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Superconducting proximity effect through a magnetic domain wall

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We study the superconducting proximity effect in a superconductor-ferromagnet-superconductor heterostructure containing a domain wall in the ferromagnetic region. For the ferromagnet we assume an alloy with an exchange splitting of the conduction bands comparable to the superconducting gaps. We calculate the modification of the density of states in the center of the domain wall as a result of the proximity effect. We show that the density of states is sensitive to domain-wall parameters due to triplet-pairing correlations created in the vicinity of the domain wall. We present a theoretical tool which in a very effective way enables retaining the *full* spatially dependent spin-space structure of the problem.

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Most promising candidates for mesoscopic devices with novel functionality are hybrid structures containing superconducting elements. The key phenomenon that controls the behavior of such systems is the proximity effect. When a superconducting material is placed in contact with a normal metal (N), the superconducting pair correlations leak over to the normal-metal side, changing its conduction properties in the vicinity of the separating interface. Quite similarly, the properties on the superconducting side are also changed (the energy gap Δ_0 is suppressed) due to the contact to a normal metal. An alternative but equivalent way of thinking about the proximity effect is through Andreev-reflection[1] processes: an incoming electron from the normal side is transmitted together with another one as a Cooper pair into the superconducting side. This phase-coherent electron-hole conversion results in a nonzero pair amplitude in the normal metal.

In the diffusive limit, the correlations relating to an incident electron with an energy E (the range of energies being set by the temperature T) above the chemical potential extend a characteristic distance of $\xi_N=\sqrt{D/E}$ into the normal metal;[2] here D is the diffusion constant in N. If the extent of the N region is finite, another energy scale, $E_T \sim D/L^2$, enters the problem; L denotes the width of N. This so-called Thouless energy has associated with it one of the generic features of diffusive superconductor–normal-metal heterostructures, the minigap: the density of states in the normal metal develops a gap around the chemical potential in a manner similar to a superconductor (S) but with a smaller magnitude.

If the normal conductor is replaced by a ferromagnet (F), a multitude of new effects arises due to the emergence of yet another energy scale, that of the exchange splitting J of the two spin bands. Both on the theoretical[3–11] and on the experimental[12–20] side, interest has grown recently in the rich physics of such systems. One source for new behavior is that, in the case with a singlet superconductor, the induced pair amplitude in the ferromagnet is oscillatory.[21] However, the exchange splitting also gives rise to dephasing, which, in turn, results in the decay of induced correlations over a characteristic distance $\xi_J=\sqrt{D/(E+J)}$.[22] Unfortunately, since J is of the order of the Fermi energy E_F in typical ferromagnetic metals, this distance is very short. Still, experimental indication of the oscillatory behavior has been obtained in thin ferromagnetic layers and, relevant to the present paper, in weakly ferromagnetic alloys with $J \ll E_F$.[12,13]

Another question of current interest in SF proximity systems is the role of equal-spin triplet correlations.[23,24] If created, e.g., near magnetic inhomogeneities, such correlations would not be affected by the exchange splitting but could penetrate considerably longer distances into F.[23] Finally, the importance of domain walls has also been stressed for the Andreev conductance.[25,26]

In this paper, we study a superconductor-ferromagnetic-superconductor (SFS) structure, shown schematically in Fig. 1, in equilibrium. The ferromagnetic region consists of two domains with magnetizations oriented in opposite directions. The domains are separated by a domain wall, where the magnetization rotates continuously between the asymptotic values. While varying in direction, the magnitude J is assumed constant throughout the F region. We show that the local density of states (LDOS) in the F region is strongly modified by the presence of the domain wall. In particular, we show that it can be very sensitive to the thickness of the domain wall in a certain parameter region.

Proximity effect is a spatially inhomogeneous phenomenon. An appropriate theoretical tool to treat such a problem is the quasiclassical theory of superconductivity,[27,28] which in its diffusive version has been formulated by Usadel.[29] In equilibrium, the physical information is contained in the retarded Green functions $\hat{G}(z,E)$. Here, we assume spatial dependence in the coordinate z only, and E denotes the energy

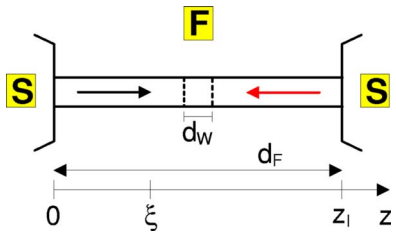

FIG. 1. (Color online) SFS structure with two magnetic domains oriented along the z axis and separated by a domain wall of width d_W; d_F denotes the length of the F region and $\xi=\sqrt{D/\Delta_0}$ is the superconducting coherence length.

as measured from the chemical potential. The 4×4 matrix structure, arising from particle-hole and spin degrees of freedom, is denoted by the hat $\hat{(\,)}$ accent,

$$\hat{G}=\begin{pmatrix}\mathcal{G} & \mathcal{F}\\ \tilde{\mathcal{F}} & \tilde{\mathcal{G}}\end{pmatrix}. \tag{1}$$

The off-diagonal elements determine the superconducting pair amplitude. Quantities denoted with the "tilde" are related to those without one through $\tilde{\mathcal{A}}(z,E)=\mathcal{A}(z,-E^*)^*$. All the elements in Eq. (1) are 2×2 spin matrices: e.g., $\mathcal{G}=\mathcal{G}_{\alpha\beta}$ with $\alpha,\beta=\{\uparrow,\downarrow\}$. The Green functions satisfy the Usadel equation,[29]

$$[E\hat{\tau}_3-\hat{\Delta}-\boldsymbol{J}\cdot\hat{\boldsymbol{\sigma}},\hat{G}]_\otimes+\frac{D}{\pi}\frac{\mathrm{d}}{\mathrm{d}z}\left(\hat{G}\otimes\frac{\mathrm{d}}{\mathrm{d}z}\hat{G}\right)=\hat{0}, \tag{2}$$

where the symbol $\otimes$ denotes matrix multiplication, and $[\hat{A},\hat{B}]_\otimes=\hat{A}\otimes\hat{B}-\hat{B}\otimes\hat{A}$. In writing Eq. (2) we have followed the standard way to describe ferromagnetic materials through a spin-dependent energy shift,[22] which has the form $E\hat{\tau}_3\rightarrow E\hat{\tau}_3-\boldsymbol{J}\cdot\hat{\boldsymbol{\sigma}}$. Here, τ_3 denotes the third Pauli matrix in Nambu space, the vector $\boldsymbol{J}$ denotes the effective exchange field of the ferromagnet, and $\hat{\Delta}$ is the superconducting order parameter (appropriate for weak-coupling spin-singlet pairing). The components of the vector $\hat{\boldsymbol{\sigma}}$ and the order parameter are given by

$$\hat{\sigma}_i=\begin{pmatrix}\sigma_i & 0\\ 0 & \sigma_i^*\end{pmatrix},\quad \hat{\Delta}=\begin{pmatrix}0 & \Delta\\ \Delta^* & 0\end{pmatrix} \tag{3}$$

where σ_i are Pauli spin matrices, $i=x,y,z$, and $\Delta=\Delta_0 i\sigma_y$. The above procedure is appropriate for describing situations for which $J\ll E_F$, which holds, e.g., for the ferromagnetic alloys used in Refs. 12 and 13. In writing Eq. (2), we have chosen the normalization according to

$$\hat{G}\otimes\hat{G}=-\pi^2\hat{1}. \tag{4}$$

The spin-dependent nature of SF proximity systems calls for a formulation of the quasiclassical theory that retains the full spin-space structure, especially in studying situations where the exchange-field orientation varies in space (such as in a domain wall). Within the Eilenberger theory, a very convenient formulation already exists,[30] employing the so-called Riccati parametrization.[31,32] The extension of this method to the Usadel theory was achieved only recently,[33] and has been applied to nonequilibrium situations,[34] and to FSF systems with homogeneous magnetizations.[35] Here we demonstrate its usefulness by applying it to an SFS system with a spatially changing magnetization in a domain wall, a case where the conventional so-called θ parametrization[36] is not applicable. The spin-dependent Riccati parametrization,[30]

$$\hat{G}=-i\pi\hat{N}\otimes\begin{pmatrix}(1+\gamma\otimes\tilde{\gamma}) & 2\gamma\\ -2\tilde{\gamma} & -(1+\tilde{\gamma}\otimes\gamma)\end{pmatrix}, \tag{5}$$

with

$$\hat{N}=\begin{pmatrix}(1-\gamma\otimes\tilde{\gamma})^{-1} & 0\\ 0 & (1-\tilde{\gamma}\otimes\gamma)^{-1}\end{pmatrix}, \tag{6}$$

automatically accounts for the normalization (4), which is essential for practical numerical calculations. It is enough to determine one 2×2 matrix in spin space, γ. The other, $\tilde{\gamma}$, follows from the above-mentioned (fundamental) symmetry. The transport equation for γ follows from Eq. (2), and reads[33]

$$\frac{d^2\gamma}{dz^2}+\left(\frac{d\gamma}{dz}\right)\otimes\frac{\tilde{\mathcal{F}}}{i\pi}\otimes\left(\frac{d\gamma}{dz}\right)=\frac{i}{D}[\gamma\otimes\Delta^*\otimes\gamma-(E-\boldsymbol{J}\cdot\boldsymbol{\sigma})\otimes\gamma-\gamma\otimes(E+\boldsymbol{J}\cdot\boldsymbol{\sigma}^*)-\Delta]. \tag{7}$$

Here, the expression for $\tilde{\mathcal{F}}$ is obtained by comparing Eq. (1) with Eqs. (5) and (6).

Additionally, boundary conditions are required for the different interfaces of the system. Such conditions have been formulated by Nazarov.[37] The outer surfaces ($z=z_o$) of the superconductors are assumed to border an insulating region, and the appropriate condition is $\partial_z\hat{G}(z_o,E)=0$, i.e.,

$$\frac{d\gamma}{dz}(z_o,E)=0. \tag{8}$$

On the other hand, the two inner SF interfaces ($z=z_i^S$ for the S side, $z=z_i^F$ for the F side) are assumed in the following fully transparent (for the case of weak transparency in a similar system, see Ref. 23). The corresponding boundary conditions in this case are $\hat{G}(z_i^S,E)=\hat{G}(z_i^F,E)$, $\sigma_S\partial_z\hat{G}(z_i^S,E)=\sigma_F\partial_z\hat{G}(z_i^F,E)$, leading to

$$\gamma(z_i^S,E)=\gamma(z_i^F,E),$$

$$\sigma_S\frac{d\gamma}{dz}(z_i^S,E)=\sigma_F\frac{d\gamma}{dz}(z_i^F,E), \tag{9}$$

where σ_S and σ_F refer to the conductivities of S and F, respectively. For simplicity, we have assumed $\sigma_S=\sigma_F$, implying the continuity of the derivative at the interface. We expect a qualitatively similar picture when assuming a finite, however not too strong, mismatch in electronic properties. With the boundary conditions (8) and (9), we have solved Eq. (7) numerically by an iterative procedure (relaxation method) in the entire SFS system.

We apply the outlined theory to study the SFS structure of Fig. 1 in equilibrium. Lengths are given in units of the superconducting coherence length, $\xi=\sqrt{D/\Delta_0}$. The spin-singlet superconductors are chosen to have the same gap magnitude. The contact areas at the SF interfaces are assumed to be small enough, so that any gap suppression can be neglected. The two superconducting regions and the intermediate ferromagnet are taken to have fixed lengths of $d_S=5\xi$ and $d_F=2\xi$. We model the domain wall by a varying direction of $\mathbf{J}=(J_x,J_y,J_z)$ (keeping the magnitude $J=|\mathbf{J}|$ constant), with $J_y=0$ and

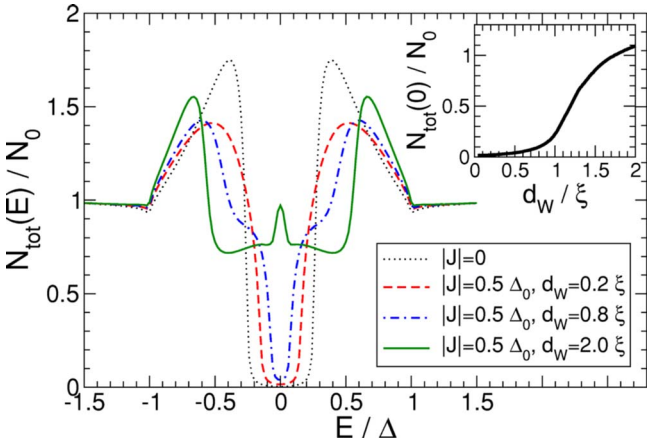

FIG. 2. (Color online) LDOS as a function of energy for the system of Fig. 1, calculated in the middle of the F region for different widths d_W of the domain wall. Here, $J=0.5\ \Delta_0$, $d_F=2.0\ \xi$. For comparison, the dotted line shows the corresponding dependence in the normal-metal case ($J=0$). The inset shows for $J=0.5\ \Delta_0$ the value of the LDOS at the chemical potential ($E=0$) as a function of d_W.

$$\begin{pmatrix} J_x \\ J_z \end{pmatrix} = J \begin{pmatrix} \cos\,\theta(z) \\ \sin\,\theta(z) \end{pmatrix}, \quad \theta(z) = -\arctan\frac{z-z_0}{d_W}. \tag{10}$$

Here, d_W is an effective domain-wall width parameter. In the following, we study the influence of the width d_W of a domain wall centered in F ($z_0=d_F/2$) on the density of states in the center of the domain wall ($z=z_0$).

Knowing $\gamma(z,E)$ from the solution of Eq. (7) with boundary conditions (8) and (9), the quasiclassical Green function and the (total) LDOS

$$N_{\rm tot}(z,E) = -\frac{N_0}{2\pi}\,{\rm Im\ Tr}\,\mathcal{G}(z,E) \tag{11}$$

is determined via Eqs. (5) and (6); N_0 is the normal-state density of states and Tr denotes the spin trace.

An important characteristic of the value of the LDOS in superconductor–normal-metal proximity systems is the minigap: the density of states in the normal-metal region shows a gap of width $E_g<\Delta_0$ induced by proximity to a superconductor. The energy E_g can be thought of as that of the lowest-energy Andreev bound state in a finite normal-metal layer.

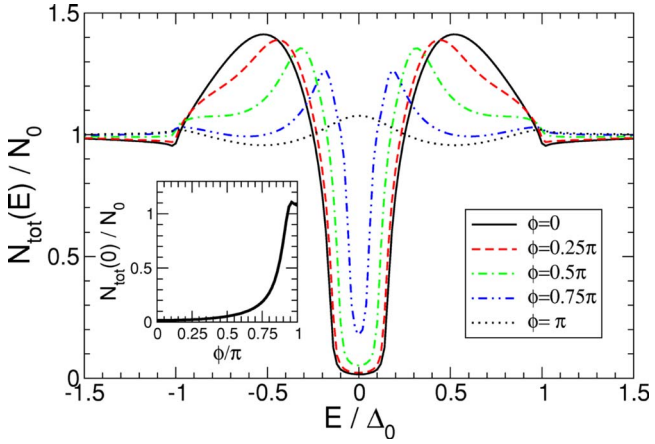

FIG. 3. (Color online) LDOS as a function of energy for several phase differences ϕ between the superconductors, calculated in the middle of F. Here, $J=0.5\ \Delta_0$, $d_F=2.0\ \xi$. The width of the domain wall is $d_W=0.2\ \xi$. The inset shows the corresponding LDOS at the chemical potential ($E=0$) as a function of ϕ.

This convenient physical picture can easily be extended to single-domain ferromagnets.[38] The corresponding spin-dependent energy shift of the quasiparticle and the Andreev-reflected quasihole by $\pm J$ leads to a reduction of the energy of the lowest-lying bound state, and correspondingly of the minigap, from the expression for a normal metal by J, vanishing altogether when $J \geqslant E_{g,J=0}$. This picture is confirmed by our numerical calculations.

In the inhomogeneously magnetized case of Fig. 1, the above picture is modified. The effect of the domain wall on the LDOS is summarized in Fig. 2, which shows $N_{\rm tot}$ as a function of energy for different domain-wall widths d_W. Although the value of $J=0.5\Delta_0$ is larger here than the value of the normal-state minigap $E_g \approx 0.25\Delta_0$ (as seen from the dotted curve in Fig. 2 for $J=0$), the minigap is reduced to zero only for larger domain-wall widths $d_W \approx 2\xi$. For the smallest width $d_W=0.2\xi$, the minigap is only reduced by about 40%. The additional states which fill the minigap with increasing domain-wall width are due to spin triplet correlations, which are sensitive to the *direction* of **J**. Our calculations show that the influence of equal-spin pairing components created by the domain wall increases. This is reflected by the appearance of additional Andreev bound states inside the gap,

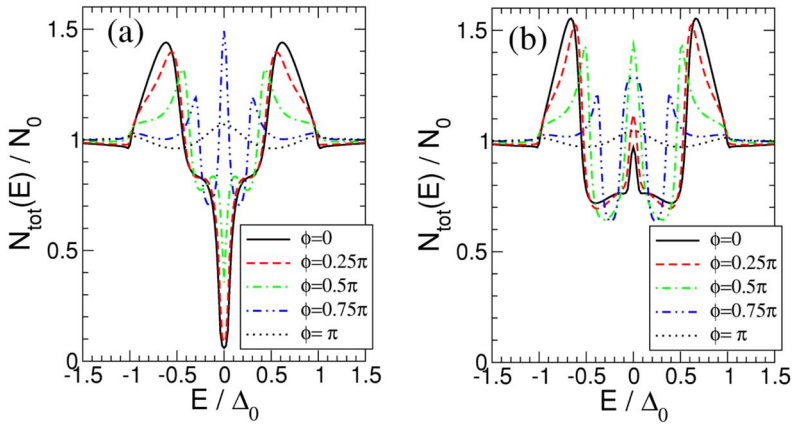

FIG. 4. (Color online) LDOS as a function of energy for several phase differences ϕ between the superconductors, calculated in the middle of F, and for $J=0.5\ \Delta_0$, $d_F=2.0\ \xi$. The domain-wall width is in (a) $d_W=1.0\ \xi$ and in (b) $d_W=2.0\ \xi$.

modifying the LDOS. The relative importance of the triplet correlations depends on J and d_W: as clearly manifested by Fig. 2, the efficiency of the triplet-inducing mechanism grows with increasing d_W. The inset of Fig. 2 shows the value of the LDOS at $E=0$ as a function of d_W. The interesting observation here is that the LDOS at the chemical potential is very sensitive to the domain-wall width when the latter is comparable to ξ.

Finally, with a view toward studying the possible effects of the domain walls on the supercurrent flowing in an SFS structure, we have studied the LDOS in the case where there is a phase difference ϕ between the two superconductors. This phase difference adds to the one accumulated by the quasiparticles and quasiholes in the ferromagnetic region and, thus, modifies the spectrum of Andreev bound states. Figures 3 and 4 present the LDOS in the middle of the F region for three domain walls with different widths. In the inset of Fig. 3, we also show the zero-energy LDOS for a domain wall of width $d_W=0.2\xi$ as a function of the phase difference. As can be seen in Fig. 4, by a possible tuning of the domain-wall width d_W one can always find a region of strongest sensitivity for a given phase difference ϕ and vice versa. This increases the possibilities of controlling the zero energy density of states in the domain wall. The rich structure exhibited by these results could easily result in highly nontrivial behavior of the transport current both in equilibrium and nonequilibrium situations.

We have studied numerically the LDOS in a heterostructure consisting of a ferromagnetic alloy sandwiched between two singlet superconductors. We find strong modifications of the LDOS caused by the presence of a domain wall. As only triplet superconducting correlations are sensitive to the direction of the exchange field, the strong variations in the LDOS result from the presence of triplet correlations induced by the spatially varying magnetization. We also find a strong dependence of the density of states in the domain wall on a possible phase difference between the superconducting order parameters, giving an additional tool to control its value. This motivates future studies of the interplay of a supercurrent and the domain wall (Josephson effect). We hope that the variety of features observed in our calculations motivates further experimental research on proximity systems involving weak ferromagnets.

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