Deconfinement in a 2D Optical Lattice of Coupled 1D Boson Systems

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We show that a two-dimensional (2D) array of 1D interacting boson tubes has a deconfinement transition between a 1D Mott insulator and a 3D superfluid for commensurate fillings and a dimensional crossover for the incommensurate case. We determine the phase diagram and excitations of this system and discuss the consequences for Bose condensates loaded in 2D optical lattices.

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Loading optical lattices with ultracold atoms is providing new types of many-body systems as well as new ways to look at hard problems such as quantum-phase transitions in strongly interacting systems. The recent observation of the superfluid-Mott insulator transition [1] has demonstrated the large degree of tunability offered by these setups. This will allow the exploration of, e.g., spin-1 atoms, fermions, and Bose-Fermi mixtures, where strong interactions may lead to novel quantum many-body states. They offer also the possibility to realize anisotropic traps and thus to explore the physics of interacting particles in reduced dimensions [2,3]. Recently, two-dimensional (2D) optical lattices have been realized, where at each lattice site a 1D tube of ultracold atoms is trapped [4,5]. Such systems thus provide a unique way to obtain strongly interacting 1D Bose gases, of which the Tonks gas is the best known example (see, e.g., [6,7]).

In 1D, interactions cause more drastic effects than in higher dimensions and lead, both for fermions and bosons, to a very peculiar state known as the Tomonaga-Luttinger liquid (TLL) [7–9], which by now is theoretically well understood. However, despite the intense theoretical and experimental activity in fermionic systems or spin chains [9], much less is known when a collection of such 1D systems is coupled. In general, one can expect a dimensional crossover where the system goes from a TLL at high temperature to an anisotropic 3D system with more conventional properties (e.g., a Fermi liquid for fermions, a superfluid for bosons) at low temperature.

The transition is even more drastic and less well understood when, due to the existence of an axial periodic potential (thereafter called "Mott potential"), the 1D system becomes a Mott insulator (MI) [10–12]. Quite generally, the competition between Mott localization and superfluidity leads to interesting phenomena [13]. For coupled 1D chains, the competition between the Mott insulating physics and interchain tunneling changes the dimensional crossover into a deconfinement transition where the system goes from a 1D insulator towards a 3D metal or superfluid. For fermions this transition remains a theoretical challenge [14] and has been observed in coupled chain systems such as the organic conductors [15]. For bosons, much less is known even though the problem has been tackled in the context of spin chains (i.e., hard core bosons) [16] or for a bosonic ladder [17]. In

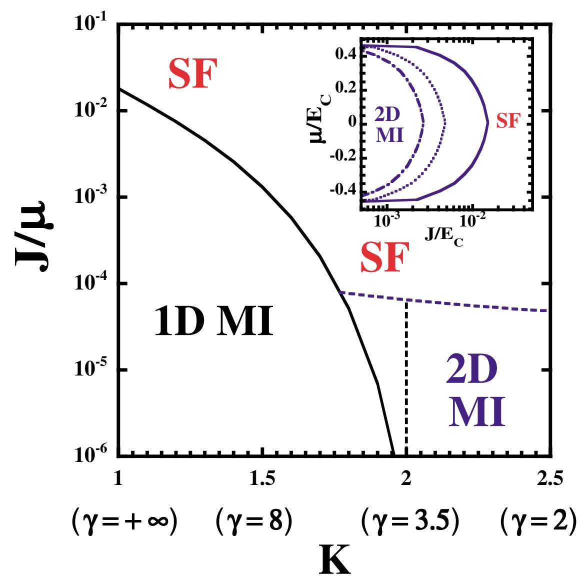

FIG. 1 (color online). Zero-temperature phase diagram of a 2D lattice of coupled 1D boson systems. The values on the vertical axis are defined up to a factor of order unity. K is the TLL parameter. The corresponding values of $\gamma = Mg/\hbar^2\rho_0$ are also given. A periodic potential of amplitude u_0, commensurate with the density ρ_0, is applied longitudinally ($u_0 \approx 0.03\mu$ for $K = 1$ and $u_0 \approx 0.02\mu$ for $K = 2.5$). For infinite 1D systems only two phases exist: a 1D Mott insulator (1D MI) and a 3D (but anisotropic) superfluid (SF). For finite tubes, a 2D Mott insulator (2D MI) can exist for J/μ below the horizontal dashed curve (we have taken $N_0 = 100$ atoms per 1D tube). The vertical dashed line (schematic) indicates that the transition to the 1D MI should take place for $K \simeq 2$. The inset is the phase diagram of a 2D optical lattice of finite 1D systems for $u_0 = 0$ and $N_0 = 100$, as a function of chemical potential μ and the hopping J, and for $K = 1$ (continuous curve), $K = 2$ (dotted curve), $K = 4$ ($\gamma = 0.71$, dash-dotted curve).

general, however, comparison between theory and experiments is rendered difficult by the complex nature of the solid-state materials.

In this Letter, motivated by the *clean* and *tunable* realization of coupled 1D gases offered by 2D optical lattices [4,5], we investigate their phases and phase diagram (see Fig. 1). For the lattice without the axial Mott potential, we show that the system exhibits, as a function of temperature, a dimensional crossover from a strongly anisotropic normal gas to a 3D Bose-Einstein condensate (BEC) or superfluid, which can have a large quantum depletion. For this 3D superfluid (SF), we obtain the condensation temperature, the zero-temperature condensate fraction, the excitation spectra, and the momentum distribution in the thermodynamic limit. When an axial Mott potential is present, we show that its tendency to lead to a Mott insulator competes with the Josephson coupling between the 1D tubes, which tries to delocalize the atoms. Changing the hopping amplitude between the tubes drives the deconfinement transition, and we determine its properties and the phase diagram. In addition, since in the experiments [4,5] the 2D optical lattices are confined in a harmonic trap, we have considered the effect of finite-size tubes. For small enough hopping, a 2D lattice behaves as a 2D array of Josephson junctions which can undergo a transition to a 2D Mott insulating state where tunneling between tubes is suppressed by the "charging energy" of each tube.

We consider a 2D square lattice of 1D tubes of length L containing $N_0 = \rho_0 L$ bosons with the Hamiltonian

$$
\begin{aligned}
H = &\sum_{\mathbf{R}} \int_0^L dx \left[\frac{\hbar^2}{2M} \partial_x \Psi_{\mathbf{R}}^\dagger(x) \partial_x \Psi_{\mathbf{R}}(x) + u(x)\rho_{\mathbf{R}}(x) \right] \\
&+ \sum_{\mathbf{R}} \frac{1}{2} \int_0^L dx \int_0^L dx' \, v(x-x') \rho_{\mathbf{R}}(x) \rho_{\mathbf{R}}(x') \\
&- \frac{J}{2} \sum_{\langle \mathbf{R},\mathbf{R}' \rangle} \int_0^L dx [\Psi_{\mathbf{R}}^\dagger(x) \Psi_{\mathbf{R}'}(x) + \text{H.c.}]. \qquad (1)
\end{aligned}
$$

$\Psi_{\mathbf{R}}(x)$ is the bosonic field operator at x (the axial coordinate), and at the lattice site $\mathbf{R} = (m, n)b$, where m, n are integers and b the lattice parameter; $\rho_{\mathbf{R}}(x) = \Psi_{\mathbf{R}}^\dagger(x)\Psi_{\mathbf{R}}(x)$ is the density operator and $u(x)$ is the Mott potential. For cold bosonic atoms confined to 1D, the interaction $v(x) = g_{1\mathrm{D}}\,\delta(x)$, where $g_{1\mathrm{D}} = 2\hbar^2 a_{3\mathrm{D}}[(1 - Ca_{3\mathrm{D}}/\sqrt{2}\ell_\perp)M\ell_\perp^2]^{-1}$ [18], with $a_{3\mathrm{D}}$ the 3D scattering length, $C \simeq 1.4603$, and $\ell_\perp \simeq bV_{0\perp}^{-1/4}/\pi$ the transverse oscillator length [1] ($V_{0\perp}$ is the strength of the transverse optical potential in units of the recoil energy $E_R = \hbar^2\pi^2/2Mb^2$). In the last term of (1), i.e., the Josephson coupling of the 1D tubes, $\langle \mathbf{R}, \mathbf{R}' \rangle$ stands for sum over nearest neighbors. Furthermore, we assume that the 2D optical lattice is deep enough for the hopping $J \ll \mu$, μ being the 1D chemical potential. We first focus on the thermodynamic properties (i.e., $N_0 \to \infty$, and $L \to \infty$); finite-size and trap effects will be considered at the end.

For the low-temperature properties of (1), it is convenient to use the so-called bosonization technique [7–9]. Using $\Psi_{\mathbf{R}}(x) \simeq \sqrt{A\rho_0}\, e^{i\phi_{\mathbf{R}}(x)}$ and $\rho_{\mathbf{R}}(x) = [\rho_0 + \partial_x\theta_{\mathbf{R}}(x)/\pi]\sum_m e^{2im[\theta_{\mathbf{R}}(x) + \pi\rho_0 x]}$, Eq. (1) becomes

$$
\begin{aligned}
H_{\text{eff}} = &\frac{\hbar v_s}{2\pi} \sum_{\mathbf{R}} \int_0^L dx \left[\frac{1}{K} [\partial_x \theta_{\mathbf{R}}(x)]^2 + K[\partial_x \phi_{\mathbf{R}}(x)]^2 \right] \\
&+ \frac{\hbar v_s g_u}{2\pi a^2} \sum_{\mathbf{R}} \int_0^L dx \cos[2\theta_{\mathbf{R}}(x) + \delta\pi x] \\
&- \frac{\hbar v_s g_J}{2\pi a^2} \sum_{\langle \mathbf{R},\mathbf{R}' \rangle} \int_0^L dx \cos[\phi_{\mathbf{R}}(x) - \phi_{\mathbf{R}'}(x)], \qquad (2)
\end{aligned}
$$

where $\delta = 2\rho_0 - G/\pi$ is the mismatch between the density ρ_0 and the periodicity of $u(x) = u_0\cos(Gx)$. The dimensionless couplings $g_J = 2\pi J(\rho_0 a)^2 A/\hbar v_s\rho_0$, and $g_u = 2\pi u_0(\rho_0 a)^2/\hbar v_s \rho_0$, where $a \approx \hbar v_s/\mu$ is the short-distance cutoff and $A \approx (K/\pi)^{1/2K}$ [7]. Results and asymptotic expressions for K and the sound velocity v_s in terms of $\gamma = Mg_{1\mathrm{D}}/\hbar^2\rho_0$ can be found in Ref. [7]. Interestingly, an anisotropic version of the Bose-Hubbard model [13] (i.e., with axial hopping $J_x \gg J$) also leads to Eq. (2) [19], but the relationship of the microscopic parameters (namely, J_x, J, and U) to K, v_s, g_u, and g_J is not easy to estimate. But if instead K and g_u are regarded as phenomenological parameters, our results directly apply to this model as well.

First, we consider the case $u_0 = 0$ (i.e., $g_u = 0$). This includes the incommensurate case $\delta \neq 0$ for which the periodic potential $u(x)$ has no effect at low temperatures [10–12]. In the absence of tunneling, the system is then a set of isolated TLL's with no true condensate and a power-law decay of phase correlations [10]. Tunneling thus induces, at a temperature T_c, a dimensional crossover from the TLL behavior at $T > T_c$, towards a 3D superfluid or BEC at low temperatures. To study the crossover, we treat the tunneling term in a mean-field (MF) approximation, taking $\psi_0 = \langle \Psi_{\mathbf{R}}(x) \rangle$ to be the (square root of the) condensate fraction. The problem then reduces to a sine-Gordon (SG) model [16,19], describing an effective 1D system. Above T_c, $\psi_0 = 0$. Therefore, T_c is found by the existence of a nonzero solution for ψ_0. We get, in agreement with [20],

$$
\left(\frac{2\pi T_c}{\hbar v_s \rho_0} \right)^{2-1/2K} = f(K) \frac{4J}{\hbar v_s \rho_0}, \qquad (3)
$$

where $f(K) = A\sin(\frac{\pi}{4K})B^2(\frac{1}{8K}, 1 - \frac{1}{4K})$ and $B(x)$ is the beta function. Below T_c, ψ_0 is found by minimizing the free energy density. At $T = 0$, one minimizes $\mathcal{E}_{\mathrm{MF}}(\psi_0) = 4J\psi_0^2 - \{[\Delta_s^2(\psi_0)]/(4\hbar v_s)\}\tan\{\pi/[2(8K-1)]\}$, where the second term is the SG model energy density [21], and $\Delta_s(\psi_0)$ the soliton gap (the excitations of the SG model are gapped solitons and breathers [8,9,22]). Thus, we find that the condensate fraction $\psi_0^2(T=0) \sim \rho_0(J/\mu)^{1/(4K-1)}$, whereas the soliton gap $\Delta_s \sim \mu(J/\mu)^{2K/(4K-1)}$ [23].

In the broken-symmetry (superfluid) phase the system possesses two modes. The Goldstone mode (i.e., oscillations of the order parameter phase) is gapless as a consequence of (global) gauge invariance. The other mode corresponds to oscillations of the order parameter amplitude. To obtain these modes, we compute the Gaussian fluctuations around the MF solution (random-phase approximation). Using known properties of correlation functions of the SG model [8,9,21], and within the single-mode approximation (SMA: taking into account only the lowest breather mode) [16], we obtain

$$\omega_-^2(q, \mathbf{Q}) = v_s^2 q^2 + \frac{\Delta_1^2}{2\hbar^2} F(\mathbf{Q}), \tag{4}$$

$$\omega_+^2(q, \mathbf{Q}) = \Delta_+^2 + v_s^2 q^2 + \left(\frac{\Delta_+^2/2\hbar^2}{8K-2}\right) F(\mathbf{Q}), \tag{5}$$

where $\Delta_+^2 = \Delta_2^2(8K-2)/(8K-1)$ and q (respectively, $\mathbf{Q}$) is the momentum along (respectively, perpendicular to) the chains. $\Delta_n = \Delta_s \sin[n\pi/2(8K-1)]$ is the energy gap for the nth breather ($n = 1, 2, \ldots$), and $F(\mathbf{Q}) = \sum_{j=y,z}(1 - \cos Q_j b)$.

For weakly interacting bosons ($K \gg 1$), the use of SMA may be questioned because the breather excitations of the mean-field SG model proliferate [8,9,22]. Therefore, we have employed a variational approach previously used for coupled spin chains [24], and found the same dependence on J/μ for the dispersion as in (4). With this approach, we have also calculated the momentum distribution of the superfluid phase (in the absence of a trap). At $T = 0$ and small momenta ($\ll \pi/b$), $\Psi_{\mathbf{R}}(x) \simeq \rho_0^{1/2} e^{i\phi_{\mathbf{R}}(x)}$ and $\omega_-^2(q, \mathbf{Q}) \simeq v_s^2 q^2 + v_\perp^2 \mathbf{Q}^2$ [$v_\perp \sim \mu b (J/\mu)^{2K/(4K-1)}/\hbar$ cf. Eq. (4)], we find an anisotropic generalization of the Bogoliubov result [25]:

$$\frac{n(q, \mathbf{Q})}{|w(\mathbf{Q})|^2} \simeq \psi_0^2 \delta(\mathbf{Q})\delta(q) + \frac{\pi b^2 \psi_0^2/2K}{[q^2 + (v_\perp \mathbf{Q}/v_s)^2]^{1/2}}, \tag{6}$$

which is valid for arbitrary interactions within the tubes. $w(\mathbf{Q})$ is the Fourier transform of the Wannier orbital, which varies slowly with $\mathbf{Q}$.

We next consider the case of a finite axial Mott potential, i.e., $u_0 \neq 0$ and $\delta \simeq 0$. In the absence of tunneling, the individual tubes would be 1D MIs: Even for a weak Mott potential, an integer number of bosons are localized to each well of the Mott potential [10–12]. Increasing the tunneling will thus lead to deconfinement (i.e., 1D MI to 3D SF transition). To study the competition between the Mott potential (g_u) and the Josephson coupling (g_J), we have used the renormalization group (RG) method. Since the Hamiltonian in (2) defines an effective field theory, its couplings (K, g_u, g_J, etc.) depend on the cutoff energy scale, which is typically set by the temperature for $T < \mu$. Thus, as the temperature is decreased, the couplings K, g_u, g_J are renormalized, due to virtual transitions to high energy states. The nature of the ground state is thus determined by the dominant coupling as $T \to 0$. Using perturbative RG, we find the RG flow equations:

$$\frac{dg_F}{d\ell} = \frac{g_J^2}{K}, \qquad \frac{dg_J}{d\ell} = \left(2 - \frac{1}{2K}\right) g_J + \frac{g_J g_F}{2K}, \tag{7}$$

$$\frac{dg_u}{d\ell} = (2 - K) g_u, \qquad \frac{dK}{d\ell} = 4g_J^2 - g_u^2 K^2, \tag{8}$$

where $\ell \approx \ln \mu/T$. The coupling g_F is generated by the RG and describes an interaction between bosons in neighboring tubes; g_u and g_J compete for $1/4 < K < 2$: The relative magnitude of their initial (i.e., bare) values determines which coupling grows faster. The faster growth of one coupling inhibits the other's growth via the renormalization of K: If g_u grows faster, K flows towards 0, leading to the 1D MI, while if g_J grows faster, K also grows and leads to the 3D SF. To estimate the phase boundary between the 1D MI and the 3D SF, we integrate these equations [at fixed $g_u(0)$] with various $g_J(0)$ until the couplings are of order 1. The resulting phase boundary is the continuous curve in Fig. 1. Via a separate mean-field calculation at a specific K value (see [19]), we showed that the condensate fraction ψ_0^2 grows continuously from zero at the phase boundary.

Finally, we turn to finite-size effects. First, the finite extent of trapped cloud (either longitudinally or transversally) limits the minimum momenta of the modes. Thus, the energy of the lowest modes in the 3D SF phase can be directly estimated from our results by using the minimum available momentum in (4) and (5). For instance, for a finite 2D lattice containing $M_y \times M_z$ tubes (i.e., an atom cloud of size $L \times M_y b \times M_z b$), the lowest available momentum is $\sim \pi/(M_i b)$. Putting this value into (4) and (5) shows that the frequency of the lowest transverse modes decreases with decreasing J. This is in qualitative agreement with the hydrodynamic analysis of Ref. [26]. However, unlike this hydrodynamic analysis, our results apply to the strongly interacting regime of the superfluid near the transition where depletion of the condensate can be large [as shown, e.g., by the power-law behavior of the condensate fraction, $\psi_0^2 \sim \rho_0 (J/\mu)^{1/(4K-1)}$]. In addition, we are also able to describe the transition to the Mott insulating regime. Note that for finite-size systems the phase transitions described above become crossovers. Furthermore, for harmonically trapped systems in the MI regimes, an inhomogeneous state arises where the SF and MI coexist [1,12], and ψ_0 decreases slowly across the phase boundaries.

A second type of finite-size effects comes from the finite size and, hence, discrete spectrum of each tube. In the experiments in 2D optical lattices loaded with ultracold atoms [4,5], there are typically $N_0 \sim 10^2$–10^3 atoms per tube. At low temperatures, if J is made very small (i.e., the lattice potential very deep), each 1D tube will behave as an atomic "quantum dot" characterized by a charging energy [27] $E_C = \hbar\pi v_s/KL$ [7]. This is to be balanced against the hopping energy E_J, which is not just JN_0 as the hopping amplitude is renormalized by the creation of virtual sound waves within the 1D tubes. Thus, to study this effect for $u_0 = 0$, we integrate out the phonons perturbatively in J and obtain a quantum-phase model:

$$H_{\rm QP} = -E_J \sum_{\langle \mathbf{R},\mathbf{R}'\rangle} \cos(\phi_{0\mathbf{R}} - \phi_{0\mathbf{R}'}) + \frac{E_C}{2}\sum_{\mathbf{R}}(N_{\mathbf{R}} - N_0)^2 - \mu\sum_{\mathbf{R}} N_{\mathbf{R}}, \quad (9)$$

where $N_{\mathbf{R}}$ (respectively, $\phi_{0\mathbf{R}}$) is the particle-number (respectively, phase) operator of tube $\mathbf{R}$ (hence, $[N_{\mathbf{R}}, \phi_{0\mathbf{R}'}] = i\delta_{\mathbf{R},\mathbf{R}'}$). The renormalized hopping $E_J = E_J(N_0) \simeq JN_0^{1-[1/(2K)]}$. The model (9) has been extensively studied in the literature in connection with 2D arrays of Josephson junctions. It exhibits a 2D MI-SF at a critical value of E_J/E_C. For commensurate filling N_0 ($\mu = 0$), a Monte Carlo (MC) calculation [28] gives $(E_J/E_C)_c \simeq 0.15$. In our case, this reduces to $(J/\mu)_c \simeq 0.3N_0^{-3/2}$ in the Tonks limit ($K \simeq 1$) and to $(J/\mu)_c \simeq 0.15N_0^{-2}$ for weakly interacting bosons ($K \gg 1$). Away from commensurate filling ($\mu \neq 0$), the critical J/μ is reduced as shown in the inset of Fig. 1, obtained using the MC data from Ref. [28]. Note that, in the 2D MI, only the phase coherence *between* different tubes of the 2D lattice is lost. There is no gap (apart from the finite-size gap) to excitations *within* each 1D system. This is different from the 1D MI described above, where all excitations are gapped and any order (even quasilong range) in the phase is absent. For finite u_0, when $K \gtrsim 2$, g_u flows to zero and the consideration above applies (hence, the horizontal and vertical dashed lines in Fig. 1). Otherwise ($K \lesssim 2$), g_J flows to zero and the 1D MI is as described above.

The above predictions can be directly tested in 2D optical lattices [4,5]. In the presence of the Mott potential, the deconfinement transition is of course the easiest to check. Even without the Mott potential, ψ_0^2 and T_c as functions of J/μ and the change in the momentum distribution from the 1D TLL to the 3D superfluid should be accessible by releasing the trap and the optical lattice, then measuring the expansion images. Finally, the excitation spectra (4) and (5) of the superfluid can be measured with Bragg spectroscopy at low momentum transfer.

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Note added.—After this Letter was submitted, we received a preprint from Esslinger [29] reporting on experimental results for the system we have considered here. The reported results are at least qualitatively consistent with our predictions here.

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