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Individually-Rational Collective Choice under Random Preferences.

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Abstract

In this paper I consider the following problem: there is a collection of exogenously given socially feasible sets, and for each one of them, each one of a group of individuals chooses from an individually feasible set. The fact that the product of the individually feasible sets is larger than the socially feasible set notwithstanding, there arises no conflict between individuals. Assuming that individual preferences are random, I here characterize collective choices in terms of the way in which individual preferences must co-vary in order to explain them. I do this by combining standard revealed preference theory and its counterpart under random preferences. I also argue that there exist collective choices that cannot be rationalized, and hence that the individual rationality assumption can be refuted.

Keywords: Revealed preference, random utility; collective choice; individual rationality.

JEL classification: D70, D74, D12.

When one is traveling by plane, if the flight is long enough, chances are that one is going to be offered a (rather limited) menu of meal choices, say "beef or chicken." It seems reasonable to assume that each passenger makes her own meal choice based only on what she feels like eating, without giving any consideration whatsoever to whether or not there will be enough of both choices to attend the demands of all the passengers in the flight. Quite likely, if all the passengers chose to order the same meal, there would be not enough of that one meal to serve all of them. However, it is also true that in many occasions the choices of the passengers are such that everyone can be served her own choice.

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The latter is just an example of a situation in which a group of individuals face a collective choice problem, where the decision corresponds to a vector of multiple dimensions, each individual chooses some of the dimensions of the vector considering only an individual feasibility constraint, but where the set that is feasible, from the perspective of the whole group, is strictly smaller than the Cartesian product of the individually feasible sets. Indeed, as Debreu (1952) has pointed out:

> "In a wide class of social systems each agent has a range of actions among which he selects one. His choice is not, however, entirely free and the actions of all the other agents determine the subset to which his selection is restricted. Once the action of every agent is given, the outcome of the social activity is known. The preferences of each agent yield *his* complete ordering of the outcomes and each one of them tries by choosing his action in his restricting subset to bring about the best outcome according to his own preferences."

In the example, suppose that there are I passengers in the plane, and for simplicity assume that all of them want to eat something, either beef or chicken. Denumerate the passengers from 1 to I and denote by 0 the decision to order beef and by 1 the decision to order chicken. Then, for each individual the feasible set is $\{0,1\}$ and the collective choice is a vector $x = (x_i)_{i=1}^{I} \in \{0,1\}^I$. However, if the number of meals available is X^C for the choice chicken and X^B for the choice beef (hopefully, $X^C + X^B \geqslant I$), then the socially feasible set is just $\left\{ x \in \{0,1\}^I \middle| I - X^B \leqslant \sum_{i=1}^{I} x_i \leqslant X^C \right\}$, which may very well be a proper subset of $\{0,1\}^I$ (for example if $I = 2$, $X^C = X^B = 1$).

If one observes a choice situation like this and the collective choice is socially feasible, one can argue at least two explanations. The first one is that enough members of the group did actually take into account the collective constraints and were able to accommodate them by means of their own choices. An alternative explanation, however, does not require one to drop the assumption of pure individual rationality: if individual preferences just happen to be such that the profile of purely individually-rational choices lies within the feasible set, then no individual really needs to consider the collective constraints when making her or his own choice. In the example, if it just so happens that the number of passengers who at the time when meals are being served prefer chicken to beef is between $I - X^B$ and X^C, then no passenger will see her choice unserved nor will she have to accommodate her choice to the available number of meals.

The problem is indeed very simple when only one collective choice problem is being studied. Now, consider a more general and interesting situation in which there is a family of collectively feasible sets, which I will call collective budgets, and in which each individual has to choose from the projection of these budgets into her own choice set. Suppose that for each collective budget, a collective choice has been observed, and one wants to test the hypothesis that such choice was made from purely individually-rational decisions. If one assumes that all

the individuals have tastes that have not varied along the observation, then the test is again somewhat simple: all one needs to do is check the satisfaction of the Strong Axiom of Revealed Preferences individual by individual, using as individual budgets the corresponding projections of the collective budgets.

This solution to the problem, in particular, and Revealed Preference Theory, in general, can be harshly criticized for their assumption that individual preferences are an invariant concept. Specifically, observation of human behavior, in experiments as in real life, and even simple introspection suggest that preferences change in a random manner. Mathematical Psychologists have since long ago observed in their laboratories that a correct explanation of human choice has to take into account the random character that individual preferences have, and have established the so called "random utility theory," which seems to have received less attention in economics.

When the problem of collective choice is studied under the assumption of random preferences, a new and interesting dimension of analysis appears: one can no longer just require that individual choices be such that their profiles lie within the collective budgets; now, preferences have to be correlated in a way such that these profiles cannot be outside the collective budgets with positive probability. Of course, one can dismiss this aspect of the problem by assuming that individual preferences are determined independently and restricting the analysis to data sets consisting with such assumption. In this paper I have chosen to concentrate precisely on the joint determination of preferences, which allows consistency with a larger class of data sets. One can think of reasons why the assumption of independence may not be suitable for analysis in all cases. Changes in weather wuite likely affect individual preferences in a common direction. Or fashion: it usually affects significant gorups of people in a similar way, while some others may rebel against it (still not being independent). The problem under independence is indeed interesting but will not be dealt with here.

In this paper I study the joint distribution of random preferences that can explain, together with pure individual rationality, probabilistic distributions of collective choices over collective budgets. In order to avoid confusion, an important distinction must be made here: I deal with a collective-choice problem but not with an aggregate problem. That is, my interest is in explaining profiles of choices, not aggregations of choices. moreover, in the general cases I deal with, individual choices need not even be physically comparable (which is not the case of the motivational example above) in the sense that different indidvidual pick different dimensions of the collective choice and these dimensions may represent different physical features, like when a member of a group decides at what time an event that the group is organizing will take place, while some other member picks the venue for the realization of such event.

The results obtained here are based on the application of the separating hyperplane theorem given by McFadden and Richter (1990), where the finite individual random choice problem is characterized. As McFadden and Richter claim, their result is more general than the application they are explicit about. I consider the more general case of collective choice over not-necessarily-finite

domains, and impose the assumptions that are necessary to make this case compatible with the condition derived by them. When this more general setting is considered, however, an unpleasant feature of the McFadden-Richter solution makes itself obvious: their result requires the analyst to specify the finite family of (profiles of) preferences that will be allowed to have a positive probability when attempting to explain the observed data. Although this may be acceptable under finite domains (where one can use the family of all possible orders), it is quite restricting in the infinite case, as it narrows the concept or rationality tremendously: the fact that the data set appears at variance with the rationality concept may be due solely to the family of preferences and in no way means that there do not exist families of preferences (and distributions over them) that are able to explain the data via rationality. I overcome this difficulty by combining standard revealed preference theory and the McFadden-Richter condition, so as to weaken the rationalitzability requirement to just the existence of a family of preferences (controlling only its cardinality) and a probability distribution over it that are able to exactly explain the data. I also show that, regardless of the cardinality allowed, there do exist data sets which cannot be explained by individual rationality under random preferences.

The paper is organized as follows: in the next section I state the problem more formally, considering a general abstract case, and also the case of consumption choice, which by a slight difference is not a particular instance of the general case; then, in section 2, I show that one can apply the result of McFadden and Richter to the case of individually-rational collective choice, for a given set of profiles of preferences of a group of individuals; in section 3, I define and characterize individually-rational collective choice, when the set of profiles of preferences is not given, but one is allowed to construct it. In both sections I consider finite and infinite choice sets, as well as the particular case of consumption choices. I finally show examples of nonrationalizable data sets.

1 Stochastic collective choices:

1.1 The general case:

Suppose that there is a nonempty, finite set $\mathcal{I} = \{1, ..., I\}$ of decision makers. By nature, each decision maker $i \in \mathcal{I}$ chooses from a nonempty set X^i, which I will call i's choice set. In individual-choice theory, a budget is a nonempty subset of a choice set. Here, a collective budget is a nonempty subset of the Cartesian product of all the choice sets. I assume that one observes a nonempty family, $\mathcal{B}$, of collective budgets. Formally,[1]

$$\mathcal{B} \subseteq \mathcal{P}\left(\prod_{i\in\mathcal{I}} X^i\right) \setminus \{\varnothing\}, \mathcal{B} \neq \varnothing$$

Since it is over these budgets that collective choices are made, I assume

[1] For any set Z, I denote by $\mathcal{P}(Z)$ its power set, which is the collection of all of its subsets.

that for each $B \in \mathcal{B}$ one observes a probability measure. Formally, then, I endow each $B \in \mathcal{B}$ with a σ-algebra Σ^B and assume that a probability measure $\gamma_B : \Sigma^B \longrightarrow [0,1]$ has been observed. These are all the elements that I assume are being observed. They constitute a "stochastic collective choice." Formally:

Definition 1 *A stochastic collective choice is*

$$\left\{\mathcal{I}, \left(X^i\right)_{i\in\mathcal{I}}, \mathcal{B}, \left(\Sigma_B, \gamma_B\right)_{B\in\mathcal{B}}\right\}$$

Throughout the paper, for the general case I maintain the assumption that the following condition holds:

Condition 1 *B is finite, and for each $B \in \mathcal{B}$, Σ^B is finite.*

This assumption is reasonable since data collected in real life is likely to satisfy it. It must be noticed, however, that I am not assuming that the individual choice sets, X^i, are finite.

For the general case, the following notation will be used: if $Z \subseteq \prod_{i\in\mathcal{I}} X^i$ and $\mathcal{I}' \subseteq \mathcal{I}$, I denote by $\overrightarrow{Z}^{\mathcal{I}'}$ the projection of Z into $\prod_{i\in\mathcal{I}'} X^i$. Formally,

$$\overrightarrow{Z}^{\mathcal{I}'} = \left\{ x \in \prod_{i\in\mathcal{I}'} X^i \middle| \left(\exists z \in Z\right)\left(\forall i \in \mathcal{I}'\right) : z^i = x^i \right\}$$

In the case when $\mathcal{I}'$ is a singleton, I denote $\overrightarrow{Z}^{\{i\}}$ simply by $\overrightarrow{Z}^i$.

1.2 Consumers:

The specific problem of consumption choice deserves attention in its own right. I assume that there is a nonempty, finite set of consumers, $\mathcal{I} = \{1, ..., I\}$, and a finite number $L \in \mathbb{N}$ of consumption goods. Hence, each individual chooses a bundle from the nonnegative orthant of the L-dimensional space, which means that $X^i = \mathbb{R}^L_+$. I further assume that individuals face constraints of the usual form of individual budgets. Hence, I assume that there is a set of data, D, consisting of strictly positive, normalized prices and strictly positive individual endowments. Formally, this is: $D \subseteq \mathcal{S}^{L-1}_{++} \times \left(\mathbb{R}^L_{++}\right)^I$, where $\mathcal{S}^{L-1}_{++}$ is the $(L-1)$-dimensional simplex of strictly positive prices:

$$\mathcal{S}^{L-1}_{++} = \left\{ p \in \mathbb{R}^L_{++} \middle| \sum_{l=1}^{L} p_l = 1 \right\}$$

Typical elements of D will be denoted by (p, e) or $\left(p, \left(e^i\right)_{i\in\mathcal{I}}\right)$.

Individual budgets are defined as usually: for every $p \in \mathbb{R}^L_{++}$ and every $e^i \in \mathbb{R}^L_{++}$, define

$$B\left(p, e^i\right) = \left\{ x \in \mathbb{R}^L_+ \middle| \, p \cdot x \leqslant p \cdot e^i \right\}$$

Given prices $p \in \mathbb{R}^L_{++}$ and a profile of endowments $\left(e^i\right)_{i\in\mathcal{I}} \in \left(\mathbb{R}^L_{++}\right)^I$, each individual faces as constraint $B\left(p, e^i\right)$. Social feasibility, however, must take into account the aggregate endowment of commodities. Hence, given $(p, e) \in \mathcal{S}^{L-1}_{++} \times \left(\mathbb{R}^L_{++}\right)^I$ define

$$B\left(p,e\right) = \left\{ \left(x^i\right)_{i\in\mathcal{I}} \in \left(\mathbb{R}^L_{+}\right)^I \middle| \left(x^i\right)_{i\in\mathcal{I}} \in \prod_{i\in\mathcal{I}} B\left(p,e^i\right) \wedge \sum_{i\in\mathcal{I}} x^i = \sum_{i\in\mathcal{I}} e^i \right\}$$

As before, for each $(p, e) \in D$, $B(p, e)$ is endowed with a σ-algebra $\Sigma_{p,e}$ and I assume that one has observed a probability measure $\gamma_{p,e} : \Sigma_{p,e} \longrightarrow [0, 1]$, which satisfy the following straightforward condition:

$$\left(\forall \left(p,e\right), \left(p',e'\right) \in D\right) : \left(B\left(p,e\right) = B\left(p',e'\right) \Longrightarrow \left(\Sigma_{p,e} = \Sigma_{p',e'} \wedge \gamma_{p,e} = \gamma_{p',e'}\right)\right)$$

This is all the information that I assume to be observed. Then,

Definition 2 *A stochastic collective consumption choice (or demand) is*

$$\left\{\mathcal{I}, D, \left(\Sigma_{p,e}, \gamma_{p,e}\right)_{(p,e)\in D}\right\}$$

Throughout the paper, for the purposes of consumption choices, I maintain the assumption that the following condition holds:

Condition 2 *D is finite and for every $(p, e) \in D$, $\Sigma_{p,e}$ is finite.*

Remark 1 *Given the goal of this paper, I am assuming that the aggregate feasibility constraint always binds. If that is not the case, for example because society $\mathcal{I}$ is open to trade with the rest of the world (and that is why its prices are exogenously given), then the assumption can be dispensed with and one should redefine the collective budgets just as*

$$B\left(p,e\right) = \prod_{i\in\mathcal{I}} B\left(p,e^i\right)$$

2 Strong rationalizability:

2.1 The general case:

If one assumes that decision makers act noncooperatively and are individually rational, in the sense of maximizing their individual satisfaction subject only to their own constraints, then for each $B \in \mathcal{B}$ and each $C \in \Sigma^B$, $\gamma_B(C)$ is understood as the (observed) probability that if each player i chooses from $\overrightarrow{B}^i$ according to his own preferences, then the collective choice $\left(x^i\right)_{i\in\mathcal{I}}$ lies in the set $C \subseteq B$.

To formalize the concept of rationality, I then need to introduce individual preferences and define the profiles of preferences that will be allowed in

determining the rationality of the collective choice. I do this as follows. Let $S \in \mathbb{N}$, and for each $i \in \mathcal{I}$, let $\mathcal{R}^i = \left\{R^{i,1}, R^{i,2}, ..., R^{i,S}\right\}$ be a sequence of binary relations over X^i. Let $\mathcal{R}$ be the set that contains the profiles of preferences conformed by the individual sequences: $\mathcal{R} = \left\{\left(R^{1,s}, R^{2,s}, ..., R^{I,s}\right)\right\}_{s=1}^{S}$. By $R \in \mathcal{R}$, I denote the profile $\left(R^i\right)_{i\in\mathcal{I}} = \left(R^1, R^2, ..., R^I\right)$.

For definiteness, it is convenient that for each $i \in \mathcal{I}$, each $s \in \{1, ..., S\}$ and each $B \in \mathcal{B}$, the set of $R^{i,s}$-maximal elements over $\overrightarrow{B}^i$ be a singleton. That is to say:

$$(\forall i \in \mathcal{I})(\forall s \in \{1, ..., S\})(\forall B \in \mathcal{B})\left(\exists x \in \overrightarrow{B}^i\right) : \left(\begin{array}{c}\left(\forall x' \in \overrightarrow{B}^i\right) : xR^{i,s}x' \\ \wedge \\ \left(\forall x' \in \overrightarrow{B}^i \backslash \{x\}\right) : \neg x' R^{i,s} x\end{array}\right)$$

To express this idea, the following notation, which is usual in economics, is introduced. For each $i \in \mathcal{I}$, each $s \in \{1, ..., S\}$ and each $B \in \mathcal{B}$

$$Arg\max_{\overrightarrow{B}^i} R^{i,s} = \left\{ x \in \overrightarrow{B}^i \middle| \left(\left(\forall x' \in \overrightarrow{B}^i\right) : xR^{i,s}x'\right) \wedge \left(\left(\forall x' \in \overrightarrow{B}^i \backslash \{x\}\right) : \neg x' R^{i,s} x\right)\right\}$$

Under the uniqueness assumption, one can further define, implicitly,

$$\left\{\arg\max_{\overrightarrow{B}^i} R^{i,s}\right\} = Arg\max_{\overrightarrow{B}^i} R^{i,s}$$

Hence, for the sake of concreteness, for the remainder of this section I assume that one of the following cases holds:

Case $\mathbf{F}^{\mathrm{S}}$: *For every $i \in \mathcal{I}$, X^i is finite and for each $i \in \mathcal{I}$ and each $s \in \{1, ..., S\}$, $R^{i,s}$ is an order over X^i.*

Case $\mathbf{I}^{\mathrm{S}}$: *For every $i \in \mathcal{I}$, X^i is infinite and for each $i \in \mathcal{I}$ and each $s \in \{1, ..., S\}$, $R^{i,s}$ is a weak order over X^i such that for every $B \in \mathcal{B}$,*

$$\#Arg\max_{\overrightarrow{B}^i} R^{i,s} = 1$$

I can now state the concept of rationality, for a given family of profile preferences $\mathcal{R}$:

Definition 3 *The stochastic collective choice*

$$\left\{\mathcal{I}, \left(X^i\right)_{i\in\mathcal{I}}, \mathcal{B}, \left(\Sigma_B, \gamma_B\right)_{B\in\mathcal{B}}\right\}$$

is $\mathcal{R}$-rationalizable if there exists a probability measure $\delta : \mathcal{P}(\mathcal{R}) \longrightarrow [0, 1]$ such that for each $B \in \mathcal{B}$ and each $C \in \Sigma^B$,

$$\gamma_B(C) = \delta\left(\left\{ R \in \mathcal{R} | \left(\arg\max_{\overrightarrow{B}^i} R^i\right)_{i\in\mathcal{I}} \in C\right\}\right)$$

Hence, $\mathcal{R}$-rationalizability of a stochastic collective choice is obtained when one can explain the observed probabilities by pure individual rationality, via a probabilistic distribution over the profiles of preferences in $\mathcal{R}$. For the definition to be nonvacuous, I assume that the stochastic collective choice satisfies the following condition:

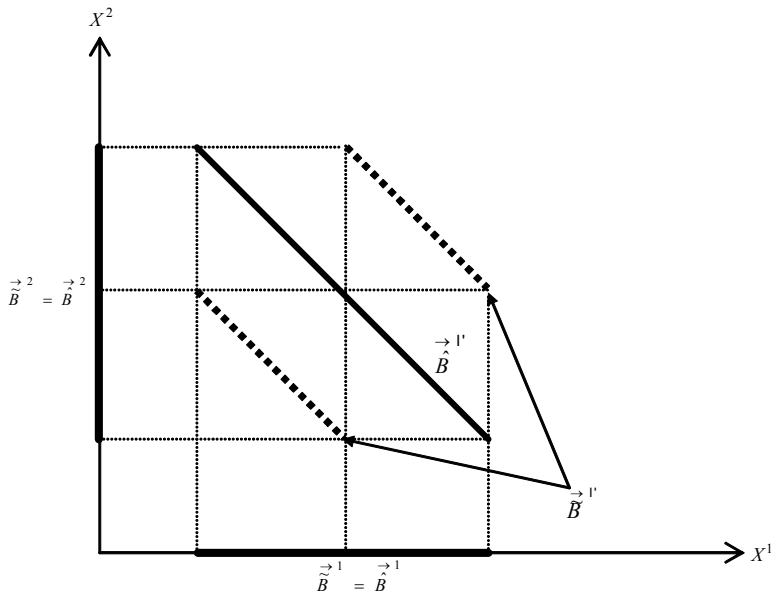
Condition 3 *Denoting for each $\mathcal{I}' \subseteq \mathcal{I}$ and each $B \in \mathcal{B}$,*

$$\mathcal{B}_{\mathcal{I}',B} = \left\{ B' \in \mathcal{B} | \left(\forall i \in \mathcal{I}' \right) : \overrightarrow{B'}^i = \overrightarrow{B}^i \right\}$$

it is true that for each $\mathcal{I}' \subseteq \mathcal{I}$ and each $B \in \mathcal{B}$,

$$\bigcap_{B' \in \mathcal{B}_{\mathcal{I}',B}} \overrightarrow{B'}^{\mathcal{I}'} \neq \varnothing$$

This condition implies that all the collective budgets leave room for players to choose by pure individual rationality, without violating the aggregate conditions that define the collective budget. The kind of problem that I want to rule out is illustrated in figure **??**. Suppose that $I \geqslant 2$. If there exist $\widehat{B}, \widetilde{B} \in \mathcal{B}$ such that $\overrightarrow{\widehat{B}}^i$ and $\overrightarrow{\widetilde{B}}^i$ are as depicted in the figure, then, irrespectively of the σ-algebras $\Sigma_{\widehat{B}}$ and $\Sigma_{\widetilde{B}}$ and, more importantly, of the actual distributions $\gamma_{\widehat{B}}$ and $\gamma_{\widetilde{B}}$, the data would be nonrationalizable, since the definition would require that with probability one, the vectors of maximizers for players 1 and 2 lie, at the same time, in one of the two dashed lines and in the solid line, which do not intersect.



Of course, the fact that individuals *can* choose individually without violating collective constraints does not mean that they *want* to do so. It is clear that

the definition of rationalizability imposes some "correlation" between the preferences of decision-makers. Hence under the null hypothesis of rationalizability, there should arise no conflict between individual and aggregate feasibility, or, better yet, this conflict should arise with probability zero and can therefore be dismissed.[2] Hence, I assume that the stochastic collective choice and the family of profiles of preferences satisfy the following condition:

Condition 4 *For each $B \in \mathcal{B}$ and each $R \in \mathcal{R}$*

$$\left(\arg\max_{\overrightarrow{B}^i} R^i\right)_{i\in\mathcal{I}} \in B$$

The following characterization of $\mathcal{R}$-rationalizability is derived from McFadden and Richter (1990), theorem 2.[3] Define the binary function

$$\alpha : \mathcal{R} \times \bigcup_{B\in\mathcal{B}} \left(\{B\} \times \Sigma_B\right) \longrightarrow \{0,1\}$$

as follows:

$$\left(\forall R \in \mathcal{R}\right)\left(\forall B \in \mathcal{B}\right)\left(\forall C \in \Sigma_B\right) : \alpha\left(R, B, C\right) = \begin{cases} 1 & \text{if} \quad \left(\arg\max_{\overrightarrow{B}^i} R^i\right)_{i\in\mathcal{I}} \in C \\ 0 & \text{otherwise} \end{cases}$$

Theorem 1 *The stochastic collective choice*

$$\left\{\mathcal{I}, \left(X^i\right)_{i\in\mathcal{I}}, \mathcal{B}, \left(\Sigma_B, \gamma_B\right)_{B\in\mathcal{B}}\right\}$$

is $\mathcal{R}$-rationalizable if, and only if, for every $K \in \mathbb{N}$ and every $\{B_k, C_k\}_{k=1}^K \overset{seq}{\subseteq} \bigcup_{B\in\mathcal{B}} \left(\{B\} \times \Sigma^B\right)$, there exists $R \in \mathcal{R}$ such that

$$\sum_{k=1}^{K} \gamma_{B_k}\left(C_k\right) \leqslant \sum_{k=1}^{K} \alpha\left(R, B_k, C_k\right)$$

Proof. Define the ternary relation Γ on

$$\mathcal{R} \times \bigcup_{B\in\mathcal{B}} \left(\{B\} \times \Sigma^B\right)$$

[2] Put another way, if this assumption does not hold, then under the hypothesis of individual rationality one could not be observing the data set that I assume is being observed.

[3] If it just so happens that for each $B \in \mathcal{B}$,

$$B = \prod_{i\in\mathcal{I}} \overrightarrow{B}^i$$

and all the binary relations $R^{i,s}$ are representable, then theorem 1 follows directly from Theorem 2 in McFadden Richter (1990), since maximizing individual relations over individual domains in then equivalent to maximizing their sum over the Cartesian product of those domains.

as follows:

$$\left(\forall R \in \mathcal{R}\right)\left(\forall B \in \mathcal{B}\right)\left(\forall C \in \Sigma^{B}\right) : \Gamma\left(R, B, C\right) \Longleftrightarrow \left(\arg\max_{\overrightarrow{B}^{i}} R^{i}\right)_{i \in \mathcal{I}} \in C$$

Then, by construction,

$$\Gamma\left(R, B, C\right) \Longrightarrow C \in \Sigma^{B}$$

whereas

$$\begin{aligned}
\Gamma\left(R, B, C\right) &\Longleftrightarrow \left(\arg\max_{\overrightarrow{B}^{i}} R^{i}\right)_{i \in \mathcal{I}} \in C \\
&\Longleftrightarrow \neg\left(\left(\arg\max_{\overrightarrow{B}^{i}} R^{i}\right)_{i \in \mathcal{I}} \in B \backslash C\right) \\
&\Longleftrightarrow \neg\Gamma\left(R, B, B \backslash C\right)
\end{aligned}$$

Also, the collective choice is $\mathcal{R}$-rationalizable if, and only if, $\exists \delta : \mathcal{P}\left(\mathcal{R}\right) \longrightarrow [0,1]$, a probability measure, such that

$$\left(\forall B \in \mathcal{B}\right)\left(\forall C \in \Sigma^{B}\right) : \gamma_{B}\left(C\right) = \delta\left\{R \in \mathcal{R} \middle| \Gamma\left(R, B, C\right)\right\}$$

and, by construction, that

$$\left(\forall R \in \mathcal{R}\right)\left(\forall B \in \mathcal{B}\right)\left(\forall C \in \Sigma^{B}\right) : \alpha\left(R, B, C\right) = 1 \Longleftrightarrow \Gamma\left(R, B, C\right)$$

Although each X^{i} need not be finite, since, by condition 1, each $\mathcal{B}$ is finite and each Σ^{B} is finite, it then follows from McFadden and Richter (1990), theorem 2 and footnote 30, that the collective choice is $\mathcal{R}$-rationalizable if, and only if, $\forall K \in \mathbb{N}$ and $\forall \left\{B_{k}, C_{k}\right\} \overset{seq}{\subseteq} \bigcup_{B \in \mathcal{B}}\left(\left\{B\right\} \times \Sigma^{B}\right)$,

$$\sum_{k=1}^{K} \gamma_{B_{k}}\left(C_{k}\right) \leqslant \max_{R \in \mathcal{R}} \sum_{k=1}^{K} \alpha\left(R, B_{k}, C_{k}\right)$$

which occurs if, and only if, $\exists R \in \mathcal{R}$ such that

$$\sum_{k=1}^{K} \gamma_{B_{k}}\left(C_{k}\right) \leqslant \sum_{k=1}^{K} \alpha\left(R, B_{k}, C_{k}\right)$$

■

The condition of this theorem is what McFadden and Richter have called the "Axiom of Stochastic Revealed Preferences." Its intuition is that events that are likely to happen should happen often. That is, if one considers the event "for each k, if a each individual chooses from $\overrightarrow{B}_{k}^{i}$, then, collectively, they choose an element of C_{k}," and such an event occurs with a "high probability", in the sense that the left-hand side of the condition of the theorem is "high," then, it should also be true that for at least one of the preferences profiles, it happens that from "many" of the budgets B_{k}, the group would choose an element of C_{k}, which would make the right-hand side of the condition "high" as well.

2.2 Consumers:

As before, in order to formalize the concept of rationality, I need to introduce individual preferences. Let $S \in \mathbb{N}$ and for each $i \in \mathcal{I}$, let $\mathcal{R}^i = \left\{R^{i,1}, R^{i,2}, ..., R^{i,S}\right\}$ be a sequence of continuous, strictly monotone and strongly convex weak orders over $\mathbb{R}^L_+$. Define again the set of profiles of preferences

$$\mathcal{R} = \left\{\left(R^{1,s}, R^{2,s}, ..., R^{I,s}\right)\right\}_{s=1}^{S}$$

In this particular case, the hypothesis of pure individual rationality means that for every pair of observed prices and endowments, $(p, e) \in D$, and for every discernible subset C of the collective budget $B(p, e)$, the observed probability $\gamma_{p,e}(C)$ is understood as the probability that if each individual chooses from $B\left(p, e^i\right)$, then the collective choice lies in C. This is precisely the formal difference for which this is not a particular case of $\mathbf{I}^{\mathrm{S}}$: now individuals choose from supersets of the projections of the socially feasible budgets.

Again, rationalizability attains when one can explain the observed stochastic choice via a distribution of profiles of preferences under the hypothesis of pure individual rationality. Then,

Definition 4 *The stochastic collective consumption choice*

$$\left\{\mathcal{I}, D, \left(\Sigma_{p,e}, \gamma_{p,e}\right)_{(p,e)\in D}\right\}$$

is $\mathcal{R}$-rationalizable if there exists a probability measure $\delta : \mathcal{P}(\mathcal{R}) \longrightarrow [0, 1]$ such that for every $(p, e) \in D$ and every $C \in \Sigma_{p,e}$,

$$\gamma_{p,e}(C) = \delta\left(\left\{R \in \mathcal{R} | \left(\arg \max_{B(p,e^i)} R^i\right)_{i\in\mathcal{I}} \in C\right\}\right)$$

Since this case does not deal with the projection of the collective budgets, one does not need to assume a condition analogous to 3. However, for the sake of feasibility it is still necessary that people do want to choose consumption bundles that satisfy aggregate feasibility. Hence, for the remainder of this subsection I assume that the data set D and the family of profiles $\mathcal{R}$ satisfy the following condition:

Condition 5 *For every $(p, e) \in D$ and every $R \in \mathcal{R}$,*

$$\sum_{i\in\mathcal{I}} \arg \max_{B(p,e^i)} R^i = \sum_{i\in\mathcal{I}} e^i$$

For this case, define the binary function

$$\alpha : \mathcal{R} \times \cup_{(p,e)\in D}\left(\{(p, e)\} \times \Sigma_{p,e}\right) \longrightarrow \{0, 1\}$$

as follows:

$$\begin{aligned}(\forall R \in \mathcal{R})(\forall (p, e) \in D)(\forall C \in \Sigma_{p,e}) \quad &: \\ \alpha(R, (p, e), C) \quad &= \quad \begin{cases} 1 \text{ if } \left(\arg\max_{B(p,e^i)} R^i\right)_{i\in\mathcal{I}} \in C \\ 0 \text{ otherwise} \end{cases}\end{aligned}$$

Theorem 2 *The stochastic collective consumption choice*

$$\left\{\mathcal{I}, D, \left(\Sigma_{p,e}, \gamma_{p,e}\right)_{(p,e)\in D}\right\}$$

is $\mathcal{R}$-rationalizable if, and only if, for every $K \in \mathbb{N}$ and every $\{(p,e)_k, C_k\}_{k=1}^{K} \overset{seq}{\subseteq} \bigcup_{(p,e)\in D}(\{(p,e)\} \times \Sigma_{p,e})$, there exists $R \in \mathcal{R}$ such that

$$\sum_{k=1}^{K} \gamma_{(p,e)_k}(C_k) \leqslant \sum_{k=1}^{K} \alpha(R, (p,e)_k, C_k)$$

Proof. The argument is similar to the one given on the proof of theorem 1, except that one now needs to define the ternary relation Γ on

$$\mathcal{R} \times \bigcup_{(p,e)\in D} (\{(p,e)\} \times \Sigma_{p,e})$$

as follows:

$$\begin{aligned} (\forall R \in \mathcal{R})(\forall (p,e) \in D)(\forall C \in \Sigma_{p,e}) \quad &: \\ \Gamma(R,(p,e),C) \iff &\left(\arg\max_{B(p.e^i)} R^i\right)_{i\in\mathcal{I}} \in C \end{aligned}$$

The argument goes through, because it is again true that, by construction,

$$\Gamma(R,(p,e),C) \Longrightarrow C \in \Sigma_{p,e}$$

whereas

$$\begin{aligned} \Gamma(R,(p,e),C) &\iff \left(\arg\max_{B(p.e^i)} R^i\right)_{i\in\mathcal{I}} \in C \\ &\iff \neg\left(\left(\arg\max_{B(p.e^i)} R^i\right)_{i\in\mathcal{I}} \in B(p,e)\setminus C\right) \\ &\iff \neg\Gamma(R,(p,e),B(p,e)\setminus C) \end{aligned}$$

■

Remark 2 *Following the point of remark 1 before, if aggregate feasibility can be disregarded, then $\mathcal{R}$ need not be assumed to satisfy condition 5, but, under the redefinition of each $B(p,e)$ and each $\Sigma_{p,e}$, theorem 2 still holds, as follows immediately from theorem 1*

3 Weak rationalizability:

In section 2, I assumed that a finite family of individual preference relations and the way in which form profiles of preferences were given, and the rationalizability

problem reduced to assigning probabilities to those profiles. Rationalizability in such a sense appears to be too strong, since one usually has no *a priori* information on preferences, and data sets may fail to be $\mathcal{R}$-rationalizable only because of the particular $\mathcal{R}$ in use. I now weaken the definition of rationalizability to test for the existence some family $\mathcal{R}$ such that the data are consistent with it, under some probability distribution and assuming purely individually-rational behavior.

3.1 The general case:

I now assume that one knows a nonempty, finite set of states of the world Ω, and that for each $i \in \mathcal{I}$, one only fixes a class $\mathcal{R}^i$ of binary relations over X^i. As before, I restrict attention only to families of preferences that give unique maximizers over the individual domains under consideration. That is, I assume that for each $i \in \mathcal{I}$, each $R^i \in \mathcal{R}^i$ and each $B \in \mathcal{B}$

$$\#Arg\max_{\overrightarrow{B}^i} R^i = 1$$

and maintain the implicit notation

$$\left\{\arg\max_{\overrightarrow{B}^i} R^i\right\} = Arg\max_{\overrightarrow{B}^i} R^i$$

Specifically, for the purposes of this subsection I now assume that one of the following cases is true:

Case $\mathbf{F}^{\mathrm{W}}$: *For every $i \in \mathcal{I}$, X^i is finite and $\mathcal{R}^i$ is the family of orders over X^i.*

Case $\mathbf{I}^{\mathrm{W}}$: *For every $i \in \mathcal{I}$, X^i is infinite and $\mathcal{R}^i$ is the family of all weak orders R^i over X^i such that for every $B^i \in \mathcal{B}^i$,*

$$\#Arg\max_{B^i} R^i = 1$$

Weak rationality is obtained if one can assign to each state of the world a profile of preferences and a probability which are able to explain the observed probabilities via pure individual rationality:

Definition 5 *The stochastic collective choice*

$$\left\{\mathcal{I}, \left(X^i\right)_{i\in\mathcal{I}}, \mathcal{B}, \left(\Sigma_B, \gamma_B\right)_{B\in\mathcal{B}}\right\}$$

is Ω-rationalizable if there exists a probability measure $\delta : \mathcal{P}(\Omega) \longrightarrow [0,1]$ and a function $R : \Omega \longrightarrow \prod_{i\in\mathcal{I}} \mathcal{R}^i$ such that for each $B \in \mathcal{B}$ and each $C \in \Sigma^B$,

$$\gamma_B(C) = \delta\left(\left\{\omega \in \Omega | \left(\arg\max_{\overrightarrow{B}^i} R^i(\omega)\right)_{i\in\mathcal{I}} \in C\right\}\right)$$

Following is the analogous to theorem 1 for this weaker definition of rationalizability. I first introduce the following notation: for every set $Z \subseteq \prod_{i\in\mathcal{I}} X^i$, the indicator functions $1_Z : \prod_{i\in\mathcal{I}} X^i \longrightarrow \{0,1\}$ is defined by

$$1_Z(z) = \begin{cases} 1 & \text{if } z \in Z \\ 0 & \text{otherwise} \end{cases}$$

Theorem 3 *The stochastic collective choice*

$$\left\{\mathcal{I}, \left(X^i\right)_{i\in\mathcal{I}}, \mathcal{B}, \left(\Sigma_B, \gamma_B\right)_{B\in\mathcal{B}}\right\}$$

is Ω-rationalizable if, and only if, for each $i \in \mathcal{I}$, each $B \in \mathcal{B}$ and each $\omega \in \Omega$ there exists $x^{i,B,\omega} \in \overrightarrow{B}^i$ such that for every $K \in \mathbb{N}$ and every $\{B_k, C_k\}_{k=1}^K \overset{seq}{\subseteq} \bigcup_{B\in\mathcal{B}} \left(\{B\} \times \Sigma^B\right)$:

1. $(\forall i \in \mathcal{I})(\forall \omega \in \Omega)$:

$$\left((\forall k \in \{1,...,K-1\}) : x^{i,B_{k+1},\omega} \in \overrightarrow{B}^i_k\right) \Longrightarrow \left(x^{i,B_K,\omega} = x^{i,B_1,\omega} \vee x^{i,B_1,\omega} \notin \overrightarrow{B}^i_K\right)$$

2. $(\exists \omega \in \Omega)$:

$$\sum_{k=1}^K \gamma_{B_k}(C_k) \leqslant \sum_{k=1}^K 1_{C_k}\left(\left(x^{i,B_k,\omega}\right)_{i\in\mathcal{I}}\right)$$

Proof. Necessity: Let $\delta : \mathcal{P}(\Omega) \longrightarrow [0,1]$ and $R : \Omega \longrightarrow \prod_{i\in\mathcal{I}} \mathcal{R}^i$ be the functions that Ω-rationalize the choice. Define, implicitly, for each $i \in \mathcal{I}$, each $B \in \mathcal{B}$ and each $\omega \in \Omega$:

$$\left\{x^{i,B^i,\omega}\right\} = Arg\max_{\overrightarrow{B}^i} R^i(\omega)$$

Clearly, $x^{i,B,\omega} \in \overrightarrow{B}^i$. Since the deterministic individual choice

$$\left(x^{i,B,\omega}, \overrightarrow{B}^i\right)_{B\in\mathcal{B}}$$

is regular-rational, it follows from Richter (1966), theorem 1, that it must satisfy the Congruence Axiom which is condition (1).

Moreover, define

$$\mathcal{R} = \{R(\omega)\}_{\omega\in\Omega}$$

and $\widetilde{\delta} = \mathcal{P}(\mathcal{R}) \longrightarrow [0,1]$ by

$$(\forall \mathcal{Q} \in \mathcal{P}(\mathcal{R})) : \widetilde{\delta}(\mathcal{Q}) = \delta(\{\omega \in \Omega | R(\omega) \in \mathcal{Q}\})$$

It is clear that $\widetilde{\delta}$ is a probability measure, while if $B \in \mathcal{B}$ and $C \in \Sigma^B$, then

$$\begin{aligned} \gamma_B(C) &= \delta\left(\left\{\omega \in \Omega | \left(\arg\max_{\overrightarrow{B}^i} R^i(\omega)\right)_{i\in\mathcal{I}} \in C\right\}\right) \\ &= \widetilde{\delta}\left(\left\{R \in \mathcal{R} | \left(\arg\max_{\overrightarrow{B}^i} R^i\right)_{i\in\mathcal{I}} \in C\right\}\right) \end{aligned}$$

which means that $\widetilde{\delta}$ stochastically $\mathcal{R}$-rationalizes the collective choice. Since $S = \#\mathcal{R} \leqslant \#\Omega < \infty$, it then follows from theorem 1 that $\forall K \in \mathbb{N}$ and $\forall \{B_k, C_k\}_{k=1}^{K} \overset{seq}{\subseteq} \bigcup_{B\in\mathcal{B}} (\{B\} \times \Sigma^B)$, $\exists \widetilde{R} \in \mathcal{R}$:

$$\sum_{k=1}^{K} \gamma_{B_k}(C_k) \leqslant \sum_{k=1}^{K} \alpha\left(\widetilde{R}, B_k, C_k\right)$$

Where α is defined as in subsection 2.1. Letting $\omega \in R^{-1}\left(\widetilde{R}\right) \subseteq \Omega$, it follows that $\exists \omega \in \Omega$:

$$\sum_{k=1}^{K} \gamma_{B_k}(C_k) \leqslant \sum_{k=1}^{K} \alpha\left(R(\omega), B_k, C_k\right)$$

from where (2) follows, since $\forall \omega \in \Omega$, $\forall B \in \mathcal{B}$ and $\forall C \in \Sigma_B$:

$$\begin{aligned} \alpha(R(\omega), B, C) = 1 &\iff \left(\arg\max_{\overrightarrow{B}^i} R^i(\omega)\right)_{i\in\mathcal{I}} \in C \\ &\iff \left(x^{i,B,\omega}\right)_{i\in\mathcal{I}} \in C \\ &\iff \mathbf{1}_C\left(\left(x^{i,B,\omega}\right)_{i\in\mathcal{I}}\right) = 1 \end{aligned}$$

Sufficiency: Fix $i \in \mathcal{I}$ and $\omega \in \Omega$. Since $\left(x^{i,B,\omega}, \overrightarrow{B}^i\right)_{B\in\mathcal{B}}$ satisfies the Congruence Axiom, which is condition 1, it follows from Richter (1966), theorem 1,[4] that $\exists R^{i,\omega} \in \mathcal{R}^i$ such that

$$(\forall B \in \mathcal{B}) : \arg\max_{\overrightarrow{B}^i} R^{i,\omega} = x^{i,B,\omega}$$

Define $\mathcal{R} = \left\{\left(R^{i,\omega}\right)_{i\in\mathcal{I}}\right\}_{\omega\in\Omega}$. Let $K \in \mathbb{N}$ and $\{B_k, C_k\}_{k=1}^{K} \overset{seq}{\subseteq} \bigcup_{B\in\mathcal{B}} (\{B\} \times \Sigma^B)$. Using condition 2, by construction, $\exists \omega \in \Omega$:

$$\begin{aligned} \sum_{k=1}^{K} \gamma_{B_{(k)}}\left(C_{(k)}\right) &\leqslant \sum_{k=1}^{K} 1_{C_k}\left(\left(x^{i,B_k,\omega}\right)_{i\in\mathcal{I}}\right) \\ &= \sum_{k=1}^{K} 1_{C_k}\left(\left(\arg\max_{\overrightarrow{B}_k^i} R^{i,\omega}\right)_{i\in\mathcal{I}}\right) \\ &= \sum_{k=1}^{K} \alpha\left(\left(R^{i,\omega}\right)_{i\in\mathcal{I}}, B_k, C_k\right) \end{aligned}$$

[4] In case $\mathbf{F}^{\mathrm{W}}$, this theorem only implies that $\exists \widetilde{R}^{i,\omega}$, a weak order, such that $\forall B \in \mathcal{B}$,

$$\arg\max_{\overrightarrow{B}^i} \widetilde{R}^{i,\omega} = x^{i,B,\omega}$$

But then one can modify $\widetilde{R}^{i,\omega}$ to make it irreflexive without losing the former property, by arbitrarily ordering the subset:

$$\left\{ x \in X^i \middle| \, (\forall B \in \mathcal{B}) : x^{i,B,\omega} \neq x \right\}$$

which implies by theorem 1, since $S = \#\mathcal{R} \leqslant \#\Omega < \infty$, that there exists a probability measure $\widetilde{\delta} : \mathcal{P}(\mathcal{R}) \longrightarrow [0,1]$ that $\mathcal{R}$-rationalizes the collective choice.

Define $R : \Omega \longrightarrow \prod_{i\in\mathcal{I}} \mathcal{R}^i$ and $\delta : \mathcal{P}(\Omega) \longrightarrow [0,1]$ by:

$$\begin{aligned} (\forall \omega \in \Omega) \quad &: \quad R(\omega) = \left(R^{i,\omega}\right)_{i\in\mathcal{I}} \\ (\forall \Phi \in \mathcal{P}(\Omega)) \quad &: \quad \delta(\Phi) = \widetilde{\delta}\left(\left\{ \left(R^{i,\omega}\right)_{i\in\mathcal{I}} \in \mathcal{R} \middle| \, \omega \in \Phi \right\}\right) \end{aligned}$$

That δ is a probability measure is straightforward. Now, let $B \in \mathcal{B}$ and $C \in \Sigma^B$. Then,

$$\begin{aligned} & \delta\left(\left\{ \omega \in \Omega | \left(\arg\max_{\overrightarrow{B}^i} R^i(\omega)\right)_{i\in\mathcal{I}} \in C \right\}\right) \\ = \; & \widetilde{\delta}\left(\left\{ \left(R^{i,\omega}\right)_{i\in\mathcal{I}} \in \mathcal{R} \middle| \left(\arg\max_{\overrightarrow{B}^i} R^{i,\omega}\right)_{i\in\mathcal{I}} \in C \right\}\right) \\ = \; & \gamma_B(C) \end{aligned}$$

which implies that δ and R Ω-rationalize

$$\left\{\mathcal{I}, \left(X^i\right)_{i\in\mathcal{I}}, \mathcal{B}, \left(\Sigma_B, \gamma_B\right)_{B\in\mathcal{B}}\right\}$$

■

3.2 Consumers:

I again assume that a nonempty, finite set of states of the world Ω is given and let $\mathcal{R}$ be the class of continuous, strictly monotone and strongly concave weak orders over $\mathbb{R}^L_+$. In this case:

Definition 6 *The stochastic collective consumption choice*

$$\left\{\mathcal{I}, D, \left(\Sigma_{p,e}, \gamma_{p,e}\right)_{(p,e)\in D}\right\}$$

is Ω-rationalizable if there exists a probability measure $\delta : \mathcal{P}(\Omega) \longrightarrow [0,1]$ and a function $R : \Omega \longrightarrow \mathcal{R}^I$ such that for each $(p,e) \in D$ and each $C \in \Sigma_{p,e}$,

$$\gamma_{p,e}(C) = \delta\left(\left\{ \omega \in \Omega | \left(\arg\max_{B(p,e^i)} R^i(\omega)\right)_{i\in\mathcal{I}} \in C \right\}\right)$$

The following result is analogous to theorem 3

Theorem 4 *The stochastic collective consumption choice*

$$\left\{\mathcal{I}, D, \left(\Sigma_{p,e}, \gamma_{p,e}\right)_{(p,e)\in D}\right\}$$

is Ω-rationalizable if, and only if, for each $i \in \mathcal{I}$, each $(p,e) \in D$ and each $\omega \in \Omega$ there exist $x^{i,(p,e),\omega} \in \mathbb{R}^L_+$, $\lambda^{i,(p,e),\omega} \in \mathbb{R}_{++}$ and $V^{i,(p,e),\omega} \in \mathbb{R}$ such that:

1. $(\forall (p,e) \in D)(\forall \omega \in \Omega)$:

$$\sum_{i \in \mathcal{I}} x^{i,(p,e),\omega} = \sum_{i \in \mathcal{I}} e^i$$

2. $(\forall i \in \mathcal{I})(\forall \omega \in \Omega)\left(\forall (p,e), (p,e)' \in D\right)$:

$$p \cdot x^{i,B\left(p,e^i\right),\omega} = p \cdot e^i$$

and

$$V^{i,(p,e)',\omega} \geqslant V^{i,(p,e),\omega} + \lambda^{i,(p,e),\omega} p \cdot \left(x^{i,(p,e)',\omega} - x^{i,(p,e),\omega}\right)$$

with strict inequality if

$$x^{i,(p,e)',\omega} \neq x^{i,(p,e),\omega}$$

3. $(\forall K \in \mathbb{N})\left(\forall \{(p_k,e_k), C_k\}_{k=1}^K \overset{seq}{\subseteq} \bigcup_{(p,e)\in\mathcal{D}} (\{(p,e)\} \times \Sigma_{p,e})\right)(\exists \omega \in \Omega)$:

$$\sum_{k=1}^K \gamma_{p_k,e_k}(C_k) \leqslant \sum_{k=1}^K \mathbf{1}_{C_k}\left(\left(x^{i,p_k,e_k,\omega}\right)_{i\in\mathcal{I}}\right)$$

Proof. The argument is similar to the one given for theorem 3.

Necessity: Let $\delta : \mathcal{P}(\Omega) \longrightarrow [0,1]$ and $R : \Omega \longrightarrow \mathcal{R}^I$ be the functions that Ω-rationalize the choice. Define, again, for each $i \in \mathcal{I}$, each $(p,e) \in D$ and each $\omega \in \Omega$:

$$x^{i,(p,e),\omega} = \arg \max_{B(p,e^i)} R^i(\omega)$$

The first part of condition (2) is straightforward, while condition (1) follows from the definition of Ω-rationalizability, since $\gamma_{p,e}(B(p,e)) = 1$. Moreover, since the deterministic demand

$$\left\{x^{i,(p,e),\omega}, B\left(p,e^i\right)\right\}_{(p,e)\in D}$$

is rationalized by a continuous, strongly concave and strictly monotone utility function, the second part of condition (2) follows from Matzkin and Richter (1991), theorem 2, (b)$\Longrightarrow$(c).

Moreover, define

$$\mathcal{R} = \{R(\omega)\}_{\omega\in\Omega}$$

and $\widetilde{\delta} = \mathcal{P}(\mathcal{R}) \longrightarrow [0,1]$ by

$$(\forall \mathcal{Q} \in \mathcal{P}(\mathcal{R})) : \widetilde{\delta}(\mathcal{Q}) = \delta(\{\omega \in \Omega |\, R(\omega) \in \mathcal{Q}\})$$

It is clear that $\widetilde{\delta}$ is a probability measure, while if $(p,e) \in D$ and $C \in \Sigma_{p,e}$, then

$$\begin{aligned}\gamma_{p,e}(C) &= \delta\left(\left\{\omega \in \Omega | \left(\arg\max_{B(p,e^i)} R^i(\omega)\right)_{i\in\mathcal{I}} \in C\right\}\right) \\ &= \widetilde{\delta}\left(\left\{R \in \mathcal{R} | \left(\arg\max_{B(p,e^i)} R^i\right)_{i\in\mathcal{I}} \in C\right\}\right)\end{aligned}$$

which means that $\widetilde{\delta}$ stochastically $\mathcal{R}$-rationalizes the collective choice. Since $S = \#\mathcal{R} \leqslant \#\Omega < \infty$, it then follows from theorem 2 that $\forall K \in \mathbb{N}$ and

$$\forall \{(p_k,e_k),C_k\}_{k=1}^K \overset{seq}{\subseteq} \bigcup_{(p,e)\in\mathcal{D}} (\{(p,e)\} \times \Sigma_{p,e})$$

$\exists \widetilde{R} \in \mathcal{R}$:

$$\sum_{k=1}^K \gamma_{p_k,e_k}(C_k) \leqslant \sum_{k=1}^K \alpha\left(\widetilde{R},(p_k,e_k),C_k\right)$$

where α is defined as in subsection 2.2. Letting $\omega \in R^{-1}\left(\widetilde{R}\right) \subseteq \Omega$, it is true that $\exists \omega \in \Omega$:

$$\sum_{k=1}^K \gamma_{p_k,e_k}(C_k) \leqslant \sum_{k=1}^K \alpha(R(\omega),(p_k,e_k),C_k)$$

from where 3 follows, since $\forall \omega \in \Omega$, $\forall (p,e) \in D$ and $\forall C \in \Sigma_{p,e}$:

$$\begin{aligned}\alpha(R(\omega),(p,e),C) = 1 &\iff \left(\arg\max_{B(p,e^i)} R^i(\omega)\right)_{i\in\mathcal{I}} \in C \\ &\iff \left(x^{i,(p,e),\omega}\right)_{i\in\mathcal{I}} \in C \\ &\iff \mathbf{1}_C\left(\left(x^{i,(p,e),\omega}\right)_{i\in\mathcal{I}}\right) = 1\end{aligned}$$

Sufficiency: Fix $i \in \mathcal{I}$ and $\omega \in \Omega$. Since $\left\{x^{i,(p,e),\omega}, B\left(p,e^i\right)\right\}_{(p,e)\in D}$ satisfies condition (2), it follows from Matzkin and Richter (1991), theorem 2, (c)$\Longrightarrow$(b) that $\exists R^{i,\omega} \in \mathcal{R}^i$ such that

$$(\forall (p,e) \in D) : \arg\max_{B(p,e^i)} R^{i,\omega} = x^{i,(p,e),\omega}$$

Define $\mathcal{R} = \left\{\left(R^{i,\omega}\right)_{i\in\mathcal{I}}\right\}_{\omega\in\Omega}$. Let $K \in \mathbb{N}$ and

$$\{(p_k,e_k),C_k\}_{k=1}^K \overset{seq}{\subseteq} \bigcup_{(p,e)\in\mathcal{D}} (\{(p,e)\} \times \Sigma_{p,e})$$

By condition (1), it follows that condition 5 is satisfied, whereas by condition (3), $\exists \omega \in \Omega$:

$$\begin{aligned}
\sum_{k=1}^{K} \gamma_{(p_k,e_k)}\left(C_{(k)}\right) &\leqslant \sum_{k=1}^{K} \mathbf{1}_{C_k}\left(\left(x^{i,(p_k,e_k),\omega}\right)_{i\in\mathcal{I}}\right) \\
&= \sum_{k=1}^{K} \mathbf{1}_{C_k}\left(\left(\arg\max_{B\left(p_k,e_k^i\right)} R^{i,\omega}\right)_{i\in\mathcal{I}}\right) \\
&= \sum_{k=1}^{K} \alpha\left(\left(R^{i,\omega}\right)_{i\in\mathcal{I}},(p_k,e_k),C_k\right)
\end{aligned}$$

which implies, by theorem 2, that there exists a probability measure $\widetilde{\delta} : \mathcal{P}(\mathcal{R}) \longrightarrow [0,1]$ that $\mathcal{R}$-rationalizes the collective choice.

Define $R : \Omega \longrightarrow \mathcal{R}^I$ and $\delta : \mathcal{P}(\Omega) \longrightarrow [0,1]$ by:

$$\begin{aligned}
(\forall \omega \in \Omega) \quad &: \quad R(\omega) = \left(R^{i,\omega}\right)_{i\in\mathcal{I}} \\
(\forall \Phi \in \mathcal{P}(\Omega)) \quad &: \quad \delta(\Phi) = \widetilde{\delta}\left(\left\{\left(R^{i,\omega}\right)_{i\in\mathcal{I}} \in \mathcal{R} \middle| \, \omega \in \Phi\right\}\right)
\end{aligned}$$

That δ is a probability measure is straightforward. Now, let $(p,e) \in D$ and $C \in \Sigma_{p,e}$. Then,

$$\begin{aligned}
&\delta\left(\left\{\omega \in \Omega \middle| \left(\arg\max_{B(p,e^i)} R^i(\omega)\right)_{i\in\mathcal{I}} \in C\right\}\right) \\
= \; &\widetilde{\delta}\left(\left\{\left(R^{i,\omega}\right)_{i\in\mathcal{I}} \in \mathcal{R} \middle| \left(\arg\max_{B(p,e^i)} R^{i,\omega}\right)_{i\in\mathcal{I}} \in C\right\}\right) \\
= \; &\gamma_{p,e}(C)
\end{aligned}$$

■

4 Nonrationalizable data sets:

The previous sections characterize stochastic, collective rationalizability. I now show examples of collective choices that are not weakly (and therefore not strongly) rationalizable, which implies that the individual rationality hypothesis is refutable in this context.

4.1 Finite choice sets:

Consider case $\mathbf{F}^{\mathrm{W}}$ first.

4.1.1 Example 1:

Let $I = \{1,2\}$. Suppose that $X^1 = X^2 = \{1,2,3,4,5\}$, and let $\mathcal{B} = \left\{\widehat{B}, \widetilde{B}\right\}$, where $\widehat{B}$ and $\widetilde{B}$ are as follows:

$$\widehat{B} = \left\{ \begin{array}{ccccc} (1,5) & & & & \\ (1,4) & (2,4) & & & \\ (1,3) & (2,3) & (3,3) & & \\ (1,2) & (2,2) & (3,2) & (4,2) & \\ (1,1) & (1,2) & (1,3) & (1,4) & (1,5) \end{array} \right\}$$

$$\widetilde{B} = \left\{ \begin{array}{ccccc} (1,5) & (2,5) & (3,5) & (4,5) & (5,5) \\ & (2,4) & (3,4) & (4,4) & (5,4) \\ & & (3,3) & (4,3) & (5,3) \\ & & & (4,2) & (5,2) \\ & & & & (5,1) \end{array} \right\}$$

Define $\widehat{C} = \{(1,5),(1,4),(2,4)\}$ and $\widetilde{C} = X^1 \times \{5\}$ and suppose that $\Sigma^{\widehat{B}} = \left\{\varnothing, \widehat{B}, \widehat{C}, \widehat{B}\backslash\widehat{C}\right\}$ and $\Sigma^{\widetilde{B}} = \left\{\varnothing, \widetilde{B}, \widetilde{C}, \widetilde{B}\backslash\widetilde{C}\right\}$. Finally, suppose that $\gamma_{\widehat{B}}\left(\widehat{C}\right) = 1/3$ and $\gamma_{\widetilde{B}}\left(\widetilde{C}\right) = 1/2$.

I now show that there cannot exist Ω such that the data set described above is Ω-rationalizable.

To see this, notice that, since $\overrightarrow{\widehat{B}}^1 = \overrightarrow{\widetilde{B}}^1 = X^1$ and $\overrightarrow{\widehat{B}}^2 = \overrightarrow{\widetilde{B}}^2 = X^2$, rationalizability would imply that

$$(\forall B \in \mathcal{B})(\forall \omega \in \Omega) : \left(\arg\max_{\overrightarrow{B}^i} R^i(\omega)\right)_{i \in \mathcal{I}} \in D$$

where

$$D = \left\{ x \in X^1 \times X^2 \middle| \, x^1 + x^2 = 6 \right\}$$

so that one needs to consider D only and see that under rationalizability,

$$\begin{aligned} 1/3 &= \gamma_{\widehat{B}}\left(\widehat{C}\right) \\ &= \delta\left(\left\{\omega \in \Omega | \left(\arg\max_{X^i} R^i(\omega)\right)_{i \in \mathcal{I}} \in \{(2,4),(1,5)\}\right\}\right) \\ &\geqslant \delta\left(\left\{\omega \in \Omega | \left(\arg\max_{X^i} R^i(\omega)\right)_{i \in \mathcal{I}} = (1,5)\right\}\right) \\ &= \gamma_{\widetilde{B}}\left(\widetilde{C}\right) \\ &= 1/2. \end{aligned}$$

an obvious impossibility.

This shows that these data are not weakly rationalizable and cannot therefore be strongly rationalized, despite the facts that $\mathcal{B}$ satisfies the first feasibility assumption of section 2, and one can construct $\mathcal{R}$ such that the second assumption of feasibility is satisfied by letting $R^{2,s}$ be the opposite order to $R^{1,s}$, for $R^{1,s} \in \mathcal{R}$.

4.1.2 Example 2:

Let $I = \{1,2\}$. Suppose that $X^1 = X^2 = \{1,2,3,4,5\}$, and let $\mathcal{B} = \left\{\widehat{B}, \widetilde{B}\right\}$, where $\widehat{B}$ and $\widetilde{B}$ are as follows:

$$\begin{aligned}\widehat{B} &= \{1,2\} \times X^2 \\ \widetilde{B} &= \{1,2,3,4\} \times X^2\end{aligned}$$

Suppose that $\{(1,1),(1,2)\} \in \Sigma^{\widehat{B}}$, $\{(1,3),(1,4),(1,5)\} \in \Sigma^{\widehat{B}}$ and that $\{1\} \times X^2 \in \Sigma^{\widetilde{B}}$. Moreover, let

$$\begin{aligned}\gamma_{\widehat{B}}(\{(1,1),(1,2)\}) &= 1/6 \\ \gamma_{\widehat{B}}(\{(1,3),(1,4),(1,5)\}) &= 1/6 \\ \gamma_{\widetilde{B}}(\{1\} \times X^2) &= 1/2\end{aligned}$$

It is straightforward that these data are not rationalizable, since rationalizability would require that

$$\begin{aligned}& \delta\left(\left\{\omega \in \Omega | \arg\max_{\{1,2\}} R^1(\omega) = 1\right\}\right) \\ = \;& \delta\left(\left\{\omega \in \Omega | \left(\arg\max_{\overrightarrow{\widehat{B}}^i} R^i(\omega)\right)_{i\in\mathcal{I}} \in \{1\} \times X^2\right\}\right) \\ = \;& \delta\left(\left\{\omega \in \Omega | \left(\arg\max_{\overrightarrow{\widehat{B}}^i} R^i(\omega)\right)_{i\in\mathcal{I}} \in \{(1,1),(1,2)\}\right\}\right) \\ & + \delta\left(\left\{\omega \in \Omega | \left(\arg\max_{\overrightarrow{\widehat{B}}^i} R^i(\omega)\right)_{i\in\mathcal{I}} \in \{(1,3),(1,4),(1,5)\}\right\}\right) \\ = \;& \gamma_{\widehat{B}}(\{(1,1),(1,2)\}) + \gamma_{\widehat{B}}(\{(1,3),(1,4),(1,5)\}) \\ = \;& 1/3\end{aligned}$$

whereas

$$\begin{aligned}& \delta\left(\left\{\omega \in \Omega | \arg\max_{\{1,2,3,4\}} R^1(\omega) = 1\right\}\right) \\ = \;& \delta\left(\left\{\omega \in \Omega | \left(\arg\max_{\overrightarrow{\widetilde{B}}^i} R^i(\omega)\right)_{i\in\mathcal{I}} \in \{1\} \times X^2\right\}\right) \\ = \;& \gamma_{\widetilde{B}}(\{1\} \times X^2) \\ = \;& 1/2\end{aligned}$$

which is impossible since when choosing from $\{1,2\}$ only, a stochastically rational individual would choose 1 at least at frequently as when choosing from $\{1,2,3,4\}$. Indeed, this behavior by agent 1 would violate the Regularity Principle of Block and Marshak (1960).

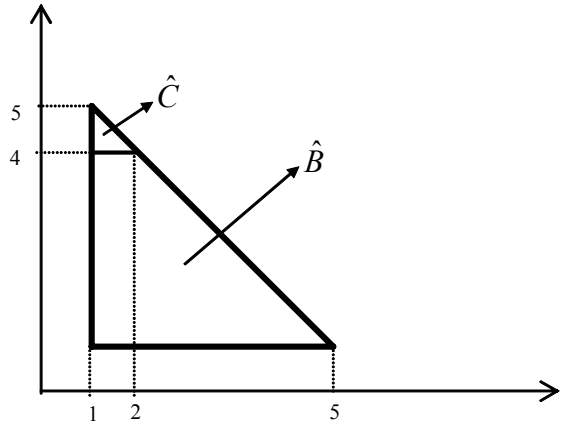
It then follows that this data set is not weakly rationalizable and, therefore, it is not strongly rationalizable, despite the fact that the feasibility assumptions of section 2 are trivially satisfied, given the shape of the collective budgets.

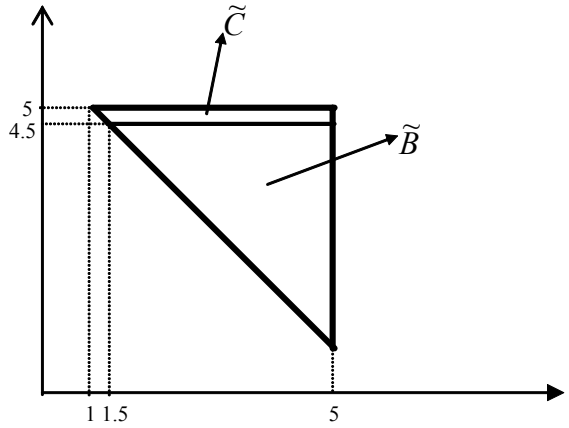
4.2 Infinite choice sets:

It is now very easy to find nonrationalizable data sets for case $\mathbf{I}^{\mathrm{W}}$, since one can simply extend the examples given for the finite case.

4.2.1 Example 1:

Let $I = \{1, 2\}$. Suppose that $X^1 = X^2 = [1, 5]$, and let $\mathcal{B} = \left\{\widehat{B}, \widetilde{B}\right\}$, where $\widehat{B}$ and $\widetilde{B}$ are as drawn in figure **??** (including their boundaries). Define $\widehat{C}$ and $\widetilde{C}$ as in those figures and suppose that $\Sigma^{\widehat{B}} = \left\{\varnothing, \widehat{B}, \widehat{C}, \widehat{B}\backslash\widehat{C}\right\}$ and $\Sigma^{\widetilde{B}} = \left\{\varnothing, \widetilde{B}, \widetilde{C}, \widetilde{B}\backslash\widetilde{C}\right\}$. Finally, suppose that $\gamma_{\widehat{B}}\left(\widehat{C}\right) = 1/3$ and $\gamma_{\widetilde{B}}\left(\widetilde{C}\right) = 1/2$.





I now show that there cannot exist Ω such that the data set described above is Ω-rationalizable. To see this, notice that, since $\overrightarrow{\widehat{B}}^1 = \overrightarrow{\widetilde{B}}^1 = X^1$ and $\overrightarrow{\widehat{B}}^2 = \overrightarrow{\widetilde{B}}^2 = X^2$, rationalizability would imply that

$$(\forall B \in \mathcal{B})(\forall \omega \in \Omega) : \left(\arg\max_{\overrightarrow{B}^i} R^i(\omega) \right)_{i \in \mathcal{I}} \in D$$

where

$$D = \left\{ x \in X^1 \times X^2 \middle| \, x^1 + x^2 = 6 \right\}$$

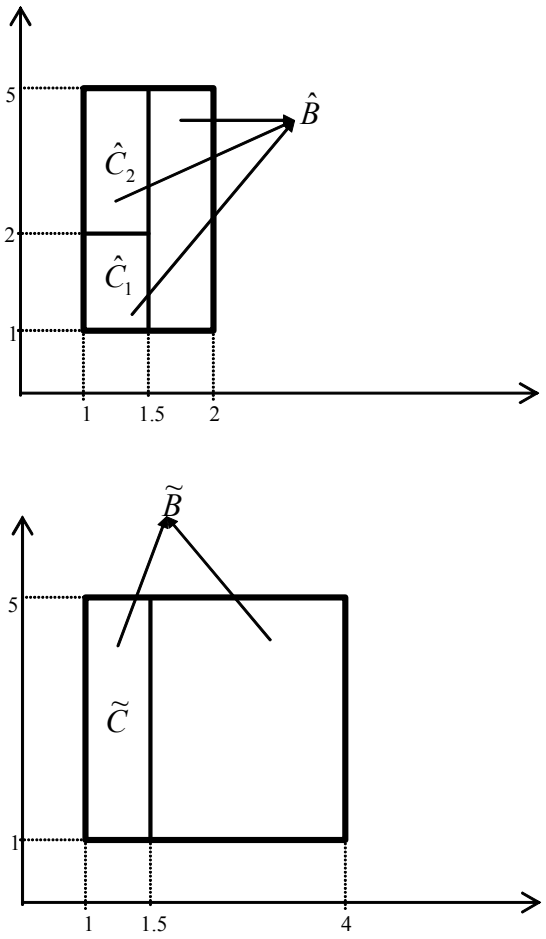
so that one can restrict attention to D only and see that under rationalizability,

$$\begin{aligned}
1/3 &= \gamma_{\widehat{B}}\left(\widehat{C}\right) \\
&= \delta\left(\left\{ \omega \in \Omega | \left(\arg\max_{X^i} R^i(\omega) \right)_{i \in \mathcal{I}} \in D \wedge \arg\max_{X^1} R^1(\omega) \leqslant 2 \right\} \right) \\
&\geqslant \delta\left(\left\{ \omega \in \Omega | \left(\arg\max_{X^i} R^i(\omega) \right)_{i \in \mathcal{I}} \in D \wedge \arg\max_{X^1} R^1(\omega) \leqslant 1.5 \right\} \right) \\
&= \gamma_{\widetilde{B}}\left(\widetilde{C}\right) \\
&= 1/2.
\end{aligned}$$

an obvious impossibility.

This shows that these data are not weakly rationalizable and cannot therefore be strongly rationalized, despite the facts that $\mathcal{B}$ satisfies the first feasibility assumption of section 2, and one can construct $\mathcal{R}$ such that the second assumption of feasibility is satisfied by letting $R^{2,s}$ be the opposite order to $R^{1,s}$, for any $R^{1,s} \in \mathcal{R}$.

4.2.2 Example 2:



Let $I = \{1, 2\}$. Suppose that $X^1 = X^2 = [1, 5]$, and let $\mathcal{B} = \left\{\widehat{B}, \widetilde{B}\right\}$, where $\widehat{B}$ and $\widetilde{B}$ are as drawn in figure **??**. Let $\widehat{C}_1$ and $\widehat{C}_2$ be defined as in the upper panel and let $\widetilde{C}$ be defined as in the lower panel.

Suppose that $\widehat{C}_1, \widehat{C}_2 \in \Sigma^{\widehat{B}}$ and that $\widetilde{C} \in \Sigma^{\widetilde{B}}$. Moreover, let

$$\begin{aligned} \gamma_{\widehat{B}}\left(\widehat{C}_1\right) &= 1/6 \\ \gamma_{\widehat{B}}\left(\widehat{C}_2\right) &= 1/6 \\ \gamma_{\widetilde{B}}\left(\widetilde{C}\right) &= 1/2 \end{aligned}$$

It is straightforward that these data are not rationalizable, since rationalizability

would require that

$$\begin{aligned}
& \delta\left(\left\{\omega\in\Omega|\arg\max_{[1,2]}R^1(\omega)\in[1,1.5]\right\}\right)\\
= \;& \delta\left(\left\{\omega\in\Omega|\left(\arg\max_{\overrightarrow{\widehat{B}}^i}R^i(\omega)\right)_{i\in\mathcal{I}}\in[1,1.5]\times X^2\right\}\right)\\
= \;& \delta\left(\left\{\omega\in\Omega|\left(\arg\max_{\overrightarrow{\widehat{B}}^i}R^i(\omega)\right)_{i\in\mathcal{I}}\in\widehat{C}_1\right\}\right)\\
& +\delta\left(\left\{\omega\in\Omega|\left(\arg\max_{\overrightarrow{\widehat{B}}^i}R^i(\omega)\right)_{i\in\mathcal{I}}\in\widehat{C}_2\right\}\right)\\
= \;& \gamma_{\widehat{B}}\left(\widehat{C}_1\right)+\gamma_{\widehat{B}}\left(\widehat{C}_2\right)\\
= \;& 1/3
\end{aligned}$$

whereas

$$\begin{aligned}
& \delta\left(\left\{\omega\in\Omega|\arg\max_{[1,4]}R^1(\omega)\in[1,1.5]\right\}\right)\\
= \;& \delta\left(\left\{\omega\in\Omega|\left(\arg\max_{\overrightarrow{\widehat{B}}^i}R^i(\omega)\right)_{i\in\mathcal{I}}\in[1,1.5]\times X^2\right\}\right)\\
= \;& \gamma_{\widetilde{B}}\left(\widetilde{C}\right)\\
= \;& 1/2
\end{aligned}$$

which again violates the Regularity Principle. Indeed, when choosing from $[1,2]$ only, a stochastically rational individual choose an element in $[1,1.5]$ at least at frequently when choosing from $[1,4]$.

It then follows that this data set is not weakly rationalizable and, therefore, it is not strongly rationalizable, despite the fact that the feasibility assumptions of section 2 are trivially satisfied, given the shape of the collective budgets.

4.3 Consumers:

I now show that there exist nonrationalizable stochastic collective consumption choices. Consider the following data for $\mathcal{I}=\{1,2\}$, $L=2$, and $\mathcal{D}=\{(\widetilde{p},\widetilde{e}),(\widehat{p},\widehat{e})\}$:

$$\begin{aligned}
\widetilde{p} &= (1,2)\\
\widetilde{e}^1 &= (1,2)\\
\widetilde{e}^2 &= (5/3,2/3)\\
\widehat{p} &= (2,1)\\
\widehat{e}^1 &= (2,1)\\
\widehat{e}^2 &= (2/3,5/3)
\end{aligned}$$

Let[5]

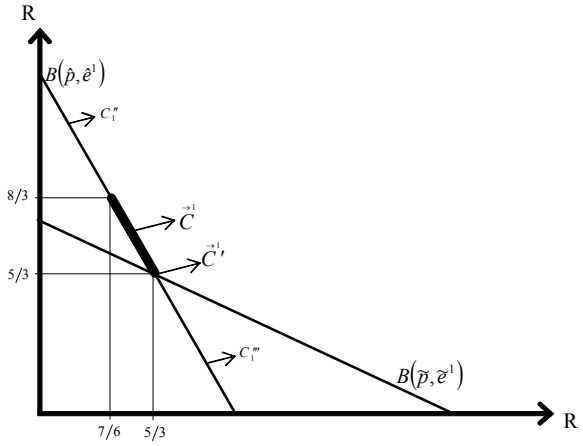
$$C=\left\{\left(\left(x_1^1,x_2^1\right),\left(x_1^2,x_2^2\right)\right)\in\mathbb{R}_+^2\,|\,\left(\begin{array}{c}x_1^1+2x_2^1\geqslant 5\\ \wedge\\ 2x_1^1+x_2^1=5\\ \wedge\\ \left(x_1^1,x_2^1\right)+\left(x_1^2,x_2^2\right)=(8/3,8/3)\end{array}\right)\right\}$$

and

$$C'=\{(5/3,5/3),(1,1)\}$$

By construction $C\subseteq B\left(\widehat{p},\widehat{e}\right)$ and $C'\subseteq B\left(\widetilde{p},\widetilde{e}\right)$, but suppose further that $C\in\Sigma_{\widehat{p},\widehat{e}}$ and $C'\in\Sigma_{\widetilde{p},\widetilde{e}}$ and

$$\begin{aligned}\gamma_{\widehat{p},\widehat{e}}\left(C\right) &= 1/3\\ \gamma_{\widetilde{p},\widetilde{e}}\left(C'\right) &= 1/2\end{aligned}$$



Now, suppose that for some Ω, the choice

$$\left\{\mathcal{I},D,\left(\Sigma_{p,e},\gamma_{p,e}\right)_{(p,e)\in D}\right\}$$

is rationalized by $\delta:\mathcal{P}\left(\Omega\right)\longrightarrow[0,1]$ and $R:\Omega\longrightarrow\mathcal{R}^2$. Then

$$\begin{aligned}\delta\left(\left\{\omega\in\Omega|\left(\arg\max_{B(\widetilde{p},\widetilde{e}^i)}R^i\left(\omega\right)\right)_{i=1}^2\in C'\right\}\right) &= \gamma_{\widetilde{p},\widetilde{e}}\left(C'\right)\\ &= 1/2\end{aligned}$$

and

$$\begin{aligned}\delta\left(\left\{\omega\in\Omega|\left(\arg\max_{B(\widetilde{p},\widetilde{e}^i)}R^i\left(\omega\right)\right)_{i=1}^2\in C'\right\}\right) &= \gamma_{\widehat{p},\widehat{e}}\left(C\right)\\ &= 1/3\end{aligned}$$

[5] I will use superindices to denote individuals and subindices to denote commodities.

Now, consider any

$$\omega' \in \left\{ \omega \in \Omega | \left(\arg \max_{B(\widehat{p},\widehat{e}^i)} R^i(\omega) \right)_{i=1}^2 \in C' \right\}$$

Suppose initially that $\delta(\{\omega'\}) > 0$.

Figure **??** shows the projection of C into the choice set of individual 1. Notice that if

$$\arg \max_{B(\widehat{p},\widehat{e}^1)} R^1(\omega') \in C_1''$$

then, by aggregate feasibility, since $\delta(\{\omega'\}) > 0$, it should be true that

$$\arg \max_{B(\widehat{p},\widehat{e}^2)} R^2(\omega') \notin \mathbb{R}^2_+$$

which is impossible. On the other hand,

$$\arg \max_{B(\widehat{p},\widehat{e}^1)} R^1(\omega') \in C_1'''$$

would violate SARP. Hence, it must be that

$$\arg \max_{B(\widehat{p},\widehat{e}^1)} R^1(\omega') \in \overrightarrow{C}^1$$

and then, by aggregate feasibility, since $\delta(\{\omega'\}) > 0$, it must be true that

$$\omega' \in \left\{ \omega \in \Omega | \left(\arg \max_{B(\widehat{p},\widehat{e}^i)} R^i(\omega) \right)_{i=1}^2 \in C' \right\}$$

The only other possibility is that $\delta(\{\omega'\}) = 0$. In any case, it follows that

$$\begin{aligned} 1/2 &= \delta\left(\left\{ \omega \in \Omega | \left(\arg \max_{B(\widehat{p},\widehat{e}^i)} R^i(\omega) \right)_{i=1}^2 \in C' \right\} \right) \\ &\leqslant \delta\left(\left\{ \omega \in \Omega | \left(\arg \max_{B(\widehat{p},\widehat{e}^i)} R^i(\omega) \right)_{i=1}^2 \in C' \right\} \right) \\ &= 1/3 \end{aligned}$$

an obvious contradiction.

5 Concluding remarks:

In this paper I have studied situations in which individuals choose from their own choice sets, subject only to their own restrictions, and no conflict arises despite the fact that the set of socially feasible choices may be strictly smaller than the product of the individually feasible sets. For this to occur, individual

preferences cannot be distributed across individuals in an arbitrary manner. The problem is relatively simple when only one collective choice situation is faced or when individual preferences are assumed to be invariant. A more interesting situation arises when there is a sequence of exogenously given social constraints and individual preferences are allowed to change randomly. In this case, if one has observed probabilistic distributions of collective choices over the socially feasible sets, one can only maintain the hypothesis of individual rationality under the assumption that preferences, however random, are not independent across individuals. The alternative would be to assume that, by chance or choice, some individuals take into account social feasibility, which amounts to dropping the usual assumption of individual rationality.

This paper characterizes collective choices in terms of the way in which individual preferences must co-vary in order to explain observed distributions of choices, maintaining individual rationality, under some finiteness assumptions. Two definitions of rationality were considered. In the first one, I assumed that one is given the set of profiles of preferences that are allowed in the rationalization, and the problem reduces to assigning probabilities to those profiles, so as to explain the observed probabilities. The main result here is that a condition defined by McFadden and Richter (1990), called the Axiom of Revealed Stochastic Preferences, characterizes rationality. This definition, however, appears to be too strong in the sense that lack of rationalizability may be due to the set of profiles of preferences and not to the observed stochastic choice. Put another way, it could be that some other set of profiles of preferences, when endowed with some particular probabilities, does explain the observed choices. This leads to the second, weaker, definition of rationalizability in which one is given a set of states of the world, and the problem consists in assigning to each one of them a profile of preferences, within certain classes, and a probability. Rationalizability in this case is characterized by a combination of the Axiom of Stochastic Revealed Preferences and several instances of the Congruence Axiom (or the Strong Axiom of Revealed Preferences) – as many as there are states of the world. I finally argued that there exist collective choices that cannot be rationalized in either sense, which means that the hypothesis of individual rationality can be refuted.

Of course, one can think of many cases in which one such collective choice situation would result in conflict between individuals. This paper does not consider such cases, nor does it consider the consequences, in terms of observed data, of assuming that individual preferences are independently determined, which is an interesting topic for future research.

6 References:

Block, H. and J. Marshak (1960), "Random orderings and stochastic theories of responses," in *Contributions to Probability and Statistics*, I. Olkin (Ed.) Stanford University Press.

Debreu, G. (1952), "A social equilibrium existence problem," *Proceedings of*

the National Academy of Sciences, 38, 886-893.

Matzkin, R. and M. Richter (1991), "Testing strictly concave rationality," *Journal of Economic Theory*, 58, 287-303.

McFadden, D. and M. Richter (1990), "Stochastic rationality and stochastic revealed preference," in *Preferences, uncertainty and optimality*, J. Chipman, D. McFadden and M. Richter (Eds.) Westview Press.

Richter, M. (1966), "Revealed Preference Theory," *Econometrica*, 34, 635-645.