-4.516E-10
			3.392E-06	3.426E-09	-1.676E-07		

TABLE C5

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) SB2(1S,1S)	SB2(1S,3P) SB2(1S,2S)	SB2(1S,4P) SB2(1S,3S)	SB2(1S,5P)
0.0	1.347E-01	-1.438E-01	1.570E-02	4.893E-02	-1.360E-03	-1.473E-04
			2.455E-03	7.690E-03	-3.690E-03	
1.0	1.341E-01	-1.331E-01	1.611E-02	4.407E-02	-1.155E-03	-1.259E-04
			2.159E-03	6.761E-03	-3.243E-03	
2.5	1.305E-01	-9.569E-02	1.734E-02	2.733E-02	-5.402E-04	-5.774E-05
			1.131E-03	3.543E-03	-1.697E-03	
5.0	1.188E-01	-4.723E-02	1.694E-02	7.908E-03	-1.910E-04	-1.303E-05
			-1.680E-04	-4.290E-04	2.126E-04	
7.5	1.022E-01	-2.413E-02	1.344E-02	1.944E-03	-2.058E-04	-2.222E-05
			-7.333E-04	-1.993E-03	9.631E-04	
10.0	8.347E-02	-1.249E-02	9.211E-03	6.389E-04	-1.318E-04	-1.745E-05
			-9.701E-04	-2.465E-03	1.184E-03	
15.0	4.947E-02	-2.485E-03	3.356E-03	1.441E-05	-2.318E-05	-2.843E-06
			-1.092E-03	-2.270E-03	1.061E-03	
20.0	2.646E-02	2.976E-04	1.129E-03	-2.388E-04	-5.409E-06	-5.619E-07
			-1.046E-03	-1.678E-03	7.343E-04	
25.0	1.352E-02	5.848E-04	4.212E-04	-2.560E-04	-1.939E-06	-2.673E-07
			-9.394E-04	-1.126E-03	4.364E-04	
30.0	6.866E-03	2.436E-04	1.790E-04	-2.097E-04	-5.496E-07	-1.252E-07
			-8.104E-04	-7.187E-04	2.283E-04	
45.0	1.053E-03	-4.770E-04	2.212E-05	-9.382E-05	3.390E-07	1.674E-08
			-4.340E-04	-1.699E-04	4.225E-06	
60.0	2.263E-04	-5.900E-04	3.648E-06	-4.255E-05	2.384E-07	2.072E-08
			-1.657E-04	-3.504E-05	-1.170E-05	
90.0	2.456E-05	-5.649E-04	2.370E-08	-1.134E-05	8.152E-08	8.052E-09
			9.938E-05	8.449E-06	1.052E-05	
135.0	3.857E-06	-5.351E-04	8.974E-09	-3.336E-06	2.993E-08	3.054E-09
			2.221E-04	8.982E-06	2.601E-05	
180.0	2.188E-06	-5.283E-04	3.304E-08	-2.256E-06	2.197E-08	2.258E-09
			2.472E-04	7.874E-06	2.942E-05	

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E= 300E.V.

THETA	B1	SR2(CLSRE)	SR2(1S,2P) / SR2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SR2(1S,5P)
0.0	1.377E-01	-1.221E-01	1.772E-02	3.777E-02	-1.223E-03	-1.324E-04
			1.725E-03	5.087E-03	-2.425E-03	
1.0	1.367E-01	-1.028E-01	1.716E-02	3.018E-02	-8.830E-04	-9.787E-05
			1.293E-03	3.809E-03	-1.816E-03	
2.5	1.312E-01	-5.646E-02	1.544E-02	1.233E-02	-2.414E-04	-2.502E-05
			2.578E-04	7.750E-04	-3.673E-04	
5.0	1.137E-01	-2.117E-02	1.139E-02	1.615E-03	-1.584E-04	-1.466E-05
			-4.929E-04	-1.315E-03	6.312E-04	
7.5	9.060E-02	-8.934E-03	6.968E-03	2.962E-04	-1.033E-04	-1.371E-05
			-7.054E-04	-1.740E-03	8.327E-04	
10.0	6.736E-02	-3.456E-03	3.722E-03	1.119E-04	-3.404E-05	-4.549E-06
			-7.660E-04	-1.686E-03	7.994E-04	
15.0	3.224E-02	2.926E-04	9.534E-04	-1.403E-04	-4.714E-06	-4.647E-07
			-7.468E-04	-1.216E-03	5.441E-04	
20.0	1.401E-02	6.199E-04	2.890E-04	-1.527E-04	-1.469E-06	-2.028E-07
			-6.637E-04	-7.523E-04	2.931E-04	
25.0	6.000E-03	2.861E-04	1.048E-04	-1.120E-04	-3.157E-07	-7.555E-08
			-5.618E-04	-4.362E-04	1.309E-04	
30.0	2.649E-03	6.933E-06	4.238E-05	-7.816E-05	7.461E-08	-1.716E-08
			-4.593E-04	-2.484E-04	4.590E-05	
45.0	3.144E-04	-2.469E-04	3.980E-06	-2.698E-05	1.393E-07	1.086E-08
			-2.139E-04	-4.784E-05	-1.195E-05	
60.0	5.937E-05	-2.589E-04	4.813E-07	-1.028E-05	6.335E-08	5.898E-09
			-7.120E-05	-8.393E-06	-6.700E-06	
90.0	5.663E-06	-2.445E-04	2.690E-08	-2.259E-06	1.615E-08	1.605E-09
			5.078E-05	2.202E-06	5.290E-06	
135.0	8.232E-07	-2.363E-04	2.308E-08	-5.953E-07	5.076E-09	5.158E-10
			1.033E-04	1.996E-06	1.144E-05	
180.0	4.581E-07	-2.346E-04	2.166E-08	-3.922E-07	3.581E-09	3.662E-10
			1.140E-04	1.702E-06	1.274E-05	

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P)
0.0	1.392E-01	-1.081E-01	1.829E-02	3.138E-02	-1.118E-03	-1.198E-04
			1.340E-03	3.807E-03	-1.809E-03	
1.0	1.377E-01	-8.073E-02	1.640E-02	2.154E-02	-6.651E-04	-7.448E-05
			8.064E-04	2.291E-03	-1.088E-03	
2.5	1.303E-01	-3.507E-02	1.267E-02	5.682E-03	-1.325E-04	-1.203E-05
			-7.672E-05	-1.929E-04	9.345E-05	
5.0	1.077E-01	-1.112E-02	7.676E-03	3.048E-04	-1.234E-04	-1.423E-05
			-5.036E-04	-1.277E-03	6.102E-04	
7.5	7.986E-02	-3.884E-03	3.840E-03	9.435E-05	-4.292E-05	-5.956E-06
			-5.924E-04	-1.342E-03	6.384E-04	
10.0	5.449E-02	-8.877E-04	1.722E-03	-1.872E-05	-1.002E-05	-1.111E-06
			-6.023E-04	-1.172E-03	5.482E-04	
15.0	2.178E-02	6.045E-04	3.818E-04	-1.188E-04	-2.065E-06	-2.378E-07
			-5.504E-04	-7.265E-04	3.076E-04	
20.0	8.093E-03	3.923E-04	1.124E-04	-8.635E-05	-5.057E-07	-8.988E-08
			-4.670E-04	-3.972E-04	1.329E-04	
25.0	3.071E-03	1.060E-04	3.852E-05	-5.563E-05	-1.195E-08	-2.158E-08
			-3.791E-04	-2.096E-04	4.333E-05	
30.0	1.240E-03	-4.408E-05	1.445E-05	-3.592E-05	9.990E-08	7.510E-10
			-2.984E-04	-1.118E-04	5.823E-06	
45.0	1.266E-04	-1.410E-04	1.052E-06	-1.042E-05	5.928E-08	5.094E-09
			-1.270E-04	-1.907E-05	-9.816E-06	
60.0	2.216E-05	-1.416E-04	1.107E-07	-3.552E-06	2.244E-08	2.136E-09
			-3.924E-05	-3.013E-06	-3.915E-06	
90.0	1.955E-06	-1.358E-04	1.598E-08	-6.972E-07	4.951E-09	4.930E-10
			3.052E-05	8.099E-07	3.112E-06	
135.0	2.719E-07	-1.332E-04	1.021E-08	-1.728E-07	1.431E-09	1.452E-10
			5.979E-05	6.755E-07	6.377E-06	
180.0	1.497E-07	-1.327E-04	8.552E-09	-1.123E-07	9.896E-10	1.007E-10
			6.576E-05	5.674E-07	7.060E-06	

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) SB2(1S,1S)	SB2(1S,3P) SB2(1S,2S)	SB2(1S,4P) SB2(1S,3S)	SB2(1S,5P)
0.0	1.401E-01	-9.804E-02	1.835E-02	2.715E-02	-1.032E-03	-1.094E-04
			1.099E-03	3.045E-03	-1.443E-03	
1.0	1.382E-01	-6.393E-02	1.504E-02	1.563E-02	-4.957E-04	-5.601E-05
			5.011E-04	1.388E-03	-6.577E-04	
2.5	1.290E-01	-2.309E-02	1.031E-02	2.604E-03	-9.796E-05	-7.808E-06
			-2.065E-04	-5.403E-04	2.578E-04	
5.0	1.016E-01	-6.491E-03	5.329E-03	3.314E-05	-8.458E-05	-1.100E-05
			-4.594E-04	-1.113E-03	5.303E-04	
7.5	7.041E-02	-1.775E-03	2.256E-03	2.972E-05	-1.765E-05	-2.310E-06
			-4.972E-04	-1.044E-03	4.940E-04	
10.0	4.442E-02	-4.323E-06	9.014E-04	-6.449E-05	-4.256E-06	-3.845E-07
			-4.884E-04	-8.502E-04	3.915E-04	
15.0	1.524E-02	5.253E-04	1.898E-04	-8.448E-05	-1.095E-06	-1.477E-07
			-4.275E-04	-4.686E-04	1.855E-04	
20.0	5.008E-03	2.285E-04	5.344E-05	-5.170E-05	-1.787E-07	-4.137E-08
			-3.498E-04	-2.333E-04	6.429E-05	
25.0	1.741E-03	3.441E-05	1.713E-05	-3.096E-05	4.293E-08	-5.654E-09
			-2.747E-04	-1.154E-04	1.306E-05	
30.0	6.615E-04	-4.610E-05	5.966E-06	-1.888E-05	7.146E-08	3.190E-09
			-2.099E-04	-5.892E-05	-4.474E-06	
45.0	6.098E-05	-8.863E-05	3.520E-07	-4.802E-06	2.841E-08	2.543E-09
			-8.392E-05	-9.208E-06	-7.219E-06	
60.0	1.012E-05	-8.835E-05	3.760E-08	-1.515E-06	9.666E-09	9.297E-10
			-2.481E-05	-1.349E-06	-2.506E-06	
90.0	8.472E-07	-8.631E-05	8.918E-09	-2.758E-07	1.950E-09	1.944E-10
			2.033E-05	3.650E-07	2.036E-06	
135.0	1.145E-07	-8.556E-05	4.725E-09	-6.571E-08	5.342E-10	5.412E-11
			3.906E-05	2.890E-07	4.055E-06	
180.0	6.261E-08	-8.543E-05	3.755E-09	-4.231E-08	3.601E-10	3.713E-11
			4.290E-05	2.404E-07	4.476E-06	

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,3D) STATE OF HELIUM BY ELECTRON IMPACT
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P)
0.0	1.418E-01	-7.163E-02	1.703E-02	1.722E-02	-7.603E-04	-7.728E-05
			5.877E-04	1.526E-03	-7.196E-04	
1.0	1.380E-01	-2.285E-02	8.701E-03	3.401E-03	-1.189E-04	-1.335E-05
			-2.919E-05	-7.164E-05	3.408E-05	
2.5	1.201E-01	-5.319E-03	4.344E-03	-2.152E-04	-7.017E-05	-7.694E-06
			-2.413E-04	-5.844E-04	2.769E-04	
5.0	7.582E-02	-8.056E-04	1.229E-03	-6.470E-06	-7.669E-06	-9.913E-07
			-2.707E-04	-5.483E-04	2.590E-04	
7.5	3.910E-02	2.475E-04	3.299E-04	-4.810E-05	-1.488E-06	-1.229E-07
			-2.573E-04	-3.967E-04	1.802E-04	
10.0	1.815E-02	3.537E-04	1.155E-04	-4.166E-05	-7.509E-07	-9.669E-08
			-2.336E-04	-2.581E-04	1.055E-04	
15.0	3.710E-03	1.248E-04	2.038E-05	-1.800E-05	-8.220E-08	-1.755E-08
			-1.785E-04	-9.720E-05	2.264E-05	
20.0	8.610E-04	2.132E-05	4.322E-06	-8.258E-06	1.409E-08	-1.111E-09
			-1.290E-04	-3.765E-05	-8.167E-07	
25.0	2.394E-04	-7.131E-06	1.029E-06	-4.009E-06	1.705E-08	1.019E-09
			-9.117E-05	-1.591E-05	-5.146E-06	
30.0	7.834E-05	-1.515E-05	2.684E-07	-2.040E-06	1.084E-08	8.774E-10
			-6.415E-05	-7.218E-06	-4.922E-06	
45.0	5.588E-06	-1.961E-05	1.009E-08	-3.608E-07	2.237E-09	2.122E-10
			-2.235E-05	-8.687E-07	-2.085E-06	
60.0	8.111E-07	-2.059E-05	2.597E-09	-9.406E-08	6.099E-10	5.970E-11
			-6.101E-06	-1.049E-07	-5.987E-07	
90.0	5.983E-08	-2.139E-05	8.677E-10	-1.440E-08	1.010E-10	1.010E-11
			5.540E-06	2.804E-08	5.271E-07	
135.0	7.584E-09	-2.177E-05	3.098E-10	-3.154E-09	2.522E-11	2.580E-12
			1.032E-05	1.989E-08	1.001E-06	
180.0	4.089E-09	-2.186E-05	2.184E-10	-1.994E-09	2.026E-11	1.780E-12
			1.131E-05	1.622E-08	1.099E-06	

TABLE C6

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P) / SB2(1S,4S)	SB2(1S,6P)
0.0	9.575E-02	-3.036E-03	-1.386E-01	-3.950E-02	2.585E-01	1.792E-02	7.203E-03
			6.440E-04	1.374E-03	-2.741E-06	-1.775E-05	
1.0	9.530E-02	-3.343E-03	-1.310E-01	-3.578E-02	2.405E-01	1.743E-02	7.086E-03
			5.839E-04	1.377E-03	-2.826E-06	-1.501E-05	
2.5	9.295E-02	-4.644E-03	-1.027E-01	-2.168E-02	1.718E-01	1.418E-02	6.064E-03
			3.696E-04	1.370E-03	-2.745E-06	-4.993E-06	
5.0	8.512E-02	-7.401E-03	-5.861E-02	-3.004E-03	7.261E-02	5.982E-03	2.718E-03
			8.350E-05	1.252E-03	4.266E-08	1.005E-05	
7.5	7.380E-02	-9.626E-03	-3.006E-02	-8.462E-04	2.879E-02	2.325E-03	1.068E-03
			-5.521E-05	1.014E-03	6.263E-06	2.036E-05	
10.0	6.091E-02	-1.077E-02	-1.254E-02	-2.646E-03	9.638E-03	7.624E-04	3.376E-04
			-1.273E-04	7.155E-04	1.461E-05	2.887E-05	
15.0	3.684E-02	-9.952E-03	7.461E-04	-8.961E-04	-1.151E-04	4.454E-07	-2.216E-05
			-2.037E-04	1.289E-04	2.911E-05	4.107E-05	
20.0	2.003E-02	-7.098E-03	1.927E-03	2.647E-04	-8.569E-05	-1.218E-05	-7.412E-06
			-2.488E-04	-2.612E-04	2.705E-05	3.874E-05	
25.0	1.035E-02	-4.366E-03	1.235E-03	2.660E-04	-3.828E-05	-1.883E-05	-5.509E-06
			-2.788E-04	-4.164E-04	8.265E-06	1.962E-05	
30.0	5.296E-03	-2.470E-03	6.965E-04	1.626E-04	-7.589E-05	-2.312E-05	-7.754E-06
			-2.961E-04	-4.148E-04	-1.171E-05	-4.727E-06	
45.0	8.198E-04	-3.168E-04	1.399E-04	5.382E-05	-1.433E-05	-1.451E-05	-5.912E-06
			-2.872E-04	-1.972E-04	-1.536E-05	-2.727E-05	
60.0	1.765E-04	6.005E-05	3.798E-05	2.616E-05	2.143E-05	-6.402E-06	-2.687E-06
			-2.305E-04	-7.693E-05	-1.926E-06	-1.224E-05	
90.0	1.918E-05	1.573E-04	4.091E-06	8.951E-06	1.446E-05	-1.650E-06	-6.910E-07
			-1.217E-04	-1.245E-05	1.343E-06	-1.227E-06	
135.0	3.012E-06	1.731E-04	-4.668E-07	3.482E-06	6.353E-06	-5.391E-07	-2.217E-07
			-4.654E-05	1.521E-07	9.362E-07	1.537E-07	
180.0	1.708E-06	1.754E-04	-7.241E-07	2.589E-06	4.825E-06	-3.855E-07	-1.492E-07
			-2.844E-05	1.256E-06	8.122E-07	2.066E-07	

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E= 300E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) SB2(1S,1S)	SB2(1S,3P) SB2(1S,2S)	SB2(1S,4P) SB2(1S,3S)	SB2(1S,5P) SB2(1S,4S)	SB2(1S,6P)
0.0	9.786E-02	-1.673E-03	-1.221E-01	-4.048E-02	2.476E-01	1.619E-02	5.982E-03
			5.267E-04	9.412E-04	-1.617E-06	-1.364E-05	
1.0	9.714E-02	-2.082E-03	-1.077E-01	-3.394E-02	2.135E-01	1.593E-02	6.115E-03
			4.199E-04	9.384E-04	-9.673E-07	-8.573E-06	
2.5	9.349E-02	-3.543E-03	-6.970E-02	-1.390E-02	1.162E-01	1.086E-02	4.834E-03
			1.549E-04	9.040E-04	9.997E-07	4.127E-06	
5.0	8.173E-02	-6.087E-03	-3.143E-02	6.030E-04	3.285E-02	2.488E-03	1.140E-03
			-5.256E-05	7.455E-04	4.552E-06	1.487E-05	
7.5	6.592E-02	-7.646E-03	-1.161E-02	-1.619E-03	9.171E-03	7.164E-04	3.294E-04
			-1.250E-04	5.109E-04	9.210E-06	2.011E-05	
10.0	4.964E-02	-7.882E-03	-2.226E-03	-1.689E-03	1.080E-03	6.726E-05	8.062E-06
			-1.610E-04	2.595E-04	1.426E-05	2.391E-05	
15.0	2.430E-02	-5.801E-03	1.435E-03	1.297E-04	-6.877E-05	-5.586E-06	-6.289E-06
			-2.023E-04	-1.276E-04	1.653E-05	2.434E-05	
20.0	1.072E-02	-3.228E-03	8.659E-04	1.879E-04	5.405E-08	-9.889E-06	-2.385E-06
			-2.269E-04	-2.781E-04	3.343E-06	1.031E-05	
25.0	4.629E-03	-1.562E-03	4.189E-04	9.861E-05	-4.530E-05	-1.400E-05	-4.692E-06
			-2.388E-04	-2.689E-04	-1.077E-05	-7.837E-06	
30.0	2.053E-03	-6.973E-04	2.050E-04	5.741E-05	-3.607E-05	-1.248E-05	-4.754E-06
			-2.390E-04	-2.063E-04	-1.452E-05	-1.755E-05	
45.0	2.447E-04	-3.789E-06	3.104E-05	1.909E-05	1.087E-05	-4.614E-06	-1.931E-06
			-1.952E-04	-6.782E-05	-2.609E-06	-9.726E-06	
60.0	4.625E-05	6.407E-05	6.243E-06	8.081E-06	1.123E-05	-1.636E-06	-6.861E-07
			-1.351E-04	-2.277E-05	5.424E-07	-2.417E-06	
90.0	4.412E-06	6.235E-05	-6.906E-08	2.299E-06	4.105E-06	-3.622E-07	-1.509E-07
			-5.360E-05	-3.089E-06	5.426E-07	-1.794E-07	
135.0	6.413E-07	5.207E-05	-3.875E-07	8.014E-07	1.502E-06	-1.110E-07	-3.581E-08
			-5.201E-06	2.020E-08	3.775E-07	-4.069E-08	
180.0	3.568E-07	4.912E-05	-3.424E-07	5.820E-07	1.102E-06	-7.572E-08	-2.253E-09
			6.163E-06	2.519E-07	3.441E-07	-3.965E-08	

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) SB2(1S,1S)	SB2(1S,3P) SB2(1S,2S)	SB2(1S,4P) SB2(1S,3S)	SB2(1S,5P) SB2(1S,4S)	SB2(1S,6P)
0.0	9.952E-02	-7.924E-04	-1.016E-01	-3.951E-02	2.241E-01	1.156E-02	3.563E-03
			3.726E-04	5.814E-04	-9.617E-07	-9.463E-06	
1.0	9.829E-02	-1.355E-03	-7.407E-02	-2.707E-02	1.610E-01	1.275E-02	4.479E-03
			2.027E-04	5.673E-04	6.733E-07	-1.362E-06	
2.5	9.210E-02	-2.880E-03	-3.595E-02	-4.322E-03	5.356E-02	5.159E-03	2.556E-03
			-9.634E-06	5.106E-04	2.874E-06	8.607E-06	
5.0	7.356E-02	-4.957E-03	-1.126E-02	-7.448E-05	9.763E-03	7.574E-04	3.509E-04
			-9.704E-05	3.488E-04	4.877E-06	1.246E-05	
7.5	5.183E-02	-5.518E-03	-1.482E-03	-1.209E-03	6.425E-04	2.702E-05	-3.378E-06
			-1.239E-04	1.542E-04	7.272E-06	1.372E-05	
10.0	3.321E-02	-4.785E-03	8.683E-04	-1.312E-04	-1.423E-04	-8.986E-06	-1.161E-05
			-1.397E-04	-1.294E-05	8.367E-06	1.357E-05	
15.0	1.164E-02	-2.379E-03	5.892E-04	1.286E-04	2.417E-05	-3.743E-06	-3.481E-07
			-1.599E-04	-1.686E-04	5.521E-07	4.418E-06	
20.0	3.866E-03	-9.041E-04	2.239E-04	5.372E-05	-2.443E-05	-7.607E-06	-2.566E-06
			-1.676E-04	-1.559E-04	-8.465E-06	-8.097E-06	
25.0	1.350E-03	-2.988E-04	8.735E-05	2.773E-05	-1.338E-05	-6.012E-06	-2.354E-06
			-1.637E-04	-1.044E-04	-7.910E-06	-1.154E-05	
30.0	5.139E-04	-7.880E-05	3.614E-05	1.654E-05	6.014E-07	-3.849E-06	-1.584E-06
			-1.514E-04	-6.435E-05	-3.939E-06	-8.732E-06	
45.0	4.743E-05	3.047E-05	3.400E-06	4.511E-06	6.031E-06	-8.853E-07	-3.720E-07
			-9.960E-05	-1.566E-05	2.388E-07	-1.479E-06	
60.0	7.873E-06	2.726E-05	1.950E-07	1.622E-06	2.786E-06	-2.662E-07	-1.102E-07
			-5.868E-05	-4.470E-06	2.969E-07	-2.570E-07	
90.0	6.588E-07	1.799E-05	-1.853E-07	3.815E-07	7.154E-07	-5.223E-08	-2.346E-08
			-1.749E-05	-5.084E-07	1.826E-07	-3.772E-08	
135.0	8.900E-08	1.398E-05	-9.465E-08	1.179E-07	2.283E-07	-1.638E-08	-5.860E-09
			2.500E-06	1.314E-09	1.299E-07	-1.659E-08	
180.0	4.868E-08	1.328E-05	-7.264E-08	8.242E-08	1.611E-07	-1.314E-08	3.927E-09
			6.761E-06	3.568E-08	1.187E-07	-1.363E-08	

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) SB2(1S,1S)	SB2(1S,3P) SB2(1S,2S)	SB2(1S,4P) SB2(1S,3S)	SB2(1S,5P) SB2(1S,4S)	SB2(1S,6P)
0.0	9.890E-02	-1.099E-03	-1.103E-01	-4.032E-02	2.368E-01	1.384E-02	4.667E-03
			4.378E-04	7.182E-04	-1.132E-06	-1.104E-05	
1.0	9.793E-02	-1.590E-03	-8.907E-02	-3.064E-02	1.862E-01	1.422E-02	5.183E-03
			2.939E-04	7.093E-04	6.394E-08	-4.231E-06	
2.5	9.299E-02	-3.101E-03	-4.901E-02	-8.129E-03	7.818E-02	7.705E-03	3.620E-03
			4.396E-05	6.594E-04	2.406E-06	7.579E-06	
5.0	7.767E-02	-5.401E-03	-1.836E-02	6.088E-04	1.712E-02	1.282E-03	5.825E-04
			-8.981E-05	4.948E-04	5.156E-06	1.410E-05	
7.5	5.846E-02	-6.411E-03	-4.499E-03	-1.675E-03	2.832E-03	1.926E-04	7.712E-05
			-1.313E-04	2.798E-04	8.504E-06	1.677E-05	
10.0	4.048E-02	-6.055E-03	3.273E-04	-6.480E-04	-1.550E-04	-1.363E-05	-2.169E-05
			-1.533E-04	7.407E-05	1.132E-05	1.819E-05	
15.0	1.655E-02	-3.624E-03	9.412E-04	1.842E-04	3.561E-05	-2.579E-06	-4.038E-07
			-1.807E-04	-1.752E-04	6.863E-06	1.240E-05	
20.0	6.227E-03	-1.643E-03	4.167E-04	9.468E-05	-2.261E-05	-9.323E-06	-2.752E-06
			-1.950E-04	-2.135E-04	-6.093E-06	-2.941E-06	
25.0	2.377E-03	-6.527E-04	1.779E-04	4.759E-05	-2.855E-05	-9.251E-06	-3.440E-06
			-1.970E-04	-1.648E-04	-1.115E-05	-1.295E-05	
30.0	9.630E-04	-2.319E-04	7.941E-05	2.856E-05	-9.828E-06	-6.749E-06	-2.711E-06
			-1.889E-04	-1.103E-04	-8.187E-06	-1.351E-05	
45.0	9.853E-05	3.108E-05	9.466E-06	8.633E-06	9.168E-06	-1.861E-06	-7.827E-07
			-1.365E-04	-3.017E-05	-1.595E-07	-3.581E-06	
60.0	1.724E-05	4.148E-05	1.272E-06	3.323E-06	5.361E-06	-5.943E-07	-2.471E-07
			-8.591E-05	-9.226E-06	4.408E-07	-6.822E-07	
90.0	1.522E-06	3.097E-05	-2.589E-07	8.449E-07	1.561E-06	-1.229E-07	-4.943E-08
			-2.872E-05	-1.126E-06	2.905E-07	-6.849E-08	
135.0	2.116E-07	2.417E-05	-1.859E-07	2.752E-07	5.228E-07	-3.663E-08	-1.513E-08
			1.376E-06	4.313E-09	2.065E-07	-2.764E-08	
180.0	1.165E-07	2.275E-05	-1.491E-07	1.969E-07	3.740E-07	-2.917E-08	-1.152E-08
			8.021E-06	8.350E-08	1.891E-07	-2.361E-08	

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P) / SB2(1S,4S)	SB2(1S,6P)
0.0	1.008E-01	-2.745E-04	-7.691E-02	-3.480E-02	1.908E-01	4.175E-03	6.651E-04
			2.121E-04	2.992E-04	-7.987E-07	-5.949E-06	
1.0	9.823E-02	-1.037E-03	-3.303E-02	-1.371E-02	7.844E-02	8.459E-03	3.011E-03
			1.859E-05	2.730E-04	1.653E-06	3.764E-06	
2.5	8.620E-02	-2.487E-03	-1.169E-02	1.074E-03	1.219E-02	6.294E-04	2.904E-04
			-5.450E-05	2.163E-04	2.300E-06	6.760E-06	
5.0	5.566E-02	-3.608E-03	-9.314E-04	-7.529E-04	4.827E-04	1.422E-05	-1.814E-06
			-7.464E-05	8.168E-05	2.480E-06	6.236E-06	
7.5	2.934E-02	-2.916E-03	6.235E-04	4.185E-05	-1.203E-05	-8.505E-07	-2.605E-06
			-8.434E-05	-3.318E-05	2.007E-06	4.632E-06	
10.0	1.383E-02	-1.758E-03	3.878E-04	8.491E-05	3.872E-05	2.479E-07	8.689E-07
			-9.135E-05	-8.605E-05	-1.015E-06	8.162E-07	
15.0	2.867E-03	-4.299E-04	9.412E-05	2.377E-05	-1.059E-05	-3.352E-06	-1.155E-06
			-9.621E-05	-7.313E-05	-5.413E-06	-6.426E-06	
20.0	6.681E-04	-7.578E-05	2.441E-05	9.973E-06	-1.386E-06	-2.052E-06	-8.309E-07
			-8.950E-05	-3.801E-05	-2.582E-06	-5.207E-06	
25.0	1.860E-04	8.428E-07	6.991E-06	4.860E-06	3.028E-06	-9.905E-07	-4.146E-07
			-7.664E-05	-1.899E-05	-4.788E-07	-2.367E-06	
30.0	6.086E-05	1.446E-05	2.134E-06	2.529E-06	2.954E-06	-4.769E-07	-2.004E-07
			-6.267E-05	-9.866E-06	7.576E-08	-9.134E-07	
45.0	4.341E-06	9.784E-06	-3.497E-08	5.076E-07	8.848E-07	-7.691E-08	-3.201E-08
			-3.153E-05	-1.755E-06	1.163E-07	-7.833E-08	
60.0	6.299E-07	5.683E-06	-7.685E-08	1.516E-07	2.831E-07	-2.070E-08	-7.756E-09
			-1.590E-05	-4.173E-07	7.608E-08	-2.128E-08	
90.0	4.645E-08	3.524E-06	-2.611E-08	2.817E-08	5.670E-08	-4.514E-09	-1.701E-09
			-3.684E-06	-4.003E-08	4.498E-08	-6.173E-09	
135.0	5.887E-09	3.079E-06	-8.956E-09	8.789E-09	1.641E-08	-1.866E-09	1.372E-08
			1.426E-06	9.201E-11	3.101E-08	-3.033E-09	
180.0	3.174E-09	3.030E-06	-6.387E-09	3.936E-09	1.321E-08	-4.876E-09	1.211E-08
			2.472E-06	2.511E-09	2.803E-08	-2.610E-09	

TABLE C7

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P) / SB2(1S,4S)
0.0	9.575E-02	-1.060E-01	2.726E-02	-3.196E-04	2.160E-02	-6.347E-04
			1.757E-03	3.908E-03	-1.365E-06	1.818E-04
1.0	9.530E-02	-9.842E-02	2.591E-02	3.887E-05	1.939E-02	-5.506E-04
			1.556E-03	3.459E-03	-1.179E-06	1.609E-04
2.5	9.295E-02	-7.149E-02	2.101E-02	1.078E-03	1.190E-02	-3.320E-04
			8.446E-04	1.875E-03	-5.403E-07	8.719E-05
5.0	8.512E-02	-3.548E-02	1.384E-02	1.275E-03	3.864E-03	-1.856E-04
			-8.751E-05	-1.538E-04	1.025E-07	-7.879E-06
7.5	7.380E-02	-1.808E-02	9.313E-03	5.192E-04	1.431E-03	-9.392E-05
			-5.058E-04	-9.848E-04	2.205E-07	-4.772E-05
10.0	6.091E-02	-9.413E-03	6.090E-03	1.956E-04	5.579E-04	-4.727E-05
			-6.845E-04	-1.250E-03	2.719E-07	-6.123E-05
15.0	3.684E-02	-2.072E-03	2.155E-03	1.450E-04	3.992E-05	-1.147E-05
			-7.806E-04	-1.174E-03	4.268E-07	-5.920E-05
20.0	2.003E-02	5.807E-06	6.413E-04	5.738E-05	-7.216E-05	-2.297E-06
			-7.512E-04	-8.780E-04	-5.315E-08	-4.574E-05
25.0	1.035E-02	2.739E-04	1.838E-04	1.585E-05	-7.393E-05	4.904E-08
			-6.765E-04	-5.930E-04	-9.820E-07	-3.218E-05
30.0	5.296E-03	6.678E-05	5.051E-05	4.369E-06	-4.679E-05	6.279E-07
			-5.847E-04	-3.792E-04	-1.451E-06	-2.154E-05
45.0	8.198E-04	-4.020E-04	-3.758E-06	-2.910E-07	-2.289E-06	4.539E-07
			-3.140E-04	-8.882E-05	-4.070E-07	-5.441E-06
60.0	1.765E-04	-4.613E-04	-2.393E-06	-4.512E-07	2.759E-06	2.063E-07
			-1.201E-04	-1.800E-05	1.003E-08	-1.080E-06
90.0	1.918E-05	-4.247E-04	-3.549E-07	-2.352E-07	1.348E-06	5.525E-08
			7.159E-05	4.461E-06	4.361E-10	2.380E-07
135.0	3.012E-06	-3.953E-04	3.825E-08	-1.083E-07	4.608E-07	1.782E-08
			1.600E-04	4.649E-06	-1.745E-08	2.661E-07
180.0	1.708E-06	-3.888E-04	6.596E-08	-8.362E-08	3.201E-07	1.261E-08
			1.780E-04	4.064E-06	-2.057E-08	2.432E-07

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E= 300E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P) / SB2(1S,4S)
0.0	9.786E-02	-8.967E-02	2.449E-02	-2.901E-04	1.705E-02	-5.413E-04
			1.238E-03	2.589E-03	2.111E-06	1.193E-04
1.0	9.714E-02	-7.608E-02	2.166E-02	2.236E-04	1.352E-02	-3.919E-04
			9.420E-04	1.969E-03	1.638E-06	9.074E-05
2.5	9.349E-02	-4.235E-02	1.444E-02	1.090E-03	5.455E-03	-1.832E-04
			2.116E-04	4.476E-04	4.253E-07	2.053E-05
5.0	8.173E-02	-1.583E-02	7.895E-03	5.654E-04	1.155E-03	-8.937E-05
			-3.399E-04	-6.470E-04	-6.150E-07	-3.073E-05
7.5	6.592E-02	-6.695E-03	4.533E-03	1.379E-04	3.482E-04	-3.446E-05
			-5.004E-04	-8.842E-04	-9.070E-07	-4.272E-05
10.0	4.964E-02	-2.688E-03	2.413E-03	1.181E-04	9.126E-05	-1.454E-05
			-5.477E-04	-8.677E-04	-8.319E-07	-4.280E-05
15.0	2.430E-02	6.292E-05	5.635E-04	4.966E-05	-3.938E-05	-2.327E-06
			-5.372E-04	-6.352E-04	-6.474E-07	-3.273E-05
20.0	1.072E-02	3.512E-04	1.329E-04	1.062E-05	-4.485E-05	-1.107E-07
			-4.787E-04	-3.963E-04	-9.632E-07	-2.149E-05
25.0	4.629E-03	1.411E-04	3.324E-05	2.703E-06	-2.364E-05	3.228E-07
			-4.059E-04	-2.304E-04	-1.041E-06	-1.319E-05
30.0	2.053E-03	-4.363E-05	7.432E-06	7.647E-07	-9.060E-06	3.305E-07
			-3.323E-04	-1.309E-04	-7.069E-07	-7.796E-06
45.0	2.447E-04	-2.028E-04	-9.189E-07	-1.536E-07	1.314E-06	1.292E-07
			-1.549E-04	-2.491E-05	-1.120E-08	-1.471E-06
60.0	4.625E-05	-2.011E-04	-3.161E-07	-1.343E-07	1.049E-06	4.755E-08
			-5.158E-05	-4.306E-06	1.967E-08	-2.430E-07
90.0	4.412E-06	-1.830E-04	1.939E-08	-5.230E-08	3.025E-07	1.044E-08
			3.667E-05	1.156E-06	-8.571E-09	6.304E-08
135.0	6.413E-07	-1.740E-04	3.439E-08	-2.009E-08	8.675E-08	2.981E-09
			7.451E-05	1.034E-06	-2.160E-08	6.795E-08
180.0	3.568E-07	-1.722E-04	2.971E-08	-1.468E-08	5.809E-08	2.048E-09
			8.220E-05	8.797E-07	-2.422E-08	6.352E-08

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) / SB2(1S,1S)	SB2(1S,3P) / SB2(1S,2S)	SB2(1S,4P) / SB2(1S,3S)	SB2(1S,5P) / SB2(1S,4S)
0.0	9.890E-02	-7.919E-02	2.256E-02	-2.112E-04	1.440E-02	-4.928E-04
			9.627E-04	1.940E-03	2.837E-06	8.896E-05
1.0	9.793E-02	-5.988E-02	1.815E-02	3.912E-04	9.768E-03	-2.846E-04
			5.954E-04	1.199E-03	1.780E-06	5.499E-05
2.5	9.299E-02	-2.633E-02	1.026E-02	9.111E-04	2.627E-03	-1.263E-04
			-3.747E-05	-6.491E-05	-7.789E-08	-3.140E-06
5.0	7.767E-02	-8.278E-03	4.999E-03	2.289E-04	4.556E-04	-4.518E-05
			-3.555E-04	-6.434E-04	-1.072E-06	-3.050E-05
7.5	5.846E-02	-2.945E-03	2.474E-03	9.011E-05	1.032E-04	-1.580E-05
			-4.237E-04	-6.884E-04	-1.160E-06	-3.345E-05
10.0	4.048E-02	-7.728E-04	1.099E-03	8.190E-05	9.192E-06	-5.514E-06
			-4.328E-04	-6.072E-04	-9.584E-07	-3.029E-05
15.0	1.655E-02	3.467E-04	2.030E-04	1.674E-05	-3.658E-05	-6.529E-07
			-3.970E-04	-3.816E-04	-8.013E-07	-2.013E-05
20.0	6.227E-03	2.288E-04	4.414E-05	3.354E-06	-2.137E-05	1.299E-07
			-3.375E-04	-2.099E-04	-8.683E-07	-1.179E-05
25.0	2.377E-03	3.912E-05	9.935E-06	9.119E-07	-7.519E-06	2.101E-07
			-2.744E-04	-1.107E-04	-5.938E-07	-6.537E-06
30.0	9.630E-04	-5.957E-05	1.822E-06	2.334E-07	-1.549E-06	1.625E-07
			-2.161E-04	-5.881E-05	-2.532E-07	-3.534E-06
45.0	9.853E-05	-1.150E-04	-2.606E-07	-7.842E-08	8.515E-07	4.829E-08
			-9.203E-05	-9.921E-06	2.515E-08	-5.624E-07
60.0	1.724E-05	-1.096E-04	-4.158E-08	-5.269E-08	4.198E-07	1.587E-08
			-2.842E-05	-1.547E-06	1.293E-08	-8.536E-08
90.0	1.522E-06	-1.014E-04	2.390E-08	-1.704E-08	9.807E-08	3.126E-09
			2.207E-05	4.242E-07	-7.487E-09	2.474E-08
135.0	2.116E-07	-9.793E-05	1.440E-08	-6.002E-09	2.579E-08	8.313E-10
			4.318E-05	3.500E-07	-1.636E-08	2.767E-08
180.0	1.165E-07	-9.725E-05	1.124E-08	-4.210E-09	1.694E-08	5.580E-10
			4.748E-05	2.935E-07	-1.817E-08	2.660E-08

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P) SB2(1S,1S)	SB2(1S,3P) SB2(1S,2S)	SB2(1S,4P) SB2(1S,3S)	SB2(1S,5P) SB2(1S,4S)
0.0	9.952E-02	-7.173E-02	2.108E-02	-1.295E-04	1.262E-02	-4.614E-04
			7.906E-04	1.553E-03	2.925E-06	7.097E-05
1.0	9.829E-02	-4.757E-02	1.523E-02	5.044E-04	7.152E-03	-2.079E-04
			3.761E-04	7.382E-04	1.416E-06	3.375E-05
2.5	9.210E-02	-1.731E-02	7.597E-03	7.023E-04	1.336E-03	-9.711E-05
			-1.370E-04	-2.549E-04	-4.935E-07	-1.187E-05
5.0	7.356E-02	-4.820E-03	3.390E-03	1.011E-04	2.063E-04	-2.554E-05
			-3.272E-04	-5.660E-04	-1.204E-06	-2.689E-05
7.5	5.183E-02	-1.386E-03	1.450E-03	7.913E-05	3.237E-05	-8.037E-06
			-3.570E-04	-5.387E-04	-1.127E-06	-2.637E-05
10.0	3.321E-02	-1.020E-04	5.579E-04	4.889E-05	-1.301E-05	-2.502E-06
			-3.518E-04	-4.427E-04	-8.836E-07	-2.235E-05
15.0	1.164E-02	3.204E-04	9.207E-05	6.914E-06	-2.549E-05	-1.914E-07
			-3.088E-04	-2.470E-04	-7.602E-07	-1.334E-05
20.0	3.866E-03	1.296E-04	1.881E-05	1.503E-06	-1.008E-05	1.320E-07
			-2.532E-04	-1.234E-04	-6.304E-07	-7.122E-06
25.0	1.350E-03	4.443E-07	3.825E-06	4.046E-07	-2.371E-06	1.277E-07
			-1.990E-04	-6.089E-05	-2.973E-07	-3.639E-06
30.0	5.139E-04	-5.112E-05	5.863E-07	8.504E-08	6.821E-08	8.607E-08
			-1.522E-04	-3.095E-05	-7.577E-08	-1.842E-06
45.0	4.743E-05	-7.212E-05	-7.835E-08	-4.293E-08	4.785E-07	2.164E-08
			-6.085E-05	-4.787E-06	2.415E-08	-2.640E-07
60.0	7.873E-06	-6.831E-05	5.169E-09	-2.454E-08	1.925E-07	6.607E-09
			-1.797E-05	-6.932E-07	8.954E-09	-3.833E-08
90.0	6.588E-07	-6.437E-05	1.441E-08	-7.150E-09	3.983E-08	1.205E-09
			1.471E-05	1.909E-07	-5.904E-09	1.222E-08
135.0	8.900E-08	-6.286E-05	6.502E-09	-1.971E-09	9.935E-09	3.164E-10
			2.823E-05	1.498E-07	-1.222E-08	1.445E-08
180.0	4.868E-08	-6.257E-05	4.849E-09	-5.845E-10	6.446E-09	2.091E-10
			3.100E-05	1.245E-07	-1.351E-08	1.425E-08

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (1S,4D) STATE OF HELIUM BY ELECTRON IMPACT
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,2P)	SB2(1S,3P)	SB2(1S,4P)	SB2(1S,5P)
			SB2(1S,1S)	SB2(1S,2S)	SB2(1S,3S)	SB2(1S,4S)
0.0	1.008E-01	-5.222E-02	1.671E-02	1.489E-04	8.280E-03	-3.746E-04
			4.240E-04	7.803E-04	2.248E-06	3.541E-05
1.0	9.823E-02	-1.717E-02	6.858E-03	5.502E-04	1.635E-03	-5.979E-05
			-1.333E-05	-2.277E-05	-6.291E-08	-1.058E-06
2.5	8.620E-02	-3.925E-03	2.677E-03	1.322E-04	9.819E-05	-2.626E-05
			-1.721E-04	-2.965E-04	-9.096E-07	-1.376E-05
5.0	5.566E-02	-6.249E-04	7.823E-04	4.633E-05	3.154E-06	-3.829E-06
			-1.951E-04	-2.834E-04	-8.906E-07	-1.373E-05
7.5	2.934E-02	1.318E-04	1.958E-04	1.790E-05	-1.239E-05	-8.040E-07
			-1.860E-04	-2.074E-04	-6.253E-07	-1.054E-05
10.0	1.383E-02	2.233E-04	5.908E-05	4.226E-06	-1.298E-05	-1.961E-07
			-1.691E-04	-1.361E-04	-4.911E-07	-7.266E-06
15.0	2.867E-03	7.460E-05	7.862E-06	6.487E-07	-3.010E-06	3.534E-08
			-1.295E-04	-5.149E-05	-2.842E-07	-3.005E-06
20.0	6.681E-04	5.926E-06	1.171E-06	1.408E-07	-1.735E-07	3.143E-08
			-9.368E-05	-1.986E-05	-5.951E-08	-1.179E-06
25.0	1.860E-04	-1.136E-05	1.668E-07	2.528E-08	2.170E-07	1.694E-08
			-6.623E-05	-8.352E-06	1.158E-08	-4.776E-07
30.0	6.086E-05	-1.522E-05	1.932E-08	-1.900E-09	1.857E-07	8.743E-09
			-4.659E-05	-3.777E-06	2.010E-08	-2.075E-07
45.0	4.341E-06	-1.599E-05	6.244E-09	-4.872E-09	4.724E-08	1.509E-09
			-1.621E-05	-4.517E-07	8.998E-09	-2.488E-08
60.0	6.299E-07	-1.588E-05	4.624E-09	-1.898E-09	1.347E-08	3.901E-10
			-4.421E-06	-5.414E-08	2.606E-09	-3.563E-09
90.0	4.645E-08	-1.591E-05	1.392E-09	-3.891E-10	2.175E-09	5.475E-11
			4.015E-06	1.463E-08	-2.126E-09	1.668E-09
135.0	5.887E-09	-1.596E-05	4.184E-10	4.056E-11	4.897E-10	1.078E-11
			7.477E-06	1.033E-08	-4.080E-09	2.472E-09
180.0	3.174E-09	-1.598E-05	2.716E-10	-4.454E-10	3.371E-10	1.409E-11
			8.189E-06	8.417E-09	-4.483E-09	2.600E-09

TABLE C8

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) SB2(2S,2P)	SB2(1S,2S) SB2(2S,3P)	SB2(1S,3S) SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	2.617E-01	-2.376E-02	6.042E-03	5.506E-03	1.943E-03	-7.947E-03	-1.929E-03
			-1.920E-02	-1.428E-03	2.505E-03		
1.0	2.599E-01	-2.375E-02	5.996E-03	5.449E-03	1.922E-03	-7.909E-03	-1.920E-03
			-1.894E-02	-1.413E-03	2.493E-03		
2.5	2.511E-01	-2.370E-02	5.765E-03	5.167E-03	1.815E-03	-7.714E-03	-1.873E-03
			-1.761E-02	-1.335E-03	2.347E-03		
5.0	2.240E-01	-2.342E-02	5.045E-03	4.291E-03	1.482E-03	-7.118E-03	-1.732E-03
			-1.364E-02	-1.086E-03	1.901E-03		
7.5	1.895E-01	-2.270E-02	4.108E-03	3.169E-03	1.053E-03	-6.366E-03	-1.554E-03
			-8.861E-03	-7.488E-04	1.344E-03		
10.0	1.549E-01	-2.140E-02	3.137E-03	2.042E-03	6.199E-04	-5.619E-03	-1.377E-03
			-4.569E-03	-3.981E-04	8.137E-04		
15.0	9.801E-02	-1.722E-02	1.416E-03	2.138E-04	-8.618E-05	-4.383E-03	-1.088E-03
			6.447E-04	1.319E-04	8.683E-05		
20.0	5.996E-02	-1.208E-02	9.510E-05	-9.193E-04	-5.126E-04	-3.513E-03	-8.867E-04
			1.864E-03	3.246E-04	-1.712E-04		
25.0	3.631E-02	-7.311E-03	-8.703E-04	-1.498E-03	-7.022E-04	-2.902E-03	-7.480E-04
			1.039E-03	2.807E-04	-1.370E-04		
30.0	2.206E-02	-3.565E-03	-1.555E-03	-1.714E-03	-7.321E-04	-2.452E-03	-6.475E-04
			-3.123E-04	1.522E-04	7.493E-06		
45.0	5.439E-03	1.894E-03	-2.514E-03	-1.458E-03	-4.554E-04	-1.555E-03	-4.478E-04
			-2.626E-03	-1.068E-04	3.302E-04		
60.0	1.642E-03	3.044E-03	-2.647E-03	-9.786E-04	-1.721E-04	-9.794E-04	-3.114E-04
			-2.678E-03	-1.415E-04	3.661E-04		
90.0	2.712E-04	2.821E-03	-2.319E-03	-4.292E-04	6.067E-05	-3.917E-04	-1.519E-04
			-1.930E-03	-1.077E-04	2.782E-04		
135.0	5.781E-05	2.259E-03	-1.899E-03	-1.852E-04	1.133E-04	-1.465E-04	-6.983E-05
			-1.409E-03	-7.997E-05	2.060E-04		
180.0	3.580E-05	2.087E-03	-1.771E-03	-1.396E-04	1.159E-04	-1.048E-04	-5.336E-05
			-1.282E-03	-7.307E-05	1.878E-04		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 300E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) / SB2(2S,2P)	SB2(1S,2S) / SB2(2S,3P)	SB2(1S,3S) / SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	3.582E-01	-1.564E-02	5.586E-03	5.685E-03	2.102E-03	-7.198E-03	-1.720E-03
			-2.583E-02	-1.914E-03	3.152E-03		
1.0	3.528E-01	-1.568E-02	5.494E-03	5.569E-03	2.058E-03	-7.110E-03	-1.700E-03
			-2.519E-02	-1.880E-03	3.084E-03		
2.5	3.271E-01	-1.585E-02	5.055E-03	5.016E-03	1.848E-03	-6.695E-03	-1.602E-03
			-2.218E-02	-1.712E-03	2.762E-03		
5.0	2.607E-01	-1.604E-02	3.907E-03	3.573E-03	1.297E-03	-5.638E-03	-1.353E-03
			-1.461E-02	-1.237E-03	1.930E-03		
7.5	1.949E-01	-1.554E-02	2.727E-03	2.118E-03	7.351E-04	-4.610E-03	-1.112E-03
			-7.699E-03	-7.047E-04	1.123E-03		
10.0	1.421E-01	-1.417E-02	1.724E-03	9.471E-04	2.770E-04	-3.795E-03	-9.212E-04
			-3.062E-03	-2.643E-04	5.319E-04		
15.0	7.355E-02	-9.774E-03	2.634E-04	-5.000E-04	-2.908E-04	-2.713E-03	-6.717E-04
			3.797E-04	1.538E-04	1.724E-06		
20.0	3.782E-02	-5.312E-03	-6.652E-04	-1.100E-03	-5.043E-04	-2.071E-03	-5.265E-04
			2.497E-05	1.475E-04	-1.740E-05		
25.0	1.968E-02	-2.058E-03	-1.237E-03	-1.242E-03	-5.145E-04	-1.648E-03	-4.322E-04
			-1.039E-03	3.286E-05	1.050E-04		
30.0	1.050E-02	-7.204E-05	-1.567E-03	-1.170E-03	-4.340E-04	-1.335E-03	-3.627E-04
			-1.771E-03	-5.463E-05	2.079E-04		
45.0	1.964E-03	1.724E-03	-1.781E-03	-6.983E-04	-1.460E-04	-7.056E-04	-2.160E-04
			-1.988E-03	-1.112E-04	2.660E-04		
60.0	5.043E-04	1.632E-03	-1.584E-03	-3.678E-04	-1.695E-06	-3.577E-04	-1.242E-04
			-1.512E-03	-8.834E-05	2.103E-04		
90.0	7.038E-05	1.090E-03	-1.149E-03	-1.117E-04	6.154E-05	-1.003E-04	-4.245E-05
			-9.476E-04	-5.709E-05	1.336E-04		
135.0	1.359E-05	7.090E-04	-8.248E-04	-3.126E-05	5.779E-05	-2.784E-05	-1.291E-05
			-6.548E-04	-4.033E-05	9.149E-05		
180.0	8.217E-06	6.174E-04	-7.414E-04	-1.949E-05	5.377E-05	-1.824E-05	-8.302E-06
			-5.877E-04	-3.642E-05	8.176E-05		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) SB2(2S,2P)	SB2(1S,2S) SB2(2S,3P)	SB2(1S,3S) SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	4.356E-01	-1.165E-02	5.343E-03	5.593E-03	2.092E-03	-6.631E-03	-1.575E-03
			-2.877E-02	-2.092E-03	3.395E-03		
1.0	4.240E-01	-1.175E-02	5.192E-03	5.404E-03	2.021E-03	-6.482E-03	-1.540E-03
			-2.765E-02	-2.034E-03	3.278E-03		
2.5	3.734E-01	-1.213E-02	4.528E-03	4.574E-03	1.706E-03	-5.833E-03	-1.388E-03
			-2.278E-02	-1.769E-03	2.764E-03		
5.0	2.660E-01	-1.262E-02	3.089E-03	2.776E-03	1.019E-03	-4.486E-03	-1.071E-03
			-1.271E-02	-1.122E-03	1.664E-03		
7.5	1.803E-01	-1.206E-02	1.882E-03	1.308E-03	4.486E-04	-3.439E-03	-8.271E-04
			-5.546E-03	-5.182E-04	8.141E-04		
10.0	1.209E-01	-1.046E-02	9.788E-04	2.991E-04	5.027E-05	-2.723E-03	-6.611E-04
			-1.785E-03	-1.168E-04	3.116E-04		
15.0	5.429E-02	-6.063E-03	-2.021E-04	-7.182E-04	-3.469E-04	-1.871E-03	-4.667E-04
			-1.329E-04	1.091E-04	1.738E-05		
20.0	2.469E-02	-2.448E-03	-8.654E-04	-9.855E-04	-4.201E-04	-1.393E-03	-3.601E-04
			-9.120E-04	2.591E-05	9.216E-05		
25.0	1.159E-02	-3.151E-04	-1.214E-03	-9.393E-04	-3.551E-04	-1.075E-03	-2.894E-04
			-1.576E-03	-5.732E-05	1.851E-04		
30.0	5.684E-03	7.259E-04	-1.368E-03	-7.915E-04	-2.547E-04	-8.341E-04	-2.348E-04
			-1.788E-03	-9.242E-05	2.248E-04		
45.0	9.011E-04	1.249E-03	-1.308E-03	-3.750E-04	-4.024E-05	-3.708E-04	-1.208E-04
			-1.389E-03	-8.298E-05	1.891E-04		
60.0	2.111E-04	9.731E-04	-1.065E-03	-1.677E-04	2.972E-05	-1.588E-04	-5.948E-05
			-9.753E-04	-5.978E-05	1.352E-04		
90.0	2.686E-05	5.628E-04	-7.138E-04	-4.019E-05	4.422E-05	-3.584E-05	-1.588E-05
			-5.991E-04	-3.792E-05	8.261E-05		
135.0	4.925E-06	3.448E-04	-4.986E-04	-9.186E-06	3.506E-05	-9.093E-06	-3.898E-06
			-4.180E-04	-2.704E-05	5.679E-05		
180.0	2.941E-06	2.981E-04	-4.476E-04	-5.459E-06	3.196E-05	-5.978E-06	-2.373E-06
			-3.775E-04	-2.456E-05	5.105E-05		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) SB2(2S,2P)	SB2(1S,2S) SB2(2S,3P)	SB2(1S,3S) SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	5.018E-01	-9.328E-03	5.155E-03	5.440E-03	2.044E-03	-6.192E-03	-1.466E-03
			-3.017E-02	-2.153E-03	3.487E-03		
1.0	4.814E-01	-9.477E-03	4.934E-03	5.170E-03	1.942E-03	-5.972E-03	-1.415E-03
			-2.849E-02	-2.068E-03	3.314E-03		
2.5	4.002E-01	-1.006E-02	4.052E-03	4.083E-03	1.530E-03	-5.106E-03	-1.211E-03
			-2.182E-02	-1.708E-03	2.615E-03		
5.0	2.578E-01	-1.068E-02	2.464E-03	2.120E-03	7.795E-04	-3.629E-03	-8.657E-04
			-1.045E-02	-9.520E-04	1.373E-03		
7.5	1.618E-01	-9.927E-03	1.324E-03	7.645E-04	2.501E-04	-2.668E-03	-6.421E-04
			-3.912E-03	-3.538E-04	5.800E-04		
10.0	1.019E-01	-8.114E-03	5.368E-04	-6.156E-05	-7.842E-05	-2.067E-03	-5.039E-04
			-1.156E-03	-3.466E-05	1.921E-04		
15.0	4.085E-02	-3.907E-03	-4.223E-04	-7.543E-04	-3.422E-04	-1.392E-03	-3.512E-04
			-5.976E-04	5.116E-05	5.963E-05		
20.0	1.688E-02	-1.050E-03	-9.052E-04	-8.332E-04	-3.353E-04	-1.016E-03	-2.678E-04
			-1.301E-03	-3.666E-05	1.472E-04		
25.0	7.334E-03	3.429E-04	-1.117E-03	-7.117E-04	-2.448E-04	-7.586E-04	-2.100E-04
			-1.593E-03	-8.204E-05	1.971E-04		
30.0	3.388E-03	8.835E-04	-1.173E-03	-5.535E-04	-1.519E-04	-5.629E-04	-1.643E-04
			-1.543E-03	-8.937E-05	2.000E-04		
45.0	4.797E-04	9.160E-04	-1.003E-03	-2.205E-04	-1.241E-06	-2.138E-04	-7.372E-05
			-1.018E-03	-6.289E-05	1.392E-04		
60.0	1.057E-04	6.398E-04	-7.736E-04	-8.708E-05	3.273E-05	-8.038E-05	-3.198E-05
			-6.956E-04	-4.407E-05	9.570E-05		
90.0	1.266E-05	3.426E-04	-4.996E-04	-1.762E-05	3.218E-05	-1.587E-05	-7.108E-06
			-4.283E-04	-2.802E-05	5.809E-05		
135.0	2.249E-06	2.069E-04	-3.486E-04	-3.662E-06	2.406E-05	-3.951E-06	-1.563E-06
			-3.031E-04	-2.022E-05	4.050E-05		
180.0	1.333E-06	1.794E-04	-3.140E-04	-2.177E-06	2.185E-05	-2.626E-06	-9.457E-07
			-2.752E-04	-1.844E-05	3.662E-05		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) SB2(2S,2P)	SB2(1S,2S) SB2(2S,3P)	SB2(1S,3S) SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	7.517E-01	-4.730E-03	4.450E-03	4.696E-03	1.769E-03	-4.868E-03	-1.146E-03
			-3.048E-02	-2.041E-03	3.361E-03		
1.0	6.474E-01	-5.151E-03	3.815E-03	3.960E-03	1.492E-03	-4.244E-03	-1.000E-03
			-2.544E-02	-1.788E-03	2.847E-03		
2.5	4.042E-01	-6.304E-03	2.315E-03	2.191E-03	8.241E-04	-2.820E-03	-6.673E-04
			-1.357E-02	-1.133E-03	1.619E-03		
5.0	1.883E-01	-6.535E-03	9.115E-04	5.268E-04	1.828E-04	-1.627E-03	-3.902E-04
			-3.820E-03	-3.512E-04	5.214E-04		
7.5	9.355E-02	-4.890E-03	2.047E-04	-1.917E-04	-1.042E-04	-1.118E-03	-2.739E-04
			-1.175E-03	-4.410E-05	1.568E-04		
10.0	4.763E-02	-2.886E-03	-2.080E-04	-4.674E-04	-2.110E-04	-8.442E-04	-2.126E-04
			-8.772E-04	-9.658E-06	9.779E-05		
15.0	1.314E-02	-3.405E-04	-5.955E-04	-4.926E-04	-1.903E-04	-5.366E-04	-1.445E-04
			-1.168E-03	-6.097E-05	1.405E-04		
20.0	4.060E-03	4.674E-04	-6.882E-04	-3.485E-04	-1.033E-04	-3.453E-04	-1.003E-04
			-1.073E-03	-6.605E-05	1.382E-04		
25.0	1.426E-03	5.920E-04	-6.590E-04	-2.216E-04	-4.054E-05	-2.146E-04	-6.763E-05
			-8.470E-04	-5.384E-05	1.128E-04		
30.0	5.633E-04	5.288E-04	-5.895E-04	-1.361E-04	-6.393E-06	-1.296E-04	-4.433E-05
			-6.579E-04	-4.242E-05	8.891E-05		
45.0	6.010E-05	2.833E-04	-3.904E-04	-3.204E-05	1.823E-05	-2.850E-05	-1.192E-05
			-3.629E-04	-2.427E-05	4.899E-05		
60.0	1.144E-05	1.634E-04	-2.748E-04	-8.841E-06	1.623E-05	-7.729E-06	-3.494E-06
			-2.480E-04	-1.701E-05	3.298E-05		
90.0	1.196E-06	8.120E-05	-1.739E-04	-1.248E-06	1.096E-05	-1.280E-06	-5.138E-07
			-1.595E-04	-1.120E-05	2.083E-05		
135.0	1.984E-07	5.080E-05	-1.253E-04	-2.541E-07	7.977E-06	-3.296E-07	-1.067E-07
			-1.173E-04	-8.319E-06	1.519E-05		
180.0	1.158E-07	4.481E-05	-1.142E-04	-1.603E-07	7.283E-06	-2.217E-07	-6.773E-08
			-1.075E-04	-7.641E-06	1.390E-05		

TABLE C9

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (3S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) / SB2(2S,2P)	SB2(1S,2S) / SB2(2S,3P)	SB2(1S,3S) / SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	-1.102E-01	1.048E-02	-3.118E-03	1.000E-03	4.478E-03	7.810E-03	1.964E-03
			-4.785E-03	-3.784E-04	2.503E-03		
1.0	-1.095E-01	1.048E-02	-3.096E-03	9.973E-04	4.433E-03	7.766E-03	1.952E-03
			-4.731E-03	-3.739E-04	2.446E-03		
2.5	-1.063E-01	1.047E-02	-2.989E-03	9.810E-04	4.206E-03	7.548E-03	1.893E-03
			-4.463E-03	-3.510E-04	2.164E-03		
5.0	-9.611E-02	1.038E-02	-2.649E-03	9.284E-04	3.492E-03	6.880E-03	1.712E-03
			-3.637E-03	-2.793E-04	1.364E-03		
7.5	-8.285E-02	1.014E-02	-2.197E-03	8.542E-04	2.550E-03	6.048E-03	1.488E-03
			-2.599E-03	-1.864E-04	5.128E-04		
10.0	-6.920E-02	9.697E-03	-1.718E-03	7.673E-04	1.574E-03	5.243E-03	1.277E-03
			-1.608E-03	-9.424E-05	-1.267E-04		
15.0	-4.585E-02	8.156E-03	-8.463E-04	5.744E-04	-7.494E-05	3.979E-03	9.786E-04
			-2.646E-04	3.733E-05	-6.132E-04		
20.0	-2.942E-02	6.094E-03	-1.612E-04	3.767E-04	-1.132E-03	3.119E-03	8.145E-04
			1.784E-04	8.172E-05	-3.769E-04		
25.0	-1.866E-02	4.016E-03	3.506E-04	1.995E-04	-1.657E-03	2.492E-03	7.047E-04
			7.863E-05	6.757E-05	1.603E-04		
30.0	-1.183E-02	2.248E-03	7.221E-04	6.098E-05	-1.803E-03	2.010E-03	6.114E-04
			-2.020E-04	3.253E-05	7.173E-04		
45.0	-3.228E-03	-7.077E-04	1.278E-03	-1.213E-04	-1.245E-03	1.151E-03	4.076E-04
			-7.252E-04	-3.668E-05	1.594E-03		
60.0	-1.039E-03	-1.529E-03	1.393E-03	-1.255E-04	-5.255E-04	7.940E-04	3.058E-04
			-7.131E-04	-4.395E-05	1.581E-03		
90.0	-1.833E-04	-1.572E-03	1.266E-03	-7.013E-05	1.439E-04	5.365E-04	2.195E-04
			-5.058E-04	-3.250E-05	1.155E-03		
135.0	-4.054E-05	-1.303E-03	1.059E-03	-3.359E-05	3.204E-04	4.058E-04	1.706E-04
			-3.681E-04	-2.404E-05	8.357E-04		
180.0	-2.533E-05	-1.211E-03	9.928E-04	-2.590E-05	3.334E-04	3.729E-04	1.578E-04
			-3.347E-04	-2.195E-05	7.566E-04		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (3S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 300E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) SB2(2S,2P)	SB2(1S,2S) SB2(2S,3P)	SB2(1S,3S) SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	-1.503E-01	6.781E-03	-2.798E-03	5.455E-04	4.810E-03	7.689E-03	1.927E-03
			-6.347E-03	-5.359E-04	5.827E-03		
1.0	-1.483E-01	6.798E-03	-2.756E-03	5.433E-04	4.717E-03	7.581E-03	1.899E-03
			-6.218E-03	-5.252E-04	5.633E-03		
2.5	-1.387E-01	6.874E-03	-2.554E-03	5.328E-04	4.271E-03	7.066E-03	1.766E-03
			-5.601E-03	-4.734E-04	4.745E-03		
5.0	-1.131E-01	6.994E-03	-2.012E-03	5.026E-04	3.068E-03	5.749E-03	1.422E-03
			-3.979E-03	-3.331E-04	2.749E-03		
7.5	-8.685E-02	6.885E-03	-1.438E-03	4.624E-04	1.803E-03	4.498E-03	1.091E-03
			-2.379E-03	-1.870E-04	1.270E-03		
10.0	-6.509E-02	6.439E-03	-9.382E-04	4.109E-04	7.407E-04	3.564E-03	8.518E-04
			-1.199E-03	-7.278E-05	4.850E-04		
15.0	-3.571E-02	4.786E-03	-1.957E-04	2.766E-04	-6.300E-04	2.438E-03	6.057E-04
			-1.489E-04	3.331E-05	1.370E-04		
20.0	-1.945E-02	2.888E-03	2.865E-04	1.347E-04	-1.201E-03	1.788E-03	4.920E-04
			-1.115E-04	3.207E-05	4.641E-04		
25.0	-1.067E-02	1.346E-03	5.914E-04	2.467E-05	-1.286E-03	1.345E-03	4.072E-04
			-3.313E-04	2.002E-06	8.629E-04		
30.0	-5.956E-03	3.048E-04	7.736E-04	-4.166E-05	-1.133E-03	1.036E-03	3.374E-04
			-4.973E-04	-2.117E-05	1.118E-03		
45.0	-1.225E-03	-8.265E-04	9.185E-04	-7.854E-05	-4.349E-04	5.788E-04	2.151E-04
			-5.264E-04	-3.430E-05	1.141E-03		
60.0	-3.313E-04	-8.695E-04	8.345E-04	-5.211E-05	-3.352E-05	4.142E-04	1.608E-04
			-3.954E-04	-2.664E-05	8.732E-04		
90.0	-4.859E-05	-6.110E-04	6.160E-04	-1.837E-05	1.649E-04	2.808E-04	1.099E-04
			-2.467E-04	-1.710E-05	5.379E-04		
135.0	-9.635E-06	-3.996E-04	4.431E-04	-5.352E-06	1.616E-04	2.071E-04	7.976E-05
			-1.700E-04	-1.205E-05	3.614E-04		
180.0	-5.861E-06	-3.472E-04	3.978E-04	-3.313E-06	1.508E-04	1.891E-04	7.221E-05
			-1.525E-04	-1.088E-05	3.217E-04		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (3S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) SB2(2S,2P)	SB2(1S,2S) SB2(2S,3P)	SB2(1S,3S) SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	-1.824E-01	5.012E-03	-2.641E-03	2.839E-04	4.766E-03	7.506E-03	1.859E-03
			-6.994E-03	-6.012E-04	7.933E-03		
1.0	-1.781E-01	5.050E-03	-2.573E-03	2.846E-04	4.616E-03	7.313E-03	1.812E-03
			-6.768E-03	-5.827E-04	7.521E-03		
2.5	-1.589E-01	5.213E-03	-2.268E-03	2.878E-04	3.944E-03	6.466E-03	1.602E-03
			-5.763E-03	-4.988E-04	5.806E-03		
5.0	-1.166E-01	5.470E-03	-1.584E-03	2.937E-04	2.424E-03	4.687E-03	1.149E-03
			-3.554E-03	-3.067E-04	2.825E-03		
7.5	-8.155E-02	5.360E-03	-9.908E-04	2.862E-04	1.116E-03	3.358E-03	8.042E-04
			-1.811E-03	-1.433E-04	1.246E-03		
10.0	-5.652E-02	4.825E-03	-5.383E-04	2.563E-04	1.763E-04	2.532E-03	6.000E-04
			-7.747E-04	-3.947E-05	5.936E-04		
15.0	-2.714E-02	3.098E-03	6.325E-05	1.495E-04	-8.086E-04	1.651E-03	4.248E-04
			-1.626E-04	2.122E-05	4.545E-04		
20.0	-1.316E-02	1.467E-03	4.090E-04	4.222E-05	-1.040E-03	1.160E-03	3.374E-04
			-2.877E-04	8.483E-07	7.579E-04		
25.0	-6.525E-03	3.793E-04	5.974E-04	-2.492E-05	-9.234E-04	8.419E-04	2.693E-04
			-4.359E-04	-2.090E-05	9.727E-04		
30.0	-3.348E-03	-2.209E-04	6.864E-04	-5.379E-05	-6.947E-04	6.381E-04	2.187E-04
			-4.800E-04	-2.969E-05	1.027E-03		
45.0	-5.783E-04	-6.386E-04	6.776E-04	-4.685E-05	-1.380E-04	3.661E-04	1.389E-04
			-3.636E-04	-2.509E-05	7.890E-04		
60.0	-1.416E-04	-5.309E-04	5.597E-04	-2.456E-05	6.775E-05	2.668E-04	1.028E-04
			-2.539E-04	-1.791E-05	5.511E-04		
90.0	-1.876E-05	-3.150E-04	3.773E-04	-6.465E-06	1.202E-04	1.795E-04	6.803E-05
			-1.555E-04	-1.133E-05	3.277E-04		
135.0	-3.512E-06	-1.915E-04	2.619E-04	-1.444E-06	9.652E-05	1.320E-04	4.849E-05
			-1.083E-04	-8.060E-06	2.226E-04		
180.0	-2.107E-06	-1.648E-04	2.344E-04	-8.234E-07	8.785E-05	1.207E-04	4.385E-05
			-9.770E-05	-7.316E-06	1.997E-04		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (3S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) / SB2(2S,2P)	SB2(1S,2S) / SB2(2S,3P)	SB2(1S,3S) / SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	-2.099E-01	3.993E-03	-2.528E-03	1.201E-04	4.645E-03	7.303E-03	1.787E-03
			-7.271E-03	-6.295E-04	9.431E-03		
1.0	-2.023E-01	4.052E-03	-2.429E-03	1.251E-04	4.430E-03	7.012E-03	1.717E-03
			-6.933E-03	-6.018E-04	8.729E-03		
2.5	-1.711E-01	4.294E-03	-2.023E-03	1.467E-04	3.546E-03	5.843E-03	1.434E-03
			-5.547E-03	-4.863E-04	6.101E-03		
5.0	-1.141E-01	4.622E-03	-1.263E-03	1.862E-04	1.864E-03	3.817E-03	9.283E-04
			-2.999E-03	-2.626E-04	2.554E-03		
7.5	-7.416E-02	4.448E-03	-6.987E-04	1.979E-04	6.347E-04	2.578E-03	6.105E-04
			-1.339E-03	-1.029E-04	1.128E-03		
10.0	-4.842E-02	3.815E-03	-3.030E-04	1.763E-04	-1.507E-04	1.896E-03	4.500E-04
			-5.213E-04	-1.926E-05	6.312E-04		
15.0	-2.093E-02	2.091E-03	1.864E-04	8.262E-05	-8.250E-04	1.201E-03	3.220E-04
			-2.225E-04	7.087E-06	6.234E-04		
20.0	-9.256E-03	7.332E-04	4.396E-04	1.176E-06	-8.546E-04	8.157E-04	2.487E-04
			-3.635E-04	-1.497E-05	8.435E-04		
25.0	-4.251E-03	-2.562E-05	5.564E-04	-3.824E-05	-6.575E-04	5.814E-04	1.941E-04
			-4.273E-04	-2.651E-05	9.160E-04		
30.0	-2.050E-03	-3.702E-04	5.937E-04	-4.811E-05	-4.307E-04	4.408E-04	1.567E-04
			-4.077E-04	-2.767E-05	8.664E-04		
45.0	-3.138E-04	-4.836E-04	5.203E-04	-2.904E-05	-2.220E-05	2.605E-04	9.945E-05
			-2.651E-04	-1.888E-05	5.728E-04		
60.0	-7.186E-05	-3.529E-04	4.050E-04	-1.290E-05	8.274E-05	1.906E-04	7.269E-05
			-1.807E-04	-1.317E-05	3.852E-04		
90.0	-8.910E-06	-1.909E-04	2.611E-04	-2.736E-06	8.749E-05	1.276E-04	4.716E-05
			-1.110E-04	-8.354E-06	2.287E-04		
135.0	-1.610E-06	-1.138E-04	1.805E-04	-5.236E-07	6.543E-05	9.406E-05	3.357E-05
			-7.840E-05	-6.019E-06	1.582E-04		
180.0	-9.578E-07	-9.810E-05	1.621E-04	-2.920E-07	5.927E-05	8.614E-05	3.043E-05
			-7.114E-05	-5.486E-06	1.429E-04		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (3S,2P) STATE OF HELIUM BY ELECTRON IMPACT
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) / SB2(2S,2P)	SB2(1S,2S) / SB2(2S,3P)	SB2(1S,3S) / SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	-3.138E-01	2.004E-03	-2.148E-03	-2.006E-04	4.002E-03	6.380E-03	1.491E-03
			-7.163E-03	-6.231E-04	1.297E-02		
1.0	-2.739E-01	2.169E-03	-1.864E-03	-1.585E-04	3.417E-03	5.504E-03	1.294E-03
			-6.138E-03	-5.389E-04	1.016E-02		
2.5	-1.765E-01	2.656E-03	-1.159E-03	-4.850E-05	1.934E-03	3.393E-03	8.116E-04
			-3.581E-03	-3.242E-04	4.307E-03		
5.0	-8.604E-02	2.883E-03	-4.692E-04	5.228E-05	4.449E-04	1.634E-03	3.833E-04
			-1.209E-03	-1.039E-04	1.362E-03		
7.5	-4.498E-02	2.321E-03	-1.156E-04	6.619E-05	-2.449E-04	1.012E-03	2.387E-04
			-4.069E-04	-2.079E-05	7.527E-04		
10.0	-2.416E-02	1.504E-03	9.307E-05	4.056E-05	-5.179E-04	7.222E-04	1.874E-04
			-2.607E-04	-8.065E-06	6.540E-04		
15.0	-7.330E-03	2.927E-04	2.938E-04	-1.291E-05	-4.989E-04	4.121E-04	1.290E-04
			-3.096E-04	-1.973E-05	6.789E-04		
20.0	-2.430E-03	-1.782E-04	3.466E-04	-2.762E-05	-2.909E-04	2.624E-04	9.202E-05
			-2.806E-04	-2.021E-05	5.936E-04		
25.0	-8.950E-04	-2.871E-04	3.359E-04	-2.370E-05	-1.257E-04	1.891E-04	7.043E-05
			-2.203E-04	-1.620E-05	4.692E-04		
30.0	-3.653E-04	-2.752E-04	3.028E-04	-1.668E-05	-2.942E-05	1.490E-04	5.673E-05
			-1.707E-04	-1.269E-05	3.644E-04		
45.0	-4.111E-05	-1.567E-04	2.016E-04	-4.519E-06	4.697E-05	9.225E-05	3.469E-05
			-9.402E-05	-7.238E-06	1.950E-04		
60.0	-8.015E-06	-9.081E-05	1.411E-04	-1.252E-06	4.344E-05	6.698E-05	2.439E-05
			-6.415E-05	-5.064E-06	1.295E-04		
90.0	-8.552E-07	-4.443E-05	8.816E-05	-1.537E-07	2.925E-05	4.495E-05	1.564E-05
			-4.119E-05	-3.327E-06	8.118E-05		
135.0	-1.431E-07	-2.738E-05	6.259E-05	-2.449E-08	2.109E-05	3.355E-05	1.139E-05
			-3.027E-05	-2.470E-06	5.909E-05		
180.0	-8.373E-08	-2.407E-05	5.732E-05	-1.417E-08	1.921E-05	3.083E-05	1.042E-05
			-2.774E-05	-2.268E-06	5.405E-05		

TABLE C10

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,3P) STATE OF HELIUM BY ELECTRON IMPACT
E= 200E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) / SB2(2S,2P)	SB2(1S,2S) / SB2(2S,3P)	SB2(1S,3S) / SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	7.444E-02	-7.712E-03	1.432E-03	1.322E-03	4.670E-04	-5.722E-04	-5.453E-03
			-5.835E-03	1.617E-03	7.920E-04		
1.0	7.397E-02	-7.706E-03	1.421E-03	1.308E-03	4.618E-04	-5.697E-04	-5.430E-03
			-5.778E-03	1.691E-03	7.842E-04		
2.5	7.159E-02	-7.671E-03	1.368E-03	1.239E-03	4.355E-04	-5.572E-04	-5.313E-03
			-5.491E-03	2.038E-03	7.449E-04		
5.0	6.413E-02	-7.519E-03	1.200E-03	1.024E-03	3.526E-04	-5.180E-04	-4.948E-03
			-4.577E-03	2.877E-03	6.216E-04		
7.5	5.442E-02	-7.199E-03	9.757E-04	7.436E-04	2.437E-04	-4.673E-04	-4.478E-03
			-3.359E-03	3.435E-03	4.619E-04		
10.0	4.446E-02	-6.678E-03	7.382E-04	4.562E-04	1.315E-04	-4.156E-04	-4.000E-03
			-2.119E-03	3.444E-03	3.029E-04		
15.0	2.772E-02	-5.134E-03	3.065E-04	-1.731E-05	-5.413E-05	-3.275E-04	-3.190E-03
			-3.007E-04	2.473E-03	6.930E-05		
20.0	1.646E-02	-3.361E-03	-2.992E-05	-3.106E-04	-1.661E-04	-2.635E-04	-2.611E-03
			3.546E-04	1.331E-03	-2.428E-05		
25.0	9.589E-03	-1.827E-03	-2.759E-04	-4.548E-04	-2.133E-04	-2.177E-04	-2.203E-03
			2.503E-04	4.798E-04	-2.118E-05		
30.0	5.592E-03	-7.130E-04	-4.483E-04	-5.005E-04	-2.164E-04	-1.836E-04	-1.903E-03
			-1.061E-04	-6.764E-05	2.080E-05		
45.0	1.240E-03	6.794E-04	-6.757E-04	-4.000E-04	-1.282E-04	-1.155E-04	-1.305E-03
			-7.598E-04	-6.141E-04	1.154E-04		
60.0	3.492E-04	8.658E-04	-6.897E-04	-2.567E-04	-4.600E-05	-7.245E-05	-9.031E-04
			-7.545E-04	-5.835E-04	1.203E-04		
90.0	5.372E-05	7.247E-04	-5.833E-04	-1.061E-04	1.655E-05	-2.894E-05	-4.397E-04
			-5.308E-04	-4.128E-04	8.818E-05		
135.0	1.100E-05	5.609E-04	-4.672E-04	-4.406E-05	2.896E-05	-1.084E-05	-2.023E-04
			-3.841E-04	-3.030E-04	6.464E-05		
180.0	6.750E-06	5.150E-04	-4.334E-04	-3.287E-05	2.933E-05	-7.752E-06	-1.546E-04
			-3.489E-04	-2.764E-04	5.881E-05		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,3P) STATE OF HELIUM BY ELECTRON IMPACT
E= 300E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) SB2(2S,2P)	SB2(1S,2S) SB2(2S,3P)	SB2(1S,3S) SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	1.039E-01	-5.177E-03	1.395E-03	1.444E-03	5.383E-04	-5.180E-04	-4.857E-03
			-7.232E-03	-1.838E-03	9.765E-04		
1.0	1.024E-01	-5.185E-03	1.373E-03	1.416E-03	5.274E-04	-5.123E-04	-4.804E-03
			-7.109E-03	-1.585E-03	9.587E-04		
2.5	9.533E-02	-5.217E-03	1.269E-03	1.278E-03	4.745E-04	-4.851E-04	-4.552E-03
			-6.505E-03	-4.956E-04	8.725E-04		
5.0	7.660E-02	-5.207E-03	9.870E-04	9.100E-04	3.322E-04	-4.137E-04	-3.894E-03
			-4.807E-03	1.414E-03	6.399E-04		
7.5	5.734E-02	-4.952E-03	6.877E-04	5.279E-04	1.829E-04	-3.418E-04	-3.233E-03
			-2.944E-03	2.012E-03	3.988E-04		
10.0	4.148E-02	-4.401E-03	4.271E-04	2.145E-04	5.875E-05	-2.832E-04	-2.696E-03
			-1.441E-03	1.732E-03	2.092E-04		
15.0	2.068E-02	-2.810E-03	4.158E-05	-1.742E-04	-9.589E-05	-2.034E-04	-1.975E-03
			-3.314E-05	7.521E-04	2.346E-05		
20.0	1.007E-02	-1.338E-03	-2.036E-04	-3.295E-04	-1.517E-04	-1.551E-04	-1.546E-03
			2.388E-06	5.856E-05	8.242E-06		
25.0	4.949E-03	-3.720E-04	-3.519E-04	-3.575E-04	-1.503E-04	-1.229E-04	-1.265E-03
			-2.932E-04	-3.155E-04	4.418E-05		
30.0	2.504E-03	1.499E-04	-4.338E-04	-3.282E-04	-1.240E-04	-9.918E-05	-1.057E-03
			-5.065E-04	-4.737E-04	7.467E-05		
45.0	4.185E-04	5.081E-04	-4.730E-04	-1.840E-04	-3.880E-05	-5.197E-05	-6.238E-04
			-5.562E-04	-4.568E-04	8.685E-05		
60.0	1.010E-04	4.324E-04	-4.100E-04	-9.256E-05	6.117E-07	-2.625E-05	-3.571E-04
			-4.154E-04	-3.396E-04	6.675E-05		
90.0	1.329E-05	2.739E-04	-2.898E-04	-2.671E-05	1.614E-05	-7.344E-06	-1.219E-04
			-2.573E-04	-2.156E-04	4.175E-05		
135.0	2.488E-06	1.771E-04	-2.059E-04	-7.314E-06	1.470E-05	-2.038E-06	-3.703E-05
			-1.771E-04	-1.516E-04	2.848E-05		
180.0	1.494E-06	1.547E-04	-1.849E-04	-4.565E-06	1.362E-05	-1.335E-06	-2.380E-05
			-1.589E-04	-1.368E-04	2.543E-05		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,3P) STATE OF HELIUM BY ELECTRON IMPACT
E= 400E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) / SB2(2S,2P)	SB2(1S,2S) / SB2(2S,3P)	SB2(1S,3S) / SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	1.273E-01	-3.889E-03	1.376E-03	1.457E-03	5.499E-04	-4.772E-04	-4.445E-03
			-7.678E-03	-4.236E-03	1.042E-03		
1.0	1.241E-01	-3.914E-03	1.340E-03	1.410E-03	5.320E-04	-4.675E-04	-4.356E-03
			-7.474E-03	-3.694E-03	1.012E-03		
2.5	1.101E-01	-4.013E-03	1.177E-03	1.200E-03	4.515E-04	-4.244E-04	-3.959E-03
			-6.537E-03	-1.619E-03	8.745E-04		
5.0	7.921E-02	-4.098E-03	8.107E-04	7.291E-04	2.696E-04	-3.315E-04	-3.105E-03
			-4.256E-03	9.217E-04	5.620E-04		
7.5	5.347E-02	-3.824E-03	4.921E-04	3.318E-04	1.136E-04	-2.565E-04	-2.419E-03
			-2.181E-03	1.190E-03	2.980E-04		
10.0	3.530E-02	-3.203E-03	2.492E-04	5.550E-05	3.130E-06	-2.038E-04	-1.941E-03
			-8.351E-04	8.016E-04	1.284E-04		
15.0	1.496E-02	-1.657E-03	-7.064E-05	-2.202E-04	-1.061E-04	-1.401E-04	-1.370E-03
			-5.303E-05	8.557E-05	1.797E-05		
20.0	6.339E-03	-5.258E-04	-2.481E-04	-2.851E-04	-1.231E-04	-1.039E-04	-1.053E-03
			-2.469E-04	-3.004E-04	3.753E-05		
25.0	2.784E-03	5.408E-05	-3.381E-04	-2.635E-04	-1.014E-04	-7.976E-05	-8.417E-04
			-4.446E-04	-4.405E-04	6.519E-05		
30.0	1.292E-03	2.923E-04	-3.742E-04	-2.163E-04	-7.086E-05	-6.162E-05	-6.800E-04
			-5.045E-04	-4.529E-04	7.566E-05		
45.0	1.841E-04	3.435E-04	-3.456E-04	-9.609E-05	-9.696E-06	-2.718E-05	-3.470E-04
			-3.827E-04	-3.226E-04	6.032E-05		
60.0	4.085E-05	2.498E-04	-2.757E-04	-4.116E-05	8.439E-06	-1.160E-05	-1.704E-04
			-2.655E-04	-2.266E-04	4.242E-05		
90.0	4.946E-06	1.406E-04	-1.819E-04	-9.482E-06	1.146E-05	-2.614E-06	-4.539E-05
			-1.620E-04	-1.425E-04	2.571E-05		
135.0	8.844E-07	8.688E-05	-1.267E-04	-2.172E-06	8.971E-06	-6.625E-07	-1.113E-05
			-1.128E-04	-1.013E-04	1.763E-05		
180.0	5.251E-07	7.555E-05	-1.138E-04	-1.308E-06	8.175E-06	-4.354E-07	-6.771E-06
			-1.018E-04	-9.197E-05	1.584E-05		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,3P) STATE OF HELIUM BY ELECTRON IMPACT
E= 500E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) / SB2(2S,2P)	SB2(1S,2S) / SB2(2S,3P)	SB2(1S,3S) / SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	1.473E-01	-3.128E-03	1.354E-03	1.439E-03	5.453E-04	-4.456E-04	-4.137E-03
			-7.777E-03	-6.045E-03	1.064E-03		
1.0	1.417E-01	-3.169E-03	1.300E-03	1.371E-03	5.194E-04	-4.312E-04	-4.004E-03
			-7.482E-03	-5.117E-03	1.018E-03		
2.5	1.190E-01	-3.330E-03	1.079E-03	1.092E-03	4.128E-04	-3.732E-04	-3.470E-03
			-6.223E-03	-1.999E-03	8.311E-04		
5.0	7.733E-02	-3.460E-03	6.635E-04	5.669E-04	2.100E-04	-2.696E-04	-2.521E-03
			-3.583E-03	6.572E-04	4.718E-04		
7.5	4.807E-02	-3.124E-03	3.545E-04	1.932E-04	6.257E-05	-1.995E-04	-1.883E-03
			-1.554E-03	6.383E-04	2.171E-04		
10.0	2.952E-02	-2.441E-03	1.383E-04	-3.577E-05	-2.957E-05	-1.549E-04	-1.480E-03
			-4.905E-04	2.922E-04	8.099E-05		
15.0	1.099E-02	-1.004E-03	-1.250E-04	-2.227E-04	-1.018E-04	-1.040E-04	-1.029E-03
			-1.518E-04	-1.983E-04	2.668E-05		
20.0	4.189E-03	-1.575E-04	-2.546E-04	-2.362E-04	-9.654E-05	-7.551E-05	-7.798E-04
			-3.593E-04	-3.981E-04	5.245E-05		
25.0	1.698E-03	1.884E-04	-3.080E-04	-1.957E-04	-6.850E-05	-5.606E-05	-6.081E-04
			-4.474E-04	-4.228E-04	6.644E-05		
30.0	7.426E-04	2.916E-04	-3.190E-04	-1.482E-04	-4.111E-05	-4.141E-05	-4.737E-04
			-4.305E-04	-3.815E-04	6.553E-05		
45.0	9.527E-05	2.428E-04	-2.645E-04	-5.536E-05	5.474E-07	-1.562E-05	-2.111E-04
			-2.781E-04	-2.402E-04	4.391E-05		
60.0	2.001E-05	1.616E-04	-2.009E-04	-2.103E-05	8.899E-06	-5.857E-06	-9.134E-05
			-1.886E-04	-1.662E-04	2.987E-05		
90.0	2.296E-06	8.565E-05	-1.287E-04	-4.143E-06	8.322E-06	-1.154E-06	-2.026E-05
			-1.156E-04	-1.050E-04	1.804E-05		
135.0	3.991E-07	5.250E-05	-8.993E-05	-8.791E-07	6.198E-06	-2.870E-07	-4.451E-06
			-8.168E-05	-7.566E-05	1.256E-05		
180.0	2.354E-07	4.579E-05	-8.109E-05	-5.324E-07	5.635E-06	-1.907E-07	-2.691E-06
			-7.412E-05	-6.897E-05	1.135E-05		

THE SCATTERING AMPLITUDE FOR EXCITATION OF THE (2S,3P) STATE OF HELIUM BY ELECTRON IMPACT
E=1000E.V.

THETA	B1	SB2(CLSRE)	SB2(1S,1S) / SB2(2S,2P)	SB2(1S,2S) / SB2(2S,3P)	SB2(1S,3S) / SB2(3S,2P)	SB2(1S,2P)	SB2(1S,3P)
0.0	2.224E-01	-1.599E-03	1.220E-03	1.281E-03	4.863E-04	-3.503E-04	-3.233E-03
			-7.145E-03	-1.091E-02	1.005E-03		
1.0	1.936E-01	-1.726E-03	1.058E-03	1.092E-03	4.146E-04	-3.088E-04	-2.852E-03
			-6.302E-03	-7.216E-03	8.708E-04		
2.5	1.229E-01	-2.073E-03	6.554E-04	6.139E-04	2.327E-04	-2.094E-04	-1.942E-03
			-4.039E-03	-1.055E-03	5.314E-04		
5.0	5.675E-02	-2.081E-03	2.605E-04	1.443E-04	5.043E-05	-1.219E-04	-1.145E-03
			-1.382E-03	-1.110E-04	1.887E-04		
7.5	2.710E-02	-1.458E-03	5.888E-05	-5.963E-05	-3.190E-05	-8.370E-05	-8.031E-04
			-3.624E-04	-2.401E-04	5.849E-05		
10.0	1.303E-02	-7.686E-04	-5.831E-05	-1.355E-04	-6.173E-05	-6.294E-05	-6.207E-04
			-2.200E-04	-3.233E-04	3.465E-05		
15.0	3.188E-03	-1.510E-05	-1.658E-04	-1.367E-04	-5.360E-05	-3.964E-05	-4.180E-04
			-3.203E-04	-3.492E-04	4.694E-05		
20.0	8.987E-04	1.650E-04	-1.889E-04	-9.304E-05	-2.781E-05	-2.533E-05	-2.883E-04
			-2.964E-04	-2.791E-04	4.480E-05		
25.0	2.964E-04	1.706E-04	-1.786E-04	-5.718E-05	-1.015E-05	-1.567E-05	-1.935E-04
			-2.321E-04	-2.103E-04	3.586E-05		
30.0	1.122E-04	1.415E-04	-1.583E-04	-3.416E-05	-1.041E-06	-9.437E-06	-1.265E-04
			-1.791E-04	-1.617E-04	2.795E-05		
45.0	1.116E-05	7.079E-05	-1.035E-04	-7.652E-06	4.924E-06	-2.067E-06	-3.388E-05
			-9.802E-05	-9.116E-05	1.523E-05		
60.0	2.059E-06	4.066E-05	-7.287E-05	-2.083E-06	4.254E-06	-5.597E-07	-9.919E-06
			-6.682E-05	-6.367E-05	1.023E-05		
90.0	2.099E-07	2.056E-05	-4.639E-05	-3.007E-07	2.867E-06	-9.253E-08	-1.456E-06
			-4.292E-05	-4.183E-05	6.448E-06		
135.0	3.439E-08	1.307E-05	-3.355E-05	-6.394E-08	2.095E-06	-2.382E-08	-3.022E-07
			-3.154E-05	-3.105E-05	4.699E-06		
180.0	2.002E-08	1.157E-05	-3.061E-05	-4.085E-08	1.915E-06	-1.602E-08	-1.918E-07
			-2.890E-05	-2.852E-05	4.298E-06		

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