

## Entanglement Cost under Positive-Partial-Transpose-Preserving Operations

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We study the entanglement cost under quantum operations preserving the positivity of the partial transpose (PPT operations). We demonstrate that this cost is directly related to the logarithmic negativity, thereby providing the operational interpretation for this entanglement measure. As examples we discuss general Werner states and arbitrary bipartite Gaussian states. Then we prove that for the antisymmetric Werner state PPT cost and PPT entanglement of distillation coincide. This is the first example of a truly mixed state for which entanglement manipulation is asymptotically reversible, which points towards a unique entanglement measure under PPT operations.

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The theory of quantum entanglement is closely intertwined with the study of those quantum operations that can be locally implemented in quantum systems consisting of more than one subsystem. If one also allows for the classical transmission of outcomes of local measurements, then one arrives at the set of local quantum operations with classical communication (LQCC). This set of quantum operations reflects on the one hand the typical physical restrictions imposed by the setup of many basic applications of quantum information theory [1]. On the other hand, the very notion of entanglement is defined by this set of operations. For example, one calls a quantum state entangled if it cannot be prepared using LQCC, in contrast to so-called separable states.

The study of entanglement manipulation is concerned with the transformation from one entangled state to another by means of LQCC. Not surprisingly, one finds that for any finite number of identically prepared quantum systems such manipulation of entanglement under LQCC is generally irreversible, both for pure and mixed states. In fact, the pure-state case can be most easily assessed, as powerful necessary and sufficient criteria for the interconvertibility of entangled states have been found [2]. In the asymptotic limit of infinitely many identical copies of a pure state, in contrast, pure bipartite entanglement can be interconverted reversibly [3]. This statement can also be cast in the language of entanglement measures. These are functions of a quantum state that cannot increase under a given set of operations (e.g., LQCC). Entanglement measures are useful mathematical and conceptual tools and several such measures have been suggested, most notably the entanglement of formation [4], of distillation [4,5], and the relative entropy of entanglement [5–8]. The distillable entanglement is, essentially, defined as the asymptotic number of pure maximally entangled states that can be extracted via LQCC from a set of identically prepared quantum systems. Analogously, the entanglement cost is defined as the asymptotic number of maximally entangled states that are required to create a given, possibly mixed state, by LQCC. The asymptotic reversibility of pure state entan-

glement is then equivalent to the statement that the entanglement cost and the entanglement of distillation are in fact equal for pure states. Then a single number, the von Neumann entropy of a subsystem, uniquely quantifies the degree of entanglement [3]. For mixed states, however, this asymptotic reversibility under LQCC operations is lost again. Examples have been found for which the entanglement cost and the entanglement of distillation are provably different [9,10].

The study of general asymptotic entanglement manipulation—while formally being at the root of a theory of entanglement—is complicated by the fact that the characterization of LQCC themselves is not well understood. However, there is a closely related set of operations that can be much more easily characterized, namely, that of positive-partial-transpose-preserving operations (PPT operations in brief). These operations are defined as those that map any state which has positive partial transpose into another state with positive partial transpose. PPT operations are more powerful than LQCC operations as they allow, for example, the creation of any bound entangled state from a product state and ensure the distillability of any *NPT* state, i.e., any state that cannot be created by PPT operations [11]. As a consequence, the set of states decomposes into two subsets, the PPT states (nondistillable) and the states that are distillable under PPT operations. This provides a significant simplification of the entanglement structure under PPT operations as compared to that under LQCC operations where at least three classes of states, disentangled, bound entangled, and distillable are known. Indeed, the results presented in the following point towards the possibility that the structure of entanglement under PPT operations is even simpler, namely, that PPT-entanglement cost and PPT-distillable entanglement may be equal, i.e., that entanglement may be asymptotically reversible under PPT operations.

We start by summarizing the main results of this paper. First, we prove that the PPT-entanglement cost for the *exact* preparation of a large class of quantum states under PPT operations is given by the logarithmic negativity

[12–14], thus providing an operational meaning to the logarithmic negativity. Second, we employ this result to show that the PPT-entanglement cost of the antisymmetric Werner state in any dimension is given by the logarithmic negativity thereby demonstrating that the PPT cost is equal to the PPT entanglement of distillation for this state. This is the first example of a truly mixed state for which the entanglement manipulations have been proven to be asymptotically reversible. We end this work with a discussion of the implications of this result, including the possibility of the reversibility of PPT-entanglement manipulations for all states.

Before we state and formally prove our results we introduce a few basic concepts, including the definitions of the PPT entanglement of distillation and the PPT-entanglement cost. We introduce the notation (following Rains [15]) where  $\Psi$  denotes a trace-preserving completely positive PPT-preserving map and  $\Phi(K)$  is the density operator corresponding to the maximally entangled state vector in  $K$  dimensions, i.e.,  $\Phi(K) = |\psi^+\rangle\langle\psi^+|$  with  $|\psi^+\rangle = \sum_{i=1}^K |ii\rangle/\sqrt{K}$ . The PPT-distillable entanglement is defined as

$$D_{\text{PPT}}(\rho) = \sup\{r: \limsup_{n \rightarrow \infty} \sup_{\Psi} \text{tr}[\Psi(\rho^{\otimes n})\Phi(2^{rn})] = 1\}.$$

For the PPT-entanglement cost of a quantum state  $\rho$  we study two definitions that correspond to different requirements in the preparation of the state  $\rho$ . The standard definition of the PPT-entanglement cost  $C_{\text{PPT}}$  requires that the quality of the approximation of the state  $\rho^{\otimes n}$  by  $\Psi(\Phi(K))$  becomes progressively better and converges in the asymptotic limit under the trace norm, or, formally

$$C_{\text{PPT}}(\rho) = \inf\{r: \liminf_{n \rightarrow \infty} \inf_{\Psi} \text{tr}|\rho^{\otimes n} - \Psi(\Phi(2^{rn}))| = 0\}.$$

However, for a more restrictive definition one requires the *exact* preparation of any finite number of copies of the state and not just the asymptotically exact preparation. This quantity,  $E_{\text{PPT}}$ , which will generally be larger than  $C_{\text{PPT}}$ , reads formally as

$$E_{\text{PPT}}(\rho) = \liminf_{n \rightarrow \infty} \{r_n: \inf_{\Psi} \text{tr}|\rho^{\otimes n} - \Psi(\Phi(2^{r_n n}))| = 0\}.$$

This quantity will later be related to the logarithmic negativity, which was defined in [12] as

$$LN(\rho) = \log_2 \text{tr}|\rho^\Gamma|,$$

where  $\rho^\Gamma$  stands for the partial transpose of the density operator  $\rho$ . While the negativity  $\text{tr}|\rho^\Gamma|$  is an entanglement monotone (including convexity) [12,14], the logarithmic negativity is a monotone only under nonselective PPT-preserving operations. Apart from the partial transposition of a density operator, another important quantity for the following will be the so-called binegativity  $|\rho^\Gamma|^\Gamma$  [8]. While its physical interpretation is not yet properly understood, it plays a significant role in the following theorems and has proven to be a useful concept in

investigations of entanglement manipulations [8]. After these basic definitions we are now in a position to present and prove the first theorem concerning the PPT-entanglement cost.

**Theorem:** *The PPT-entanglement cost  $E_{\text{PPT}}(\rho)$  for the exact preparation of the state  $\rho$  satisfies*

$$\log_2 \text{tr}|\rho^\Gamma| \leq E_{\text{PPT}}(\rho) \leq \log_2 Z(\rho),$$

where  $Z(\rho) = \text{tr}|\rho^\Gamma| + \dim(\rho) \max(0, -\lambda_{\min}(|\rho^\Gamma|^\Gamma))$ , and  $\lambda_{\min}(A)$  is the smallest eigenvalue of the operator  $A$ .

**Proof:** The lower bound follows directly from the monotonicity of the logarithmic negativity under non-selective trace-preserving completely positive maps. We wish to find a PPT map  $\Psi$  that maps the maximally entangled state  $\Phi(K_n)$  of  $K_n$  dimensions to the target state  $\rho^{\otimes n}$  for any value of  $n$ , i.e.,  $\Psi(\Phi(K_n)) = \rho^{\otimes n}$  for all  $n$ . Then we have, for any  $n$ ,

$$\begin{aligned} \log_2 \text{tr}[(\rho^\Gamma)^{\otimes n}] &= \log_2 \text{tr}[\Psi(\Phi(K_n))]^\Gamma \leq \log_2 \text{tr}|\Phi(K_n)^\Gamma| \\ &= \log_2 K_n \end{aligned}$$

so that  $\log_2 \text{tr}|\rho^\Gamma| \leq \lim_{n \rightarrow \infty} \frac{1}{n} \log_2 K_n = E_{\text{PPT}}(\rho)$ .

Now we proceed to prove the upper bound on the entanglement cost. The linear map  $\Psi$  realizing the transformation  $\Phi(K_n) \mapsto \rho^{\otimes n}$  must be completely positive and trace preserving (CPTP) and PPT (which means that  $\Gamma \circ \Psi \circ \Gamma$  is positive as well). By proposing a map that satisfies these criteria, we directly find an upper bound to the PPT-entanglement cost. Consider, thereto, maps of the form

$$\Psi(A) = \text{Tr}[A\Phi(K_n)]F + \text{Tr}[A(\mathbb{1} - \Phi(K_n))]G,$$

where

$$F = \Psi(\Phi(K_n)), \quad G = \frac{\Psi(\mathbb{1} - \Phi(K_n))}{K_n^2 - 1}.$$

The requirements on  $\Psi$  are that it must be CPTP and PPT and must convert  $\Phi(K_n)$  into the state  $\rho^{\otimes n}$ . Thus  $F = \rho^{\otimes n}$  and  $G$  must be a state. From the PPT-ness requirement,  $\Gamma \circ \Psi \circ \Gamma \geq 0$ , it follows that

$$\forall A \geq 0: \text{Tr}[A\Phi(K_n)^\Gamma]F^\Gamma + \text{Tr}[A(\mathbb{1} - \Phi(K_n)^\Gamma)]G^\Gamma \geq 0,$$

where we have made use of the self-duality of the partial transpose,  $\text{Tr}(X^\Gamma Y) = \text{Tr}(XY^\Gamma)$ . The partial transpose of  $\Phi(K_n)$  is given by  $K_n \Phi(K_n)^\Gamma = \sum_{ij} |ij\rangle\langle ji|$ . This is the flip operator for which every symmetric (antisymmetric) state is eigenvector to eigenvalue 1 (−1). As a consequence one can express  $\Phi(K_n)^\Gamma$  in terms of the projectors on the symmetric and antisymmetric subspaces  $S$  and  $\mathcal{A}$ , respectively,

$$\Phi(K_n)^\Gamma = (S - \mathcal{A})/K_n,$$

and the PPT-ness condition becomes

$$\forall A \geq 0: \text{Tr}(A\mathcal{A})(-F^\Gamma + (K_n + 1)G^\Gamma) + \text{Tr}(AS)(F^\Gamma + (K_n - 1)G^\Gamma) \geq 0.$$

Since  $\mathcal{A}$  and  $S$  are mutually orthogonal projectors and sum to the identity, this condition simplifies to the operator inequality

$$-(K_n - 1)G^\Gamma \leq F^\Gamma \leq (K_n + 1)G^\Gamma.$$

As a direct consequence, it follows that  $G$  must be a PPT state,  $G^\Gamma \geq 0$ , which was, of course, to be expected.

For PPT-states, the PPT-entanglement cost is obviously zero, so that the optimal  $K_n = 1$ . Therefore, we restrict ourselves in the following to states  $F = \rho^{\otimes n}$  that are not PPT; hence,  $E_{\text{PPT}} > 0$  and  $K_n > 1$ . Obviously,  $(1/n) \log K_n$  is a nonincreasing function of  $n$ , tending to  $E_{\text{PPT}} > 0$  in the limit. Hence, for every  $n$ ,  $K_n \geq \exp(nE_{\text{PPT}}) > 1$ . This implies that, for every non-PPT state  $\rho$ , there is a number  $N$  such that  $\forall n > N: K_n \gg 1$ . For sufficiently large  $n$ , therefore, the PPT-ness condition on the map  $\Psi$  can be approximated to arbitrary precision by the condition  $-K_n G^\Gamma \leq F^\Gamma \leq K_n G^\Gamma$ .

We now propose to use the following state  $G$ , which incorporates a correction term to ensure positivity of  $G$ :

$$G = \frac{(|\rho^\Gamma|^\Gamma + \alpha \mathbb{1})^{\otimes n}}{Z^n}, \quad \alpha = \max[0, -\lambda_{\min}(|\rho^\Gamma|^\Gamma)],$$

$$Z = \text{tr}|\rho^\Gamma| + \alpha \dim(\rho).$$

It is now easily seen that the PPT-ness condition for the map  $\Psi$  will be satisfied for the choice  $K_n = Z^n$  (if  $\alpha$  is larger than zero, a somewhat smaller value of  $K$  is possible, but we will not consider this possibility). Hence, we get the upper bound for the PPT-entanglement cost,  $E_{\text{PPT}}(\rho) \leq \log_2 Z(\rho)$   $\square$

In general, the lower and the upper bound in the theorem will not coincide unless the binegativity is positive, i.e., if  $|\rho^\Gamma|^\Gamma \geq 0$ . However, the vast majority of quantum states have this property, as numerical investigations indicate. Important examples for which  $|\rho^\Gamma|^\Gamma \geq 0$  include the set of Werner states in  $d \times d$ -dimensional systems, and all Gaussian bipartite states in infinite-dimensional systems with canonical degrees of freedom. This will be proven in the subsequent two Lemmas.

**Lemma 1:** *Let  $\rho$  be a Gaussian state defined on a bipartite system with a finite number of canonical degrees of freedom. Then the binegativity satisfies  $|\rho^\Gamma|^\Gamma \geq 0$ .*

Proof: Let  $\Gamma$  be the covariance matrix of  $\rho$  [16] and  $P := \text{diag}(1, \dots, 1, 1, -1, \dots, 1, -1)$  be the matrix corresponding to mirror reflection in one part of the bipartite system, i.e., partial transposition on the level of states [16]. Then, the normal mode decomposition [12] (the Williamson normal form) of the covariance matrix of  $\rho^\Gamma$  can be written as  $SP\Gamma PS^T =: \text{diag}(x_1, x_1, \dots, x_n, x_n)$ , with  $x_i \geq 0$  for all  $i = 1, \dots, n$ , where  $S \in Sp(2n, \mathbf{R})$  is an appropriate symplectic matrix. Therefore, the problem

of taking the absolute value has been reduced to an effective single-mode problem. Going to the Fock state basis it is then straightforward to see that the covariance matrix of  $|\rho^\Gamma|/\|\rho^\Gamma\|_1$  is given by  $S^{-1}(SP\Gamma PS^T + p)(S^T)^{-1}$ , where  $p := \text{diag}(p_1, p_1, \dots, p_n, p_n)$  is a positive diagonal matrix with entries

$$p_i = \begin{cases} 0, & \text{if } x_i \geq 1, \\ 1/x_i - x_i, & \text{if } x_i < 1. \end{cases}$$

The state  $\rho$  has a positive binegativity, i.e.,  $\rho_{\text{bi}} := (|\rho^\Gamma|/\|\rho^\Gamma\|_1)^\Gamma \geq 0$ , iff the covariance matrix  $\Gamma_{\text{bi}}$  associated with  $\rho_{\text{bi}}$  satisfies the Heisenberg uncertainty principle  $\Gamma_{\text{bi}} + i\Sigma \geq 0$ , where  $\Sigma$  is the symplectic matrix [16]. Hence,  $\rho_{\text{bi}}$  is positive iff  $PS^{-1}(SP\Gamma PS^T + p)(S^T)^{-1}P + i\Sigma \geq 0$ . But as for the covariance matrix  $\Gamma$  of the original state  $\rho$  we have  $\Gamma + i\Sigma \geq 0$ , and because  $PS^{-1}p(S^T)^{-1}P \geq 0$  this is indeed the case.  $\square$

**Lemma 2:** *For any Werner state  $\rho$  in a  $d \times d$ -dimensional system the binegativity satisfies  $|\rho^\Gamma|^\Gamma \geq 0$  [8].*

Proof: Any Werner state for a bipartite state of two  $d$ -dimensional subsystems can be written as

$$\rho = \frac{p(\mathbb{1} - F)}{d(d-1)} + \frac{(1-p)(\mathbb{1} + F)}{d(d+1)}$$

$$= q\mathbb{1} + r|\Phi(d)\rangle\langle\Phi(d)|^\Gamma,$$

with

$$q = \frac{p}{d(d-1)} + \frac{1-p}{d(d+1)}, \quad r = \frac{1-p}{d+1} - \frac{p}{d-1},$$

and  $F$  being the flip operator. Then we find

$$|\rho^\Gamma|^\Gamma = q(\mathbb{1} - |\Phi(d)\rangle\langle\Phi(d)|) + (q+r)|\Phi(d)\rangle\langle\Phi(d)|$$

and

$$|\rho^\Gamma|^\Gamma = q\left(\mathbb{1} - \frac{F}{d}\right) + \frac{|q+r|}{d}F.$$

The eigenvalues of  $F$  are  $\pm 1$  and therefore the eigenvalues of  $|\rho^\Gamma|^\Gamma$  are easily checked to be non-negative.  $\square$

As a consequence, for Werner states, Gaussian states, and for any other states for which  $|\rho^\Gamma|^\Gamma \geq 0$ , such as pure states [8], we have proven that the entanglement cost for the exact preparation of the quantum state  $\rho$  using PPT operations is given by the logarithmic negativity. This provides the previously unknown operational interpretation of the logarithmic negativity for these states. Note that the cost  $E_{\text{PPT}}$  may generally coincide with the logarithmic negativity even for states whose binegativity is negative, but we were unable to prove or disprove this possibility. Furthermore, note also the surprising fact that the PPT cost for exact preparation is a concave function on Werner states (see also Fig. 1). This implies, rather counterintuitively, that mixing, i.e., the loss of information, may increase the PPT cost for exact preparation. We proceed by using the Theorem together with Lemma 2 to provide a result on the PPT-entanglement cost for the antisymmetric Werner state.

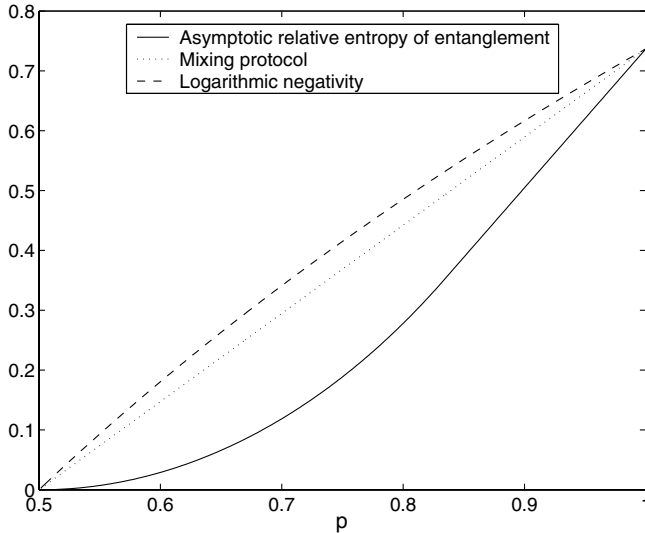


FIG. 1. Various entanglement costs plotted for Werner states of the form  $\rho = p\sigma_a + (1-p)\sigma_s$  for qutrits. The dashed line represents the PPT cost for the exact preparation of  $\rho$ , which is a concave function. The dotted line is the mixing protocol (one can create asymptotically the Werner state  $\rho = (2p-1)\sigma_a + (1-p)(\sigma_a + \sigma_s)$  by creating the state  $\sigma_a$  with probability  $2p-1$  and the state  $\sigma_a + \sigma_s$  with probability  $(1-p)/2$  at a cost given by the straight line), and represents the best known PPT cost for approximate but asymptotically exact preparation of  $\rho$ . The solid line is the asymptotic relative entropy of entanglement for PPT operations.

**Lemma 3:** *The PPT-entanglement cost  $C_{\text{PPT}}$  for the antisymmetric Werner state  $\rho = \sigma_a$  is given by  $LN(\rho)$  and coincides with its PPT-distillable entanglement  $D_{\text{PPT}}(\rho)$ .*

*Proof:* From Lemma 2 we know that the binegativity of  $\sigma_a$  is positive. As a consequence from the Theorem we conclude that  $E_{\text{PPT}}(\rho) = LN(\rho)$ . This provides an upper bound on the entanglement cost for asymptotically exact preparation of the states, i.e.,  $LN(\rho) = E_{\text{PPT}}(\rho) \geq C_{\text{PPT}}(\rho)$ . On the other hand, a lower bound is given by the PPT-distillable entanglement of  $\sigma_a$ , which has been computed in [15] and which equals the logarithmic negativity as well. Therefore we have  $LN(\rho) = E_{\text{PPT}}(\rho) \geq C_{\text{PPT}}(\rho) \geq D_{\text{PPT}}(\rho) = LN(\rho)$  and all the quantities coincide.  $\square$

This Lemma is remarkable, as it shows that asymptotic entanglement transformations can be reversible even for truly mixed states, as long as one considers the class of PPT operations. The result of Lemma 3 may still be a coincidence as it refers to an extreme point of a set of states (here the set of  $U \otimes U$  symmetric states, i.e., the Werner states) but further evidence from numerical studies suggests that PPT-entanglement cost and PPT-distillable entanglement converge towards each other on Werner states. This collection of evidence makes it plausible to ask the question as to whether the entanglement cost under PPT operations coincides with the PPT entan-

glement of distillation or, in other words, whether asymptotic entanglement transformations are reversible under PPT operations. If the answer to this question would be affirmative, the theory of entanglement would simplify considerably. As a consequence the theory of mixed state entanglement would take its simplest form in the framework of PPT operations. This and the other results in this work reveal that PPT operations are a most useful concept for the study of quantum entanglement, meriting further investigations into their properties.

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