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Quantum Thermal Conductance of Electrons in a One-Dimensional Wire

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We use an electron thermometer to measure the temperature rise of approximately 2×10^5 electrons in a two-dimensional box, due to heat flow into the box through a ballistic one-dimensional (1D) constriction. Using a simple model we deduce the thermal conductance $\kappa(V_g)$ of the 1D constriction, which we compare to its electrical conductance characteristics; for the first four 1D subbands the heat carried by the electrons passing through the wire is proportional to its electrical conductance $G(V_g)$. In the vicinity of the *0.7 structure* this proportionality breaks down, and a plateau at the quantum of thermal conductance $\pi^2 k_B^2 T/3h$ is observed.

The fabrication of tunable constrictions in high mobility two-dimensional electron gases (2DEGs) has allowed the study of transport in clean one-dimensional (1D) wires. This leads to an important result [1,2] in mesoscopic physics: the observation of quantized plateaus in units of $G_0 = 2e^2/h$ in the electrical conductance characteristics $G(V_g)$. In the only reported measurements [3] of the thermal conductance in similar GaAs based wires, plateaulike structures in $\kappa(V_g)$ were obtained as the 1D subbands were depopulated. In this Letter we refine the device and technique used previously [3] so that we can directly compare $\kappa(V_g)$ to electrical conductance measurements $G(V_g)$. Such a comparison is important because if both charge and energy are transported by electrons, there is a universal relation between κ and G known as the Wiedemann-Franz (WF) relation

$$\frac{\kappa}{GT} = \frac{\pi^2 k_B^2}{3e^2} = L_0, \qquad (1)$$

where L_0 is the Lorenz number and T is the temperature. Equation (1) predicts that the observation of conductance plateaus (in units of G_0) should be matched by steps in the thermal conductance, quantized in units of $L_0T \times G_0 = (1.89 \times 10^{-12} \text{ W/K}^2)T$. In addition it has been theoretically shown that in 1D the quantization of the thermal conductance is universal for thermal carriers that obey boson [4], fermion, or anyon statistics [5]. Careful low temperature measurements on suspended dielectric wires have demonstrated [6] this universality for phonons (bosons).

We find that over most of the gate voltage characteristics of a 1D wire the thermal conductance κ follows the electrical conductance G according to Eq. (1). In the vicinity of the *0.7 structure* there is a breakdown of the WF relation, giving an unexpected plateau in thermal conductance at $L_0T(G_0/2)$. The 0.7 structure is a reproducible feature observed near $0.7G_0$ in the conductance characteristics $G(V_g)$ of clean 1D samples. There is growing experimental

evidence that it has a spin origin: there is an enhanced g factor as the 1D subbands are depopulated [7], in a strong parallel magnetic field the 0.7 structure evolves [7–9] into the usual spin-split plateau at $G_0/2$, and the 0.7 structure moves down to $G_0/2$ as the density is decreased [10,11]. Recent shot-noise measurements [12] show that the 0.7 structure could be interpreted assuming two conducting channels which have different transmission probabilities. Unusually the 0.7 structure becomes stronger with temperature [7], a result most clearly demonstrated [13] in etched samples which have strong lateral confinement. The 0.7 structure could be interpreted [7,8] as a spontaneous static spin polarization within the lowest 1D subband, but the plateaulike structure occurs at $0.7G_0$ rather than at e^2/h, and moreover the temperature dependence suggests that the 0.7 structure is not a ground-state property. There is no single mechanism for the 0.7 structure, static [7], dynamic [9], or otherwise, that can account for all the experimental observations.

In previous thermal conductance [3] and thermopower [14,15] measurements, electrons on one side of the 1D constriction were heated above the lattice temperature T_L by passing a current I_H through the electron gas in that region. The electron-electron scattering rate is much faster than all other rates, and so electrons in the heating channel equilibrate at a local temperature $T_H = T_L + \Delta T$. We modify the device of Molenkamp *et al.* [3] by introducing a closed electron box, whose temperature T_{box} is measured from the thermopower in the linear regime [14,16]. The electrons in the closed box have a well-defined temperature and for a given I_H it produces larger thermovoltages than the more open structures, for example, when constriction C is not defined.

We present results for a sample fabricated from a wafer grown by molecular beam epitaxy. The 2DEG is 100 nm below the sample surface, with a carrier density of $3 \times 10^{11} \text{ cm}^{-2}$ and a mobility of $5 \times 10^6 \text{ cm}^2/\text{Vs}$. To avoid switching effects, the sample was cooled down with a positive voltage applied to the gates [17]. Three other

samples showed results similar to those presented here. Figure 1 shows a schematic of the 6 μm $\times$ 10 μm box, about which there are three split-gates, each with a gap that is 0.5 μm long and 0.65 μm wide. A current I_H of frequency $f = 32$ Hz heats electrons in the heating channel to a temperature $T_H = T_L + \Delta T$. Hot electrons can enter the box through A, the *sample* constriction, and leave via constrictions B and C. In steady state the heat flows in and out of the box are equal:

$$\kappa_A(T_H - T_{\text{box}}) = (\kappa_B + \kappa_C)(T_{\text{box}} - T_L), \quad (2)$$

and the increase in the box temperature is given by

$$\delta T = \frac{\kappa_A}{\kappa_A + \kappa_B + \kappa_C} \Delta T, \quad (3)$$

where κ_i is the thermal conductance of the ith constriction. B and C are known as the *thermometer* and *reference* constrictions, and are used to measure the temperature T_{box}. The temperature rise δT within the box generates a thermovoltage in both B and C; however, if constriction C is put on a plateau its contribution is zero [14]; the thermovoltage measured at frequency $2f$ is solely due to constriction B

$$V_{\text{th}}^{\text{box}} = S_B(T_{\text{box}} - T_L) = S_B\delta T. \quad (4)$$

The thermopower S_B is proportional to the average of the temperatures on either side of constriction B, allowing

Eq. (4) to be rewritten as

$$V_{\text{th}}^{\text{box}} = c_B(T_{\text{box}}^2 - T_L^2), \quad (5)$$

where the calibration constant, $c_B \approx 15$ μV/K^2, is determined by dc source-drain voltage measurements of the subband spacing [14]. When constriction A is not defined, the electron thermometer is in direct contact with the heating channel; the magnitude of thermovoltage at this gate voltage, V_{th}^H, is a measure of T_H through the relation

$$V_{\text{th}}^H = c_B(T_H^2 - T_L^2). \quad (6)$$

Equations (3) and (4) show that measurements of $V_{\text{th}}^{\text{box}}(V_g)$, and hence δT, lead directly to the thermal conductance $\kappa_A(V_g)$, provided we know ΔT and we make certain assumptions about κ_B and κ_C.

Figure 2 shows raw thermovoltage $V_{\text{th}}^{\text{box}}(V_g)$ data taken as the gate voltage V_g of constriction A is swept, while constrictions B and C are set at $G_B = 1.5G_0$ and $G_C = G_0$, respectively. The steps in $V_{\text{th}}^{\text{box}}$ are aligned with those in conductance G_A (top trace), confirming that the structure in $V_{\text{th}}^{\text{box}}$ is due to the 1D subbands. Traces (a)–(d) were obtained with heating currents $I_H = 0.2$–1 μA, and the corresponding temperature rise in the heating channel $\Delta T = 15$–100 mK was calculated using Eq. (5).

The thermovoltage $V_{\text{th}}^{\text{box}}$ shown in Fig. 2 follows the shape of the conductance characteristics $G_A(V_g)$; as well as the steps, the definition of constriction A at $V_g \approx 0.3$ V can be seen in all the traces [18]. At $V_g = 0.5$ V both constriction A and the box are not defined, and the electron thermometer is in direct contact with the heating channel;
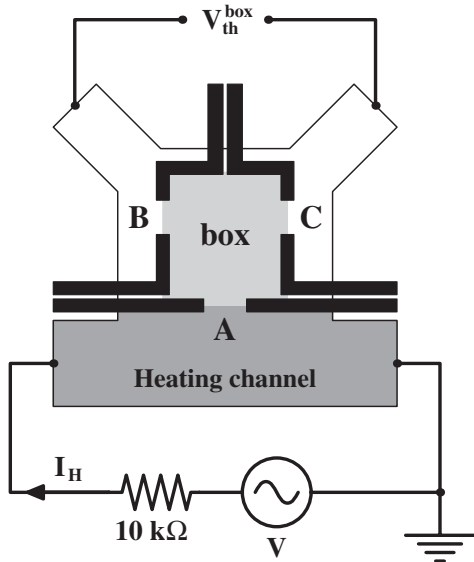

FIG. 1. Schematic of the device and setup for thermal conductance measurements. Electrons in the heating channel are heated to $T_H = T_L + \Delta T$ by a current I_H; hot electrons pass through the sample constriction (A) into the box, where the temperature $T_{\text{box}} = T_L + \delta T$ is determined by the thermovoltage $V_{\text{th}}^{\text{box}}$ generated across the thermometer and reference constrictions (B and C, respectively). Electrons can only pass through the constrictions labeled A, B, and C; the much narrower channels are pinched off.
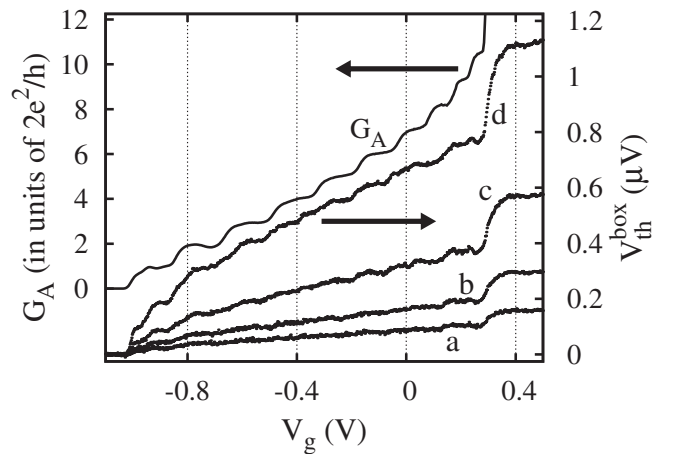

FIG. 2. The thermovoltage $V_{\text{th}}^{\text{box}}$ of the electron box together with the conductance G_A, measured at a lattice temperature $T_L = 0.27$ K using heating currents of (a) $I_H = 0.2$ μA, (b) 0.3 μA, (c) 0.5 μA, and (d) 1 μA. The corresponding temperature rise $\Delta T = 15, 30, 55,$ and 100 mK, respectively, were estimated using a source-drain calibration of the subbands. At $T_L = 0.27$ K the thermovoltage approximately scales with I_H^2 up to $I_H = 1$ μA.

we define $V_{th}^{box}(V_g = 0.5 \text{ V}) = V_{th}^H$, and T_H is obtained using Eq. (6). Energy loss from the heated electrons to the lattice via the electron-phonon interaction ($\propto T^5$) have been quantified [14], and can be neglected for $T < 0.5$ K. To ensure that the measurements of both κ_A and V_{th}^{box} are in the linear regime, the heating in the channel is limited to $\Delta T < 100$ mK.

It is not possible to measure κ_A, κ_B, and κ_C independently, and so we assume that the thermal conductance of each constriction κ_j ($j = A$, B or C) is proportional to its electrical conductance G_j,

$$\kappa_j = \alpha_j G_j \bar{T}, \tag{7}$$

where α_j is a constant and $\bar{T}$ is the average of the temperatures on either side of the jth constriction. If the WF ratio is the same for all three constrictions, $\alpha_A = \alpha_B = \alpha_C = \alpha$, Eq. (2) can be written as

$$\tilde{G}_A = (G_B + G_C)\frac{T_{box}^2 - T_L^2}{T_H^2 - T_{box}^2} = (G_B + G_C)\frac{V_{th}^{box}}{V_{th}^H - V_{th}^{box}}, \tag{8}$$

where $V_{th}^{box}(V_g)$ is measured as a function of the gate voltage on A, and V_{th}^H is the thermovoltage when constriction A is not defined. By fixing G_B and G_C at known conductances (usually constriction B is on a riser and C is on a plateau) the quantities V_{th}^H and V_{th}^{box} on the right side of Eq. (8) can be measured. The left side of Eq. (8) is written as $\tilde{G}_A$, which is the expected conductance of constriction A as derived from thermal measurements. $\tilde{G}_A(V_g)$ has the dimensions of conductance, and will be compared to the electrical conductance $G_A(V_g)$.

Because of their different grounding requirements G_A and $\tilde{G}_A$ are measured on different gate sweeps, nevertheless their pinch-off voltages agree to within 20–30 mV. Figure 3(a) shows traces of $\tilde{G}_A$ (offset horizontally) compared to the electrical conductance G_A, where the thermal data is taken with constriction B fixed at $G_B = 1.5G_0$ and the reference constriction C is set at $G_C = 5G_0$ [trace (i)], $4G_0$ [trace (ii)], and $3G_0$ [trace (iii)]. The increase in the box temperature, δT, as determined by Eq. (3) is different for each of the three traces, yet the same thermal conductance behavior for constriction A is obtained, showing the validity of the technique. In comparison to G_A, which shows conductance plateaus at multiples of $G_0 = 2e^2/h$, there are three plateaus in $\tilde{G}_A$ which occur (to within 10%–15%) at similar multiples of G_0. The conductance characteristics $G_A(V_g)$ in Fig. 3(a) show a 0.7 structure that increases in strength with temperature; there is, however, no corresponding 0.7 structure in the $\tilde{G}_A(V_g)$ traces.

Figure 3(b) shows in detail the conductances closer to pinch off, where again for clarity the $\tilde{G}_A$ traces have been shifted to the right. The thermal conductances were measured at the lattice temperature $T_L = 0.27$ K for different
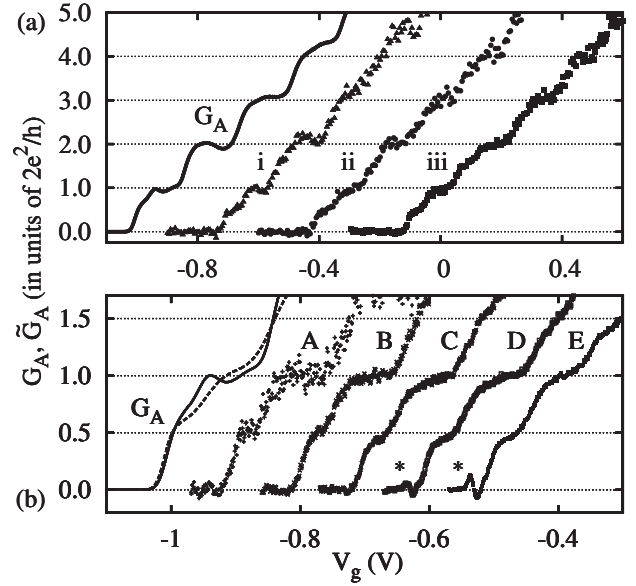

FIG. 3. (a) The thermally derived conductance $\tilde{G}_A(V_g)$ and electrical conductance $G_A(V_g)$ of constriction A at $T_L = 0.27$ K. Left solid line: conductance $G_A(V_g)$. Measurements of $\tilde{G}_A(V_g)$ were obtained with $I_H = 1$ μA, $G_B = 1.5G_0$, and $G_C = 5G_0$ [trace (i)], $G_C = 4G_0$ [trace (ii)], and $G_C = 3G_0$ [trace (iii)]. For clarity successive traces have been shifted by 0.3 V to the right. (b) Close-up of the thermal and electrical conductances near pinch off, showing that at the gate voltage corresponding to the 0.7 structure, $\tilde{G}_A$ exhibits a half plateau. Far left traces: $G_A(V_g)$ at $T_L = 0.3$ K (solid line) and $T_L = 1.2$ K (dashed line). Traces A–E: $\tilde{G}_A(V_g)$ at $T_L = 0.27$ K for heating currents $I_H = 0.2, 0.4, 1, 2$, and 3 μA; the corresponding ΔT are calculated to be 26, 66, 133, 193, and 234 mK. The $\tilde{G}_A(V_g)$ traces A and B were obtained with $G_B = 1.5G_0$ and $G_C = 3G_0$; traces C–E were obtained with $G_B = 1.5G_0$ and $G_C = 2G_0$. The small spikes in traces D and E close to pinch off (marked with a *) only occur at high I_H, and are due to a capacitive coupling that can be avoided if dc measurements are performed.

heating currents. At the lowest I_H (trace A) there is a discernible half plateau at e^2/h in $\tilde{G}_A$, which corresponds in gate voltage to the 0.7 structure in G_A. The noise on the half-plateau in trace A is due to fluctuations in the measured voltage, $V_{th}^{box} = 0.019 \pm 0.002$ μV, and the associated value of the thermal conductance is $\tilde{G}_A = (0.54 \pm 0.06)G_0$. Therefore, the plateaus in $\tilde{G}_A$ can be measured with an accuracy better than 10%; the corresponding conductance plateaus in G_A are determined with greater accuracy (<5%). With increasing I_H, traces B to E show less noise in $\tilde{G}_A$, and the half plateau in the thermal conductance is better resolved and its quantized value drifts down to $0.45G_0$, a result that was obtained in all four samples and is reminiscent of the downward trend of the height of the 0.7 structure with temperature [7,13]. This unexpected half plateau in the thermal conductance was measured both in the linear regime, $(T_H - T_{box})/T_L \sim 0.1$, but also persists into the nonlinear regime where $(T_H - T_{box})/T_L \sim 0.7$.

When the half plateau in the thermal conductance is just distinguishable, the heat flow through constriction A is $\dot{Q} = \kappa(T_H - T_{\text{box}}) \approx 6\,\text{fW}$, more than 2 orders of magnitude smaller than previous measurements [3].

In the single-particle picture a breakdown of the WF relation is possible if the transmission probability $t(E)$ of the constriction varies rapidly compared to the thermal energy difference across the constriction; however, calculations show [19] that for the realistic saddle-point potential model of the constriction there is no breakdown. First we discuss our thermal results for high conductances, $G > G_0$, where single-particle theory prevails. Figure 3(a) shows the measured behavior with no adjustable parameters—there is an alignment of the first three conductance plateaus ($G_A = G_0$, $2G_0$, and $3G_0$) with those in $\tilde{G}_A$, where the latter quantity is calculated from Eq. (8) assuming that all three constrictions behave similarly. The observed alignment cannot be interpreted as an absolute confirmation of Eq. (1) because it is not known whether the constant of proportionality α [see Eq. (7)] is equal to the Lorenz number L_0. However, in the light of theoretical predictions [5] for the quantization of κ for electrons, there is no reason why the WF relation should not be obeyed on the plateaus $G > G_0$.

Luttinger liquid behavior is expected in long clean 1D wires, and the resulting spin-charge separation causes the electrical and thermal conductances to behave differently, giving a breakdown of the WF relation which clearly demonstrates non-Fermi liquid behavior [20]. In shorter 1D constrictions there is no accepted microscopic mechanism for the 0.7 structure. One theoretical prediction [21] for the thermal conductance, that there are structures in $\kappa/(L_0 T)$ at $G_0/4$ and $3G_0/4$, has not been observed in this study. In the Landauer-Büttiker formalism the thermal conductance is calculated [19] to be $\kappa = T G_0 (k_B/e)^2 \times \int_0^\infty t(E) \partial f / \partial E \epsilon^2 dE$, where $\epsilon = (E - E_F)/k_B T$, $f(\epsilon) = [\exp(\epsilon) + 1]^{-1}$ is the Fermi function, and E_F is the Fermi energy. In a phenomenological model [8] assuming a density dependent spin gap, calculations of the electrical conductance $G = -G_0 \int_0^\infty t(E) \partial f / \partial E dE$ show some similarity to $G(V_g)$ measurements of the 0.7 structure. We find that calculations using such a model give a structure at $\kappa = L_0 T \times 0.7 G_0$; that is, the Wiedemann-Franz relation is obeyed and it is not possible to obtain simultaneous plateaus at $G = 0.7 G_0$ and $\kappa = L_0 T (G_0/2)$.

Experimentally, a plateau at $e^2/h = G_0/2$ has been measured in the electrical conductance of low density wires [10] and longer wires [8], but it is not known whether this is a full spin polarization or some other state. A mechanism for this e^2/h plateau in $G(V_g)$ has been put forward by Matveev [22], where the 1D wire is modeled as a Wigner crystal in which there are separate charge and spin degrees of freedom. The electrical resistance has contributions from both, with the spin contribution being temperature dependent and going to zero when $T \to 0$. Within this model, the thermal conductance is expected [23] to show a half plateau at $\kappa/(L_0 T) = G_0/2$.

In summary, we have demonstrated a new technique to measure the rise in temperature (typically 10–100 mK) of $\sim 2 \times 10^5$ electrons in a box, due to the heat flow in and out of the box through ballistic 1D conductors. The heat flow into the box through one of the 1D conductors is measured, and its thermal conductance characteristics $\kappa(V_g)$ have been deduced. For $G > G_0$ the Wiedemann-Franz relation is obeyed, in line with theoretical expectations. For $G < G_0$ there is no analogue in $\kappa(V_g)$ to the 0.7 structure in $G(V_g)$, but a half plateau is observed at $\kappa = L_0 T (G_0/2)$ in all four samples. This half plateau, which is measured in both the linear and nonlinear regimes, should be considered in future theoretical models of the 0.7 structure.

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