

ELECTRON INTERACTION WITH ATOMS IN INTENSE
ELECTROMAGNETIC FIELDS

by

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to Silvana and Valentina

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ABSTRACT

In the first part of the thesis we consider two theoretical problems:

(a) The Kroll-Watson result for laser assisted potential scattering of charged particle has been extended to the case where a uniform static magnetic field is also present, and the consequences examined.

(b) The problem of the transition of an incoming plane wave state of a charged particle entering such a magnetic field, to occupy Landau levels is solved in both the adiabatic and the sudden cases. The cross section for potential scattering in presence of a magnetic field and the limit for $B \rightarrow 0$ are derived.

The second part of the thesis deals with laser assisted electron impact ionisation of helium atoms. The slow electron is described in a ~~tree~~^{by}-state close-coupling formulation. Conditions for the observation of the predicted splitting of the "Erhardt" pattern of the triple differential cross section are discussed.

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CHAPTER 1

INTRODUCTION

The study of atomic collisions in presence of strong fields has received in recent years increasing attention.

The presence of an external field changes, significantly, the conditions of the electron-atom scattering processes; the electromagnetic field, exchanging energy and momentum with the projectile and the target can play the role of a third body, opening new channels and allowing the observation of electron-atom collision parameters which would not otherwise be observable (A rather complete and updated list of contributors on the topics may be found in the review paper of Clark et al ⁽¹⁾ and in the proceedings of the International Colloquium on Atomic and Molecular Physics Close to Ionisation Threshold in High Fields ⁽²⁾).

In this thesis we will consider different collision processes involving strong magnetic fields and/or radiation fields. Usually in laboratory conditions the energy changes caused by a magnetic interaction are small compared with the characteristic energies of the system, so that the scattering processes are not affected by the presence of the field while its interaction with the target atom can be treated perturbatively. However, in experiments with highly excited atoms, in solid state physics and in astrophysics, situations are encountered where perturbation theory is not applicable to the target and the effect of the field on the collision process is not negligible.

The same is true for the interaction with a radiation field except that it is easier to meet the conditions at which perturbation theory becomes inapplicable, for example, using very intense radiation fields (lasers) or in vicinity of resonances.

Magnetic fields are of much interest in astrophysics. This interest dates from the discovery by Hale in 1908 of magnetic fields in sunspots from the Zeeman splitting of their spectral lines. More recently the discovery by Kemp et al ⁽³⁾ of circularly polarised continuum radiation from a white dwarf and its interpretation as being due to a magnetic field of 10^7 Gauss has led to renewed interest in the study of atomic properties in strong fields. Since then the existence of large magnetic fields in pulsars, thought to be up to 10^{12} Gauss at the surface, has also been demonstrated ⁽⁴⁾.

The situation in a laboratory context is very different. In fact the strongest magnetic field used in *the* laboratory is of about 10^6 Gauss. Most classical Zeeman effect studies have been performed routinely at fields typically in the range $2 \cdot 10^4 - 4 \cdot 10^4$ Gauss. Pauthenet and Dransfeld ⁽⁵⁾ reported that several laboratories could produce fields of $20 \cdot 10^4$ Gauss over a useful distance of 5cm. Higher magnetic fields up to $1.6 \cdot 10^6$ Gauss have been produced transiently by discharging a large capacitance through a single turn coil by Furth et al ⁽⁶⁾. Considerably higher fields have been produced by implosion techniques. A field of between 10^4 and 10^5 Gauss is produced and then the field is rapidly compressed in a time of microseconds by an explosive device. Fowler et al ⁽⁷⁾ reported a field of $1.4 \cdot 10^7$ Gauss lasting for $2 \mu s$. This technique has the disadvantage of being self-destructive.

Due to the intense stress in the containing chambers and to the thermal heating caused by eddy currents during pulses an upper limit exists for the production of steady fields in laboratory. This limit depends somewhat on the available materials but is typically about 10^6 Gauss. These intensities are not very high; in fact, in atomic units, the unit of the magnetic field is *equal to* $4.96 \cdot 10^9$ Gauss.

However, in some particular cases the effect of high fields may be observed at low fields. For instance, in solids i) the mass of an electron in motion is represented by the effective mass m^* , which may be several orders of magnitude smaller than the mass of the electron in free space m , and ii) the dielectric constant is not equal to 1, as in the case of free space, but may have a value in the range 10 to 50 ⁽⁸⁾. Both of these facts contribute significantly to the change in the ratio of the magnetic energy to the Coulomb energy (denoted by γ) from the case where the atom exists in free space.

We have

$$\gamma = \frac{\hbar \omega_c}{2 R_y^*} \quad \text{a. u.}$$

where $\omega_c = eB/m^*$ is the cyclotron gyro-frequency and

$$R_y^* = \frac{m^* e^4}{2 \hbar^2 D^2}$$

is the effective Rydberg with D the dielectric constant. Now, if we suppose $D=50$ and $m^*=0.1 m$, then γ is a factor of $2.5 \cdot 10^5$ greater than for the case where $D=1$ and $m^*=m$.

In other words, if a magnetic field of strength 10^4 Gauss (a fairly weak field) was applied to the solid the effects observed could be those of a field of $2.5 \cdot 10^9$ Gauss (a strong field) in free space. In free space $D=1$, the energy corresponding to $\hbar \omega_c$ is 1 Rydberg when $\gamma=1$ i.e. $B=2.48 \cdot 10^9$ Gauss.

From now on, a weak magnetic field will be referred to as one in which the ionisation energy I_x of the target dominates the Landau energy ($\hbar \omega_c$) of the magnetic field so that the magnetic potential may be treated as perturbation and both the projectile and the target as unperturbed system.

The region of field strengths in which this occurs for atoms in their ground state is $\gamma \ll 1$ or $B \ll 10^9$ Gauss.

We will define a strong magnetic field as one in which the magnetic interaction becomes dominant ($\hbar \omega_c > I_x$). For the case of ^{the} scattering of ^a charged particle by a Coulomb potential the unperturbed system is given by the projectile embedded in the magnetic field, while the Coulomb potential is considered perturbatively. In this case $\gamma > 1$ and $B > 10^9$ Gauss.

Moreover, a magnetic field affects also the structure of the atomic target. For $\gamma \ll 1$ it causes the ordinary Zeeman level splitting, for $\gamma \approx 1$ ^{also} the quadratic Zeeman term becomes important and for $\gamma > 1$ the magnetic field completely dominates the Coulomb field ^{except in the field direction} and we move into what is known as the quasi-Landau regime where the motion of the atomic electrons is close to that of free electrons in a magnetic field and the atom takes a characteristic cigar shape along the magnetic field.

These effects on the atomic levels are described more fully by Garstang ⁽⁹⁾, and in more recent reviews ^(1,2).

For collisions in presence of electromagnetic radiation, we have a very different situation. The ratio of the radiation electric field \mathcal{E}_0 to the atomic (static Coulomb) electric field \mathcal{E}_a is

$$\eta = \frac{\mathcal{E}_0}{\mathcal{E}_a} = \left[\frac{4\pi I}{c} \right]^{1/2} \frac{a_0^2}{e},$$

where I is the intensity of the laser and a_0 is the Bohr radius, and η becomes equal to 1 for an intensity $I = 7 \cdot 10^{16} \text{ W/cm}^2$.

Before the advent of high power lasers, all laboratory radiation sources provided very weak fields ($\eta \ll 1$), and the semi-classical theory of the interaction of radiation with matter was formulated treating the electron and the atomic target as unperturbed system and the electron interaction with the radiation field as perturbation.

With the use of today's lasers it is easy to obtain magnitudes of the applied fields comparable with or greater than the mean atomic field ($\eta \geq 1$); the interaction of the charged particle with the target can be considered as a perturbation, with the projectile embedded in the laser field as the unperturbed system.

The interaction of the laser with the atomic target can be neglected if the laser frequency is off resonance with any of the unperturbed atomic levels and for not too intense radiations. In fact, since the target atom will be in the laser field for some time before the collision takes place, the probability of multiphoton ionisation (which increases with intensity) during this time may be important and then the collision cross section will depend upon the time spent by the target in the laser field before the collisional process. This fact limits the intensity of the laser beam used in the experiments.

A great deal of theoretical work has been done in collision theory in presence of such intense magnetic and/or laser fields.

For collisions in presence of strong magnetic fields, early treatments are due to Tannenwald⁽¹⁰⁾, Goldman⁽¹¹⁾ and Ventura⁽¹²⁾; more recently Onda⁽¹³⁾, Brandi et al⁽¹⁴⁾, and Ferrante et al⁽¹⁵⁾ have renewed the effort to provide a viable theoretical treatment for potential scattering in presence of a strong magnetic field. Onda has developed a nonrelativistic variable amplitude method for charged particle scattering by a screened Coulomb potential in an uniform constant magnetic field, while Brandi et al have given a derivation based on the Green's function formalism; finally, Ferrante et al have found that the optical theorem for this kind of collision process has a different form from the usual one. They found that, for collisions in the presence of a magnetic field, the total cross section for transitions into all the allowed final states is expressed through the real part of the elastic scattering amplitude. Very recently Ohsaki^(16,17) has reconsidered the problem of potential scattering going to higher order in a simplified Born series and derived a threshold law for the Landau state excitations, as well as the energy dependence of the cross section.

One of the problems in the scattering of electrons in the presence of a strong magnetic field is that in the limit $B \rightarrow 0$ it is not obvious how to recover the formulae derived for the magnetic field free case. This difficulty has its origin in the different boundary conditions implied for the two different cases. In the first case the particle motion is bound in the plane per-

pendicular to the field, in the second case the particle is free. It is then necessary to look for a particular combination of Landau states (eigenfunctions of a free particle in a constant magnetic field) which goes into the free particle wavefunction when the magnetic field goes to zero.

For this reason we consider in chapter 2, the (18) problem, first posed by McDowell of the expansion of a plane wave in Landau states. The electron states to be used in calculations of collision transition probabilities for electron scattering in presence of a magnetic field, are derived for two different experimental situations: adiabatic and sudden, switching on of the field. In chapter 3, using such states we derive the cross section for potential scattering and we show that, as required, it goes to the field free value when $B \rightarrow 0$.

The study of the joint action of a strong magnetic and a radiation field is, also, an interesting problem. Ferrante et al (19) have developed a formalism to treat the potential scattering of a charged particle in the simultaneous presence of a magnetic and a laser field in the particular case of light polarisation parallel to the direction of the magnetic field. They found that during the scattering process the particle may emit or absorb integer multiples of light quanta $\hbar\omega$ and make transitions between different Landau levels at the same time. However they treated the problem only to

First Born Approximation; (FBA) *level*.

In chapters 4 and 5 we generalise the above derivation to the case of laser light of arbitrary polarisation.

In chapter 4 we briefly outline the generalization of the solution of the Schrödinger equation ⁽¹⁹⁾ for a particle in a magnetic and a laser field of arbitrary polarisation. In chapter 5 we develop a formalism for the calculation of transition amplitudes and cross sections. The constant magnetic field is assumed to lie along the z axis, and the following polarisations are treated in detail:

- a) linear polarisation along the z axis
- b) right hand circular polarisation in the xy plane
- c) left hand circular polarisation in the xy plane
- d) linear polarisation along x
- e) right hand circular polarisation in the xz plane.

It is found that in ^{the} FBA *for* cases a) and b) with weak field and high frequency, the cross section maintains the form obtained by Kroll and Watson ⁽²⁰⁾ for collisions ^{the} in presence of a laser field alone. Moreover, an important feature appears for any polarisation when at least one component of the radiation field is in the plane of the magnetically confined motion. In this case, when the laser frequency matches the cyclotron frequency a resonant denominator switches on multiphoton processes, but in view of the work of Ohsaki ^(16,17) our results must be treated with some reserve.

Despite the great amount of theoretical work present in the literature not many experiments for collisions in strong fields have been performed.

To the best of our knowledge the only experiment reported for collisions in a magnetic field is that of

Blumberg et al⁽²¹⁾; they reported photodetachment cross sections of S^- in the presence of a magnetic field of about 15 kG. The application of a magnetic field produces structure in the cross section with a periodic dependence on the light frequency. The authors⁽²²⁾ interpret such structure as due to the excitation of the detached electron to discrete cyclotron (Landau) levels in the magnetic field. A more recent discussion together with results on SeH^- is given by Larson and Stoneman⁽²³⁾ while Clark⁽²⁴⁾ advances a different interpretation.

Also for laser assisted collision processes, only a few experimental results exist.

Weingartshofer et al^(25,26) reported measurements of multiphoton free-free transitions for electron Argon collisions in presence of a CO_2 laser.

On the other hand, theoretical studies of collisions in the presence of laser radiation has become a well established branch of the theory of collisions. See for reviews on the subject Ferrante⁽²⁷⁾ and Mittelman⁽²⁸⁾.

In our opinion more efforts must be devoted to suggest to the experimentalists particular processes where the presence of an external radiation field can give rise to some particular measurable effect. With this problem in mind, we derive in chapter 6 the triple differential cross section for ionisation of helium by electron impact in ^{the} presence of a laser radiation. We observe that, for a simple laser model and a polarisation perpendicular to the momentum transfer and for particular ejected electron angles, the single photon absorption process dominates the other ones and this itself may be interesting experimentally.

Finally, in chapter 7 we investigate the observational conditions under which the predicted splitting of the ^{for} laser assisted electron impact ionisation of atoms triple differential cross section occurs. Different laser models are considered: homogeneous single mode, inhomogeneous single mode, multimode laser. Particular attention is also paid to the variation of the cross section with the laser polarisation, and it is found that the splitting is always present when the polarisation is perpendicular to the momentum transfer.

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CHAPTER 2

THE WAVE FUNCTION OF THE ELECTRON IN A HOMOGENEOUS MAGNETIC FIELD AND THE PLANE-WAVE STATE: ADIABATIC AND SUDDEN APPROXIMATIONS

1. INTRODUCTION

Faisal ⁽¹⁾ and, independently, Ohsaki ⁽²⁾, have considered the problem, first posed by McDowell ⁽³⁾, of the expansion of a plane wave in Landau states. The situation arises from the conceptual scattering experiment illustrated in Fig. 1. A beam of electrons of fixed energy E represented in the field free region (I) by a plane wave enters a region (II) in which there is a uniform static magnetic field, \underline{B} , along the z-axis

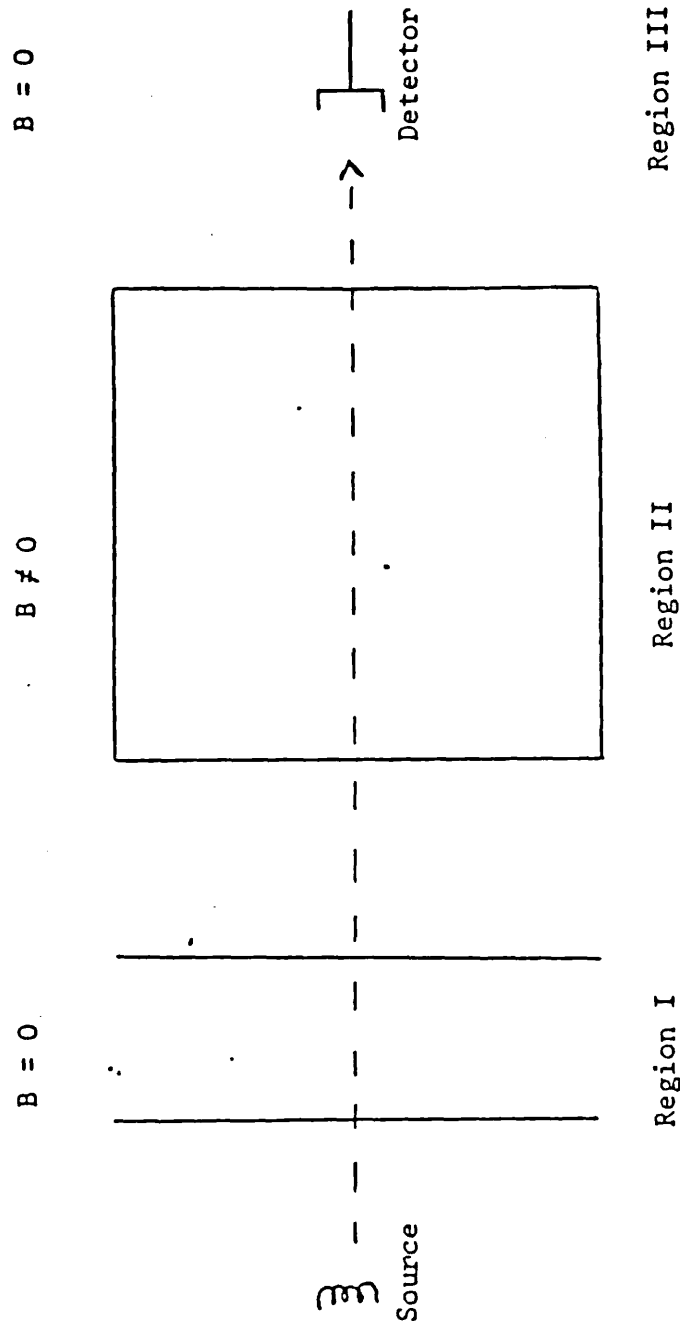
$$\underline{B} = B\hat{z} .$$

In order to calculate cross sections of reaction in region II, one needs to know the occupation numbers (or better, the occupation amplitudes) of the Landau states before any collision. Faisal and Ohsaki give apparently different results, though Ohsaki gives no details of his analysis. Ohsaki's result (eqs. 6, 8, Ohsaki ⁽²⁾) can be written in the present notation (see section 2) as

$$\chi(\underline{r}) = \sum_{m=-\infty}^{n(o)} i^{|m|} \frac{e^{ik_z z}}{\sqrt{L_z}} \left[\frac{n!}{(n+|m|)!} \right] \frac{1}{2} \frac{\phi_{nm}(o)}{(L_x L_y)^{1/2}} e^{im(\phi-\phi_K)} \quad (1)$$

where $n(o) = \frac{K_{\perp}^2}{4\gamma} - \frac{1}{2}$ and that of Faisal (1982, eq. 11) as

$$\chi(\underline{r}) = \sum_{m=-|M|}^{n(o)} i^{|m|} \frac{e^{ik_z z}}{\sqrt{L_z}} \left[\frac{n!}{(n+|m|)!} \right]^{1/2} \frac{\phi_{nm}(o)}{(L_x L_y)^{1/2}} e^{im(\phi-\phi_K)} \quad (2)$$



F I G. 1

with (integer) $|M| < \infty$. Direct substitutions of the above expansions show immediately that both (1) and (2) exactly satisfy the Schrödinger equation of the electron in a constant magnetic field with a fixed energy, provided of course, the sums over the degenerate m -states exist and satisfy the constrain

$$n = \left[\frac{K_{\perp}^2}{4\gamma} - \frac{1}{2} (|m| + m + 1) \right] \geq 0 \quad (3)$$

for all m . In the next section we reanalyse the adiabatic problem and show that the expansion coefficients identical to that of Ohsaki are obtained, provided region II has a large but finite extent $L_x L_y \sim \frac{1}{\gamma}$ in the plane perpendicular to the field. To satisfy this requirement his m -sum should be restricted to within a finite range, with a lower limit

$$m_L^- = \gamma L^2 - \frac{K_{\perp}^2}{4\gamma}$$

(see eq. (42) below), and this is in fact imposed by conservation of energy perpendicular to the field.

We then consider the completeness relation of Faisal's result and compare it with that of Ohsaki. The normalisation constant of Faisal's solution is easily found to be

$$L_x L_y = \frac{\pi S(\gamma)}{\gamma} \quad (4)$$

where

$$S(\gamma) = \sum_{m = -|M|}^{n(0)} T_{n,m} \quad (5)$$

with

$$T_{n,m} = \frac{(n(0) - \frac{m-1}{2})^{|m|} (n(0) - \frac{|m|+m}{2})!}{(n(0) + \frac{|m|-m}{2})!} \quad (6)$$

(see eq. (48) below).

It will be seen that $T_{n,m} = 1$ for m small and falls off rapidly as $(\frac{1}{2})^{|m|}$ for large $|m| > n(0)$. Thus the m -sum in Faisal's expansion is effectively and automatically confined to the same range of states. Furthermore, as $\gamma \rightarrow 0$ their expansion coefficients become identical and both the solutions reduce to the plane-wave state. The two (alternative) adiabatic expansions (1) and (2), therefore, contain essentially the same information and differ apparently due to the different choice of normalisations of the basis set (Landau states) used. The detailed analysis of the adiabatic case is presented in section 2. In section 3 we analyse the case when the magnetic field is, instead, applied suddenly and we derive an exact expansion of the plane wave in Landau states valid for any region of free-space and for any value of the magnetic field.

2. ADIABATIC CASE

In region I we suppose the beam of electrons is represented by a plane wave

$$\phi(\underline{r}) = (L)^{-3/2} e^{i\underline{k} \cdot \underline{r}} \quad (7)$$

with energy

$$E = \frac{1}{2}k^2 = \frac{1}{2}(K_z^2 + K_\perp^2) \quad (8)$$

in an obvious notation. Using cylindrical polar co-ordinates we can write (7) as ⁽⁴⁾

$$\psi(\underline{r}) = (L)^{-3/2} e^{ik_z z} \sum_{m=-\infty}^{\infty} i^m e^{im(\phi-\phi_k)} J_m(k_{\perp}\rho) \quad (9)$$

where ϕ_k is the initial azimuth.

In region II, the non-relativistic Schrodinger equation is separable (Landau and Lifshitz ⁽⁵⁾), and the Landau wave functions are

$$\psi_{n,m,k_z}(\underline{r}) = \frac{1}{\sqrt{L_z}} \left(\frac{\gamma}{\pi}\right)^{1/2} \left[\frac{n!}{(n+|m|)!} \right]^{1/2} \phi_{nm}(\rho) e^{im\phi} e^{ik_z z} \quad (10)$$

with

$$\phi_{nm}(\rho) = e^{-\sigma/2} \sigma^{|m|/2} L_n^{|m|}(\sigma) \quad (11)$$

Here we have measured the field in units of a reference field B_0 ,

$$\gamma = \frac{B}{B_0} = \frac{\hbar\omega_c}{2} \text{ a.u.}, \quad B_0 = 2.35 \times 10^9 \text{ G}, \quad (12)$$

where ω_c is the cyclotron frequency, $L_{\alpha}^{\beta}(x)$ is an Associated Laguerre Polynomial, and

$$\sigma = \gamma\rho^2. \quad (13)$$

Of course (10) corresponds to region II being the whole space. However, the Landau states are in fact confined perpendicularly to the field (see equation 33 below), having exponentially small

density (for fixed (n, m) and B) for ρ greater than some finite value. So we shall argue that we can use these solutions in a region which is not the whole space, provided n and m are appropriately restricted.

We wish to find a linear combination of Landau solutions which reduces to the plane wave (9) when the field tends adiabatically to zero, i.e. to find coefficients $C_m(n(\gamma))$ such that

$$\lim_{\gamma \rightarrow 0} \{\chi(\underline{r}) = \sum_{m=-\infty}^{\infty} C_m(n) \psi_{n,m,k_z}(\underline{r})\} = \psi(\underline{r}). \quad (14)$$

Writing (10) as

$$\psi_{n,m,k_z}(\underline{r}) = \frac{1}{\sqrt{L_z}} \left(\frac{\gamma}{\pi}\right)^{\frac{1}{2}} \left[\frac{n!}{(n+|m|)!}\right]^{\frac{1}{2}} A_{nm} P_{nm}(\rho) e^{im\phi} e^{ik_z z} \quad (15)$$

where A_{nm} is a normalization constant, then $P_{nm}(\rho)$ satisfies

$$\frac{d^2 P_{nm}}{d\rho^2} + \frac{1}{\rho} \frac{dP_{nm}}{d\rho} + \left[\alpha_{nm}^2 - \frac{m^2}{\rho^2} - \gamma^2 \rho^2 \right] P_{nm}(\rho) = 0 \quad (16)$$

with

$$\alpha_{nm}^2 = \frac{2\mu E_{\perp}}{\hbar^2} - 2\gamma m. \quad (17)$$

We restrict the motion to the interval $(0, L) \sqrt{\frac{im\rho}{\gamma}}$. Then as $\gamma \rightarrow 0$, we have, for $m \neq 0$

$$P_{nm}'' + \rho^{-1} P_{nm}' + \left[\alpha_{nm}^2 - \frac{m^2}{\rho^2} \right] P_{nm} = 0 \quad (18)$$

with

$$P_{nm}(0) = 0, \quad P_{nm}(L) = 0 \quad (19)$$

and n denotes the number of nodes in $P_{nm}(\rho)$ on $(0, L)$ and goes to infinity as $\gamma \rightarrow 0$; here

$$\alpha_{nm}^2(L) = \frac{2\mu E_{\perp}}{\hbar^2} = K_{\perp}^2 .$$

The required solutions in the limit $\gamma \rightarrow 0$ are regular Bessel functions of integer order $|m|$, and argument $K_{\perp}\rho$ except in the case $m=0$, when the paramagnetic term, $-2\gamma m$, in equation (17) can be ignored. Thus in this limit,

$$P_{nm}(\rho) = J_{|m|}(K_{\perp}\rho) \tag{20}$$

with

$$\lim_{L \rightarrow \infty} J_{|m|}(K_{\perp}L) = 0 . \tag{21}$$

If we retain the paramagnetic term, then the argument of the Bessel function is

$$q\rho = (K_{\perp}^2 - 2\gamma m)^{\frac{1}{2}}\rho = 2\gamma^{\frac{1}{2}} v^{\frac{1}{2}} \rho \tag{22}$$

where

$$v = n + \frac{1}{2}(|m| + 1). \tag{23}$$

Now we know (Erdelyi ⁽⁶⁾ Vol. 1 pp 277-281) that for arbitrary γ , from (16)

$$P_{nm}(\rho) = e^{-\sigma/2} \sigma^{|m|/2} L_n^{|m|}(\sigma) \quad (24)$$

with

$$\sigma = \gamma \rho^2 \quad (25)$$

and provided

$$\sigma = \nu^\lambda, \quad \lambda < 1/3 \quad (*)$$

then the leading term in the asymptotic expansion of (24) is

$$\frac{(n+|m|)!}{n!} \nu^{-|m|/2} J_{|m|}(q\rho), \quad (26)$$

provided $\nu \gg 1$ and $|m|$ is bounded.

We can therefore write our expansion for small γ as

$$\chi(\underline{r}) = \sum_{m=-\infty}^{\infty} C_m(n) \frac{1}{\sqrt{L_z}} \left(\frac{\gamma}{\pi}\right)^{\frac{1}{2}} \left\{ \frac{n!}{(n+|m|)!} \right\}^{\frac{1}{2}} \frac{(n+|m|)!}{n!} \nu^{-|m|/2} J_{|m|}(q\rho) e^{ik_z z} e^{im\phi} \quad (27)$$

$$= \frac{1}{\sqrt{L_z}} \left(\frac{\gamma}{\pi}\right)^{\frac{1}{2}} \sum_{m=-\infty}^{\infty} C_m(n) \left\{ \frac{(n+|m|)!}{n!} \right\}^{\frac{1}{2}} \nu^{-|m|/2} J_{|m|}(q\rho) e^{ik_z z} e^{im\phi} \quad (28)$$

Since K_{\perp}^2 is constant,

$$K_{\perp}^2 = 4\gamma(n + \frac{1}{2}(|m| + m + 1)) \text{a.u.} \quad (29)$$

then for any finite m , $n = n_0(m)$ is a constant and

$n_0 \rightarrow \infty$ as $\gamma \rightarrow 0$. Moreover

$$\lim_{n_0 \rightarrow \infty} \left[\frac{(n_0 + |m|)!}{n_0!} \right]^{\frac{1}{2}} \nu^{-|m|/2} = 1, \quad (30)$$

hence, provided (*) holds,

$$\lim_{\gamma \rightarrow 0} \chi(\underline{r}) = \frac{1}{\sqrt{L_z}} \left(\frac{\gamma}{\pi}\right)^{\frac{1}{2}} \sum_{m=-\infty}^{\infty} C_m(n_0) J_{|m|}(q_0 \rho) e^{ik_z z} e^{im\phi} \quad (31)$$

where $q\rho \rightarrow q_0\rho = K_{\perp}\rho$ as $\gamma \rightarrow 0$. Comparing (31) and (9)

$$C_m(n_0) = \frac{1}{\sqrt{L_x L_y}} \left(\frac{\pi}{\gamma}\right)^{\frac{1}{2}} i^{|m|} e^{-im\phi_k} \quad (32)$$

which is Ohsaki's result.

It remains to show that (*) holds. We note that radial distribution of Landau states is such that the mean square radius is ⁽⁷⁾

$$\langle \rho \rangle^2 = \frac{2}{\gamma} \nu \quad (33)$$

and this sets an approximate upper bound on ν ,

$$\nu_{\max} = \frac{1}{2} \gamma L^2 = \frac{1}{2} m_L. \quad (34)$$

Now for (*) to hold with K_{\perp} constant, then since when $\gamma \rightarrow 0$, ν behaves as γ^{-1} , we must have

$$L^2 \leq \gamma^{-4/3}, \quad \lambda = 1/3. \quad (35)$$

Now equation (33) says that if we fix K_{\perp} and hence L , then the states present

are those with $\nu \leq \nu_{\max}$. These all satisfy (35) if $\nu_{\max} < \nu_L = \gamma^{-1}$, but from (34) and (35), $\nu_{\max} = \frac{1}{2} \gamma^{-1/3}$, so this is true.

The plane wave expansion (9) is such that for $\rho \leq L$, terms with $m \geq M^*$ are negligible, where M^* is given by the requirement

$$\frac{J_{|m|+1}(K_{\perp}L)}{J_{|m|}(K_{\perp}L)} = \frac{e K_{\perp}L}{2m} \ll 1 \quad (36)$$

or
$$M^* = \left[\frac{eK_{\perp}L}{2} \right]. \quad (37)$$

The maximum value of m occurring in $(0,L)$ is $2v_L$ if (35) is to be satisfied. That is $m_L = 2\gamma^{-1}$. But $M^* = \frac{1}{2} eK_{\perp} \gamma^{-2/3}$, again from (35), so

$$M^* < m_L$$

if

$$K_{\perp} < \frac{4}{e\gamma^{1/3}}$$

or provided the cyclotron radius R_c satisfies

$$R_c = \frac{K_{\perp}}{2\gamma} < L. \quad (38)$$

This is of course an obvious requirement. What we have shown is that, provided (38) holds, all the states included in $(0,L)$ are sufficient to reproduce at least the leading $2M^*$ terms in (9). Moreover, since the squared moduli of the expansion coefficients

$$|C_m(n_0)|^2 = \frac{1}{L_x L_y} \frac{\pi}{\gamma} \quad (39)$$

are independent of m , the occupation numbers of the Landau states considered in our expansion are also independent of m , and thus as expected, are proportional to $1/N$ where N is the number of Landau states degenerate in energy in a unit area perpendicular to the field⁽⁵⁾

The normalisation of our expansion (14) leads to the completeness relation

$$\begin{aligned}
 S &= \sum_m |c_m(n_0)|^2 = \\
 &= \sum_m \frac{1}{L_x L_y} \frac{\pi}{\gamma} = \sum_m \frac{1}{\gamma L^2} = 1
 \end{aligned}
 \tag{40}$$

as said before, the sum in (31) and in (40) is restricted to values of the quantum number m such that the corresponding wavefunctions for a given energy are confined inside the area L^2 . The maximum positive value of m from eq (29) and eq (33) is given by

$$m_L^+ = \frac{K_L^2}{4\gamma}
 \tag{41}$$

while the minimum negative value is

$$m_L^- = \gamma L^2 - 2m_L^+
 \tag{42}$$

and the sum in (40) becomes

$$S = 1 - \frac{K_L^2}{4\gamma^2 L^2}
 \tag{43}$$

If the adiabatic condition eq (38) is verified, i.e. if

$$\frac{K_L^2}{4\gamma^2 L^2} \ll 1, \text{ then } S = 1.
 \tag{44}$$

By choosing L^2 to satisfy (35) then for fields ($\gamma^{2/3} > \frac{1}{4} K_L^2$) strong compared with the total energy, we get the completeness relation

$$S = 1.$$

We note that the size L of the box must increase faster than $1/\gamma$ as $\gamma \rightarrow 0$.

Finally we briefly discuss Faisal's expansion coefficients and the completeness relation of his expansion. We note that his expression corresponding to $C_m(n_0)$ is due to the use of a different normalisation of the basis states (Landau states) and goes over to eq.(32) in the limit $\gamma \rightarrow 0$. In fact, writing his expansion (eq.(11), Faisal⁽¹⁾) in terms of the Laguerre polynomials and taking into account the normalisation constant $L_x L_y = L^2$) the ratio R_m of each term of his expansion (see eq.(2)) to that of eqs. (1) or (14), is

$$R_m = \left[\frac{n_0!}{(n_0 + |m|)!} \right]^{1/2} v^{|m|/2} \quad (45)$$

Taking the limit for $\gamma \rightarrow 0$ of eq. (45) we have

$$\lim_{\gamma \rightarrow 0} R_m = 1 \quad (46)$$

Thus as $\gamma \rightarrow 0$, both sets of expansion coefficients become equal and the results reduce to the plane wave state, as desired.

The completeness relation for Faisal's expansion, taking into account the different normalisation of his wavefunctions is

$$S' = \sum_m \frac{1}{\gamma L^2} \frac{n!}{(n + |m|)!} v^{|m|} \quad (47)$$

Again for $\gamma \rightarrow 0$ (or $n \rightarrow \infty$) we have $S' \rightarrow S$, provided again

that $L \sim \frac{1}{\gamma^{1+\epsilon}}$, $\epsilon > 0$. For $\gamma \neq 0$ the difference between

S and S' is in the factor

$$T = \frac{n!}{(n + |m|)!} v^{|m|} \quad (48)$$

For fixed n and small values of m we have

$$T_{n,m} = 1$$

and for large $m \gg 1$

$$T_{n,m} \sim \frac{1}{2^m} .$$

so the factor T gives automatically a cut-off in the sum necessary to get convergence in the completeness relation. But, in any case, using the Tricomi expansion with the leading term only, required us to impose an upper bound on $|m|$, so there is little difference in the approach between the two expansions. The actual difference arises from the fact that in taking Faisal's approach the limit $\gamma \rightarrow 0$ of eq (30) is not taken. In consequence, Faisal ⁽¹⁾ requires a stronger sufficiency condition on the way the field vanishes at the boundary; that is in the present chapter we allow the field to exist with a constant value throughout the box, while in the earlier paper by Faisal the field was required to behave as $\rho^{-(1+\epsilon)}$ at large ρ . Both sets of coefficients are closely equal for small ρ (i.e. small $|m|$) and for all ρ when $\gamma \rightarrow 0$.

3. SUDDEN APPROXIMATION

In this section we consider a very different situation from that treated in section 2. We consider the case when the field (initially $B = 0$) is suddenly switched to a finite value $B \neq 0$. Then the electron undergoes a rapid "sudden" change in the Hamiltonian operator. The plane wave becomes a linear combination of Landau states. Of course, in this case it will not be possible to return to a plane wave if the field is then adiabatically reduced to zero.

The appropriate expansion is, again,

$$\phi(\underline{r}) = (L)^{-3/2} \exp(i \underline{k} \cdot \underline{r}) = \sum_{n,m,k_z} C'_{n,m} \psi_{n,m,k_z}(\underline{r}) \quad (49)$$

where $\psi_{n,m,k_z}(\underline{r})$ are the Landau wavefunctions eq (10). Owing to the orthonormality of the Landau wavefunctions we have

$$C'_{n,m} = \int \psi_{n,m,k_z}^*(\underline{r}) \phi(\underline{r}) d\underline{r} . \quad (50)$$

Substituting eq (10) and eq (9) in eq. (50) and performing the integration we get

$$C'_{n,m} = i^{|m|} \frac{(-1)^n}{\pi^{3/2}} \left[\frac{n!}{(n+|m|)!} \right]^{1/2} \gamma^{-1/2} (|m|+1) \frac{2\pi}{\sqrt{L_x L_y}} \\ k_{\perp}^{|m|}(o) \exp\left\{-\frac{k_{\perp}^2(o)}{2\gamma}\right\} L_n^{|m|} \left(\frac{k_{\perp}^2(o)}{\gamma} \right) \exp\{-i\pi\phi_k\} \quad (51)$$

where $k^2 = k_z^2(o) + k_{\perp}^2(o)$, and in (49) $k_z = k_z(o)$.

It is of interest to note that these coefficients have the form of Landau wavefunctions with the spatial variable $\gamma\rho^2$ replaced by the constant $k_{\perp}^2(o)/\gamma$. They are in fact the Fourier Transforms of the corresponding Landau functions.

As we said, the Landau wavefunctions are localised in the plane perpendicular to the magnetic field around the value of the mean square radius ^{given by} eq. 33, and almost totally confined to the classically accessible region, i.e.

$$\gamma\rho_{\min}^2 < \gamma\rho^2 < \gamma\rho_{\max}^2 \quad (52)$$

with

$$\gamma\rho_{\min}^2 = \left[(n+|m|+\frac{1}{2})^2 - (n+\frac{1}{2})^2 \right] \quad (53)$$

and

$$\gamma \rho_{\max}^2 = \left[(n + |m| + \frac{1}{2})^2 + (n + \frac{1}{2})^2 \right]^2, \quad (54)$$

with, of course, $\gamma \langle \rho \rangle = [\gamma \rho_{\min}^2 + \gamma \rho_{\max}^2]/2$.

Similarly it follows from (51) that when the magnetic field is switched on the significantly occupied states are those with quantum numbers n or m satisfying the relation

$$\frac{k_{\perp}^2(o)}{\gamma} = 2n + |m| + 1 \quad (55)$$

that is, the occupancy of a degenerate n^{th} Landau level decreases rapidly with increasing n up to $n_{\max} = k_{\perp}^2(o)/2\gamma$.

4. CONCLUSION

The adiabatic expansions considered in sections 1 and 2 and the sudden-switching expansion in section 3, give alternative descriptions of the electron states involved (depending on their mode of preparation) in the calculations of collision transition probabilities for electron scattering in presence of ^{the} a magnetic field. The physical difference between the adiabatic expansion and the sudden-switching expansion is that the former consists only of the degenerate Landau-states. The sudden-switching expansion derived in section 3 is an exact expansion of the plane wave and, instead, contains all the Landau eigenstates including those not degenerate in energy with the field-free plane wave. However, it gives the correct result when the field is suddenly switched on. If the field then later goes adiabatically to zero, it does not go over to a plane wave. Moreover the two expansions correspond to different experimental situations.

The sudden description in section 3 is valid if we assume that the magnetic field is switched on in a time t much shorter than the period of oscillation of the electron in the field $T = 2\pi/\omega_c$, where ω_c is the cyclotron frequency.

Now we consider the region of validity of the two models. Suppose that the magnetic field changes smoothly from zero to a given value $B (\neq 0)$ in a spatial region of dimension x . In this case in order that the sudden approximation be valid the relation

$$x \ll \frac{\pi k}{\gamma} \quad (\text{a.u}) \quad (56)$$

must hold. That is the contrary of the adiabatic condition (38).

Since x is in general, for any experiment, a macroscopic quantity, condition (56), it is readily achieved at weak laboratory fields [see (12)above]. However, in this picture the "sudden" approximation is not valid except for very fast electrons at the highest laboratory fields in current use (~ 20 KG). The correct expansion is then an adiabatic one given in sections 1 and 2.

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CHAPTER 3

CROSS SECTION FOR POTENTIAL SCATTERING IN PRESENCE OF A MAGNETIC FIELD AND LIMIT FOR $B \rightarrow 0$.

In this chapter we derive the cross section for potential scattering in presence of a strong magnetic field utilising the scattering electron wavefunctions obtained in the previous chapter in the adiabatic expansion.

As shown in chapter 2 the linear combination of Landau states

$$\chi(\underline{z}) = \sum_{m=-\infty}^{\infty} C_m(m) \psi_{m, m, k_z}(\underline{z}) \quad (1)$$

goes over ^{into} the plane wave state as $\gamma \rightarrow 0$. Consequently a cross section derived with the use of wavefunctions (1) must go over ^{into} the field free Rutherford formula when the magnetic field goes to zero adiabatically.

In eq. (1) $\psi_{m, m, k_z}(\underline{z})$ are the Landau wavefunctions and the expansion coefficients are given by

$$C_m(m) = \frac{1}{\sqrt{L_x L_y}} \left(\frac{\pi}{\gamma}\right)^{1/2} i^{|m|} \exp(-i m \varphi_k) \quad (2)$$

where φ_k is the azimuth angle of the vector \underline{k} .

In the S-matrix approach the cross section is given by the probability transition per unit time \tilde{P}_{fi} , divided by the incident current density J_{inc} and summed over all the final states and the degenerate initial states

$$\sigma_{TOT} = \sum_{m_f, m_f, k_{z_f}} \sum_{m_i} \frac{\tilde{P}_{fi}}{J_{inc}} \quad (3)$$

where

$$\tilde{P}_{fi} = \frac{2\pi}{\hbar} |\langle \chi_f | T | \chi_i \rangle|^2 \delta(\epsilon_f - \epsilon_i) \quad (4)$$

The sum in eq. (3) is taken over all the final quantum numbers corresponding to all the Landau levels with a final energy given by

$$\epsilon = K_z^2 + K_\perp^2 = K_z^2 + 4\gamma \left[m + \frac{1}{2} (|m| + m + 1) \right] R_y \quad (5)$$

Following usual procedures, the matrix element for a transition between the initial state $\chi_i(\underline{z})$ and the final state $\chi_f(\underline{z})$ caused by the potential $V(\underline{z})$ is given by

$$\begin{aligned} \langle \chi_f | T | \chi_i \rangle &= \sum_{m_i, m_f} C_{m_i}(m_i) C_{m_f}^*(m_f) \langle \psi_{m_f, m_f, K_{z_f}}(\underline{z}) | T | \psi_{m_i, m_i, K_{z_i}}(\underline{z}) \rangle = \\ &= \sum_{m_i, m_f} \frac{1}{L_x L_y} \frac{\pi}{\gamma} i^{|m_i|} i^{-|m_f|} \exp\{-i m_i \varphi_{k_i}\} \times \\ &\times \exp\{i m_f \varphi_{k_f}\} \langle \psi_{m_f, m_f, K_{z_f}}(\underline{z}) | T | \psi_{m_i, m_i, K_{z_i}}(\underline{z}) \rangle \quad (6) \end{aligned}$$

For a potential with axial symmetry the magnetic quantum number is conserved during the collision, then

$$m_i = m_f = m$$

and eq. (6) becomes

$$\begin{aligned} \langle \chi_f | T | \chi_i \rangle &= \sum_m \frac{1}{L_x L_y} \frac{\pi}{\gamma} \exp\{i m [\varphi_{k_f} - \varphi_{k_i}]\} \times \\ &\times \langle \psi_{m, m, K_{z_f}}(\underline{z}) | T | \psi_{m, m, K_{z_i}}(\underline{z}) \rangle \quad (7) \end{aligned}$$

Putting $\phi = \varphi_{k_f} - \varphi_{k_i}$, and taking the squared modulus of the matrix element, we obtain

$$|\langle \chi_f | T | \chi_i \rangle|^2 = \sum_{m m'} \frac{1}{L_x^2 L_y^2} \frac{\pi^2}{\gamma^2} \exp \{ i(m - m') \phi \} \times$$

$$\times \langle \Psi_{m_f m' k_{z_f}}^{(z)} | T | \Psi_{m_i m' k_{z_i}}^{(z)} \rangle \langle \Psi_{m_f m k_{z_f}}^{(z)} | T | \Psi_{m_i m k_{z_i}}^{(z)} \rangle \quad (8)$$

Due to the uniform distribution of the scattering centers and to the axial symmetry of the magnetic field it is possible to average the squared matrix element over the azimuthal angle ϕ obtaining (Cf. Ohsaki⁽¹⁾)

$$|\langle \chi_f | T | \chi_i \rangle|^2 = \frac{1}{2\pi} \int_0^{2\pi} |\langle \chi_f | T | \chi_i \rangle|^2 d\phi =$$

$$= \sum_{m m'} \frac{1}{L_x^2 L_y^2} \frac{\pi^2}{\gamma^2} \langle \Psi_{m_f m' k_{z_f}}^{(z)} | T | \Psi_{m_i m' k_{z_i}}^{(z)} \rangle \langle \Psi_{m_f m k_{z_f}}^{(z)} | T | \Psi_{m_i m k_{z_i}}^{(z)} \rangle \delta_{m, m'}$$

$$= \frac{1}{L_x^2 L_y^2} \frac{\pi^2}{\gamma^2} \sum_m |\langle \Psi_{m_f m k_{z_f}}^{(z)} | T | \Psi_{m_i m k_{z_i}}^{(z)} \rangle|^2 \quad (9)$$

To get the cross section we need the current density of the incident particles along z:

$$J_{inc} = \frac{i\hbar}{2\mu} \left(\chi_i \frac{\partial}{\partial z} \chi_i^* - \chi_i^* \frac{\partial}{\partial z} \chi_i \right) =$$

$$= \frac{\hbar}{\mu} k_{z_i} \sum_{m m'} C_m^{(m)} C_{m'}^{*(m)} \Psi_{m_i m k_{z_i}} \Psi_{m_i m' k_{z_i}}^* \quad (10)$$

Averaging over the azimuthal angle

$$J_{inc} = \frac{1}{2\pi} \int_0^{2\pi} J_{inc} d\varphi_{k_z} =$$

$$= \frac{\pi}{\gamma} \frac{1}{L_x L_y} \frac{\hbar k_{z_i}}{\mu} \sum_m |\Psi_{m_i m k_{z_i}}^{(z)}|^2 \quad (11)$$

and using (Ventura ⁽²⁾)

$$\sum_m |\Psi_{m m k_z}^{(2)}|^2 = \frac{1}{L_z} \frac{\delta}{\pi}$$

we obtain

$$J_{inc} = \frac{1}{L^3} \frac{\hbar k_{zi}}{\mu} \quad (12)$$

This expression is equal to the current density for a free particle normalised in a box of dimensions

$$L^3 = L_x L_y L_z.$$

Substituting eq. (12) and eq. (9) into eq. (3)

we get the cross section

$$\begin{aligned} \sigma_{TOT} = \sum_{m_f k_{zf}} \sum_m \frac{2\pi}{\hbar} \frac{1}{L_x L_y} \frac{\pi^2}{\delta^2} \frac{L_z \mu}{\hbar k_{zi}} \times \\ \times \left| \langle \Psi_{m_f m k_{zf}}^{(2)} | T | \Psi_{m_i m k_{zi}}^{(2)} \rangle \right|^2 \delta(\epsilon_f - \epsilon_i) \quad (13) \end{aligned}$$

Introducing the density of final states along z, we transform the sum over k_{zf} to an integral

$$\sum_{k_{zf}} \rightarrow \int dk_{zf} = \int \rho(\epsilon_f) d\epsilon_f \quad (14)$$

where

$$\rho(\epsilon_f) = \left[\frac{d(\epsilon_f)}{dk_{zf}} \right]^{-1} = \frac{L_z \mu}{2\pi \hbar^2 k_{zf}} \quad (15)$$

Finally, substituting eqs. (14) and (15) into eq.

(13) we get for the cross section the expression

$$\sigma_{TOT} = \sum_{m_f m} \frac{L_z^2}{L_x L_y} \left(\frac{\pi}{\delta} \right)^2 \frac{\mu^2}{\hbar^4 k_{zi} k_{zf}} \left| \langle \Psi_{m_f m k_{zf}}^{(2)} | T | \Psi_{m_i m k_{zi}}^{(2)} \rangle \right|^2 \quad (16)$$

We will now prove that this cross section goes over ^{into} the field free value when the magnetic field goes slowly to zero.

From eq (5) we see that for a fixed energy K_{\perp}^2 the limit for $\gamma \rightarrow 0$ implies $m \rightarrow \infty$ for any bounded value of m . Consequently the sum over n can be transformed into an integral

$$\sum_{m_{\perp}} \rightarrow \int \rho_{m_{\perp}}(K_{\perp}) dK_{\perp}$$

where $\rho_{m_{\perp}}(K_{\perp})$ is the density of states in the plane perpendicular to the magnetic field

$$\rho_{m_{\perp}}(K_{\perp}) = \frac{L_x L_y}{(2\pi)^2} K_{\perp}$$

and

$$\begin{aligned} \sum_{m_{\perp}} &\rightarrow \frac{L_x L_y}{(2\pi)^2} \int K_{\perp} dK_{\perp} d\varphi = \\ &= \frac{L_x L_y}{(2\pi)^2} \int K^2 \cos \theta d\Omega \end{aligned} \quad (17)$$

where we expressed K_{\perp} as

$$K_{\perp} = K \sin \theta \quad (18)$$

being the scattering angle.

Substituting eq. (17) into eq. (16), the cross section for $\gamma \rightarrow 0$ is given as

$$\sigma_{TOT} = \lim_{\gamma \rightarrow 0} \left(\frac{\pi}{\delta} \right)^2 \frac{1}{(2\pi)^2} \frac{\mu^2 L_z^2}{\hbar^4} \int d\Omega \left| \langle \psi_{m_{\perp} m_{K_{\perp}}}^{(2)} | \Pi | \psi_{m_i m_{K_{2i}}}^{(2)} \rangle \right|^2 \quad (19)$$

As the field free Rutherford differential cross section for Coulomb scattering with $V(\underline{z}) = A_0 / \underline{z}$ is

$$\frac{d\sigma}{d\Omega} = \left\{ \frac{2 A_0 \mu}{4 p^2 \sin^2 \theta/2} \right\}^2 \quad (20)$$

to prove that eq. (16) goes into eq. (20) when the magnetic field goes to zero we must prove that

$$\begin{aligned} \lim_{\delta \rightarrow 0} \sum_m \left| \langle \psi_{m_f, m, k_{z_f}}(\underline{z}) | V | \psi_{m_i, m, k_{z_i}}(\underline{z}) \rangle \right|^2 &= \\ = \frac{1}{L_z^2} A_0^2 \frac{\delta^2}{k^4 \sin^4 \theta/2} &\quad (21) \end{aligned}$$

The matrix element for transitions from a particular initial Landau state m_i, m, k_{z_i} , to a final Landau state m_f, m, k_{z_f} induced by a Coulomb potential $V(\underline{z})$ in the FBA is ⁽³⁾

$$\begin{aligned} \langle \psi_{m_f, m, k_{z_f}}(\underline{z}) | V | \psi_{m_i, m, k_{z_i}}(\underline{z}) \rangle &= \frac{A_0}{L_z} \left[\frac{(m_i + |m|)! (m_f + |m|)!}{m_i! m_f!} \right]^{1/2} \\ \psi(m_i + |m| + 1, m_i - m_f + 1, x) L_{m_i}^{m_f - m_i}(-x) &\quad (22) \end{aligned}$$

where $\psi(a, b, c)$ are confluent hypergeometric functions, $L_n^p(x)$ are Laguerre polynomials and $x = q_{fi}^2 / 4 \delta$ with $q_{fi} = k_{z_f} - k_{z_i}$, the transferred momentum along z ; (the quantum numbers n and m used in our formulation are related to the quantum numbers N and s used by Ferrante et al ⁽³⁾ by the relations $m = N - s$ and $n = N - (|m| + m) / 2$).

Considering a well collimated incident beam we can assume $K_{1z} = 0$ and therefore $m_i = 0$, $m = 0$.

In this case

$$\langle \psi_{m_f, 0, K_{2f}} | v | \psi_{0, 0, K_{2i}} \rangle = \frac{A_0}{L_z} \psi(1, 1 - m_f, x) \quad (23)$$

The hypergeometric function $\psi(1, 1 - m_f, x)$ is related to the incomplete gamma function $\Gamma(-m_f, x)$ through the relation

$$\psi(1, 1 - m_f, x) = x^{m_f} e^x \Gamma(-m_f, x) \quad (24)$$

(Erdelyi et al ⁽⁴⁾, vol 2 pag. 133).

Using the expansion

$$\Gamma(-m_f, x) = \frac{e^{-x} x^{-m_f}}{x + m_f + 1} \left[1 - \frac{m_f + 1}{(x + m_f + 1)^2} + \dots \right] \quad (25)$$

(Erdelyi et al ⁽⁴⁾, vol 2 pag. 140), we have

$$\begin{aligned} \lim_{\gamma \rightarrow 0} \psi(1, 1 - m_f, x) &= \lim_{m_f \rightarrow \infty} \psi(1, 1 - m_f, x) = \\ &= \lim_{m_f \rightarrow \infty} \frac{1}{x + m_f + 1} \left[1 - \frac{m_f + 1}{[x + m_f + 1]^2} + \dots \right] \end{aligned} \quad (26)$$

Neglecting the terms of order m_f^{-2} and putting

$$x = \frac{q_{fi}^2}{4\gamma} = Q m_f \quad (27)$$

with

$$Q = \frac{q_{fi}^2}{K_{1f}^2}$$

we get

$$\lim_{m_f \rightarrow \infty} \psi(1, 1-m_f, x) = \frac{4\gamma}{q_{fi}^2 + k_{\perp f}^2} \quad (28)$$

Substituting eq (28) into eq (23) and using the relation

$$q_{fi}^2 + k_{\perp f}^2 = (k_{zf} - k)^2 + k^2 \sin^2 \theta = 4k^2 \sin^2 \theta / 2 \quad (29)$$

we finally obtain the equality:

$$\lim_{\gamma \rightarrow 0} |\langle \psi_{m_f 0 k_{zf}}^{(2)} | T | \psi_{0 0 k_{zi}}^{(2)} \rangle|^2 = \frac{A_0^2}{L_z^2} \frac{\gamma^2}{k^4 \sin^4 \theta / 2} \quad (21)$$

Eq. (21) proves that the cross section for potential scattering in presence of a magnetic field goes over the field free one provided that we use as unperturbed wavefunctions the linear combination of Landau states derived in chapter 2.

We note that as the second term in the expansion of the hypergeometric function (eq. 26) is

$$-\frac{(m_f + 1)}{(x + m_f + 1)^2} = -\frac{1}{m_f (Q + 1)^2} = -\frac{4\gamma k_{\perp f}^2}{[q_{fi}^2 + k_{\perp f}^2]^2}$$

the leading correction to the cross section is proportional to γ ^{agreement} in $\sqrt{\gamma}$ with Bivona and Ferrante⁽⁵⁾.

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CHAPTER 4

WAVEFUNCTIONS OF A FREE PARTICLE IN PRESENCE OF A MAGNETIC FIELD AND A LASER FIELD OF ARBITRARY POLARISATION.

In this chapter we derive the solution of the Schrödinger equation for a free particle in presence of a magnetic field and a monochromatic radiation ("laser"). These solutions will be used in chapter 5 to evaluate transition probabilities and cross sections for electron scattering in presence of a strong magnetic field and a laser of arbitrary polarisation.

We assume that (a) the magnetic field is constant, uniform and directed along the z axis, (b) the radiation field is taken in the dipole approximation (c) with arbitrary polarisation, (d) homogeneity.

The Schrödinger equation for a particle of charge Q and mass μ in presence of an external field is given by

$$\frac{1}{2\mu} \left[\underline{p} - \frac{Q}{c} \underline{A}(\underline{r}, t) \right]^2 \psi(\underline{r}, t) = i\hbar \dot{\psi}(\underline{r}, t) \quad (1)$$

where

$$\underline{A}(\underline{r}, t) = A_L(t) + A_M(\underline{r}) \quad (2)$$

$A_L(t)$ is the vector potential of the radiation field in the dipole approximation, and $A_M(\underline{r})$ is the vector potential of the constant magnetic field; the electric field $\underline{E}(t)$ and the magnetic field \underline{B} are obtained as:

$$\underline{E}(t) = -\frac{1}{c} \frac{\partial A_L(t)}{\partial t}; \quad \underline{B} = \underline{\nabla} \times A_M \quad (3)$$

Since $\underline{p} \cdot \underline{A} - \underline{A} \cdot \underline{p} = -i\hbar \nabla \cdot \underline{A}$ putting $\nabla \cdot \underline{A} = 0$, equation (1) becomes

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + i \frac{Q\hbar}{\mu c} \underline{A}_M \cdot \nabla + \frac{Q^2}{2\mu c^2} A_M^2 + \frac{Q}{\mu c} \underline{A}_M \cdot \underline{A}_L + i \frac{Q\hbar}{\mu c} \underline{A}_L \cdot \nabla + \frac{Q^2}{2\mu c^2} A_L^2 \right] = i\hbar \dot{\psi}(\underline{r}, t) \quad (4)$$

Introducing the unitary transformation (Cohen-Tannoudji et al ⁽¹⁾)

$$T = \exp \left[-\frac{iQ}{\hbar c} \underline{A}_L \cdot \underline{r} \right] \quad (5)$$

we get rid of the terms $\frac{Q}{\mu c} \underline{A}_M \cdot \underline{A}_L$ and $\frac{Q^2}{2\mu c^2} A_L^2$ in eq. (4) and the transformed wavefunction

$$\bar{\psi}(\underline{r}, t) = T \psi(\underline{r}, t) \quad (6)$$

satisfies the new Schrödinger equation

$$H \bar{\psi} = i\hbar \dot{\bar{\psi}} \quad (7)$$

where

$$H = -\frac{\hbar^2}{2\mu} \nabla^2 + i \frac{Q\hbar}{\mu c} \underline{A}_M \cdot \nabla + \frac{Q}{2\mu c^2} A_M^2 - Q \underline{E}(t) \cdot \underline{r} \quad (8)$$

At this point it is convenient to specify the geometry of the problem. Putting

$$\begin{cases} \underline{A}_M = (-By, 0, 0) \\ \underline{E}(t) = [\underline{E}_x(t), \underline{E}_y(t), \underline{E}_z(t)] \end{cases} \quad (9)$$

we get a configuration where the magnetic field is along the z axis and the electric field has an arbitrary polarisation. In such a geometry the Schrödinger equation

(7) becomes

$$\left[\frac{1}{2\mu} \left(\frac{\hbar}{i} \nabla_x + \frac{Q}{c} B y \right)^2 - \frac{\hbar^2}{2\mu} (\nabla_y^2 + \nabla_z^2) - Q \underline{\underline{\epsilon}} \cdot \underline{\underline{r}} \right] \bar{\Psi} = i \hbar \dot{\bar{\Psi}} \quad (10)$$

Making the following transformations:

a) the coordinates transformation

$$y_1 = y - Y(t), \quad z_1 = z - Z(t) \quad (11)$$

b) the unitary transformation

$$\bar{\bar{\Psi}} = \exp \left[\frac{iQ}{\hbar c} Y B x \right] \bar{\Psi} \quad (12)$$

c) one more coordinate transformation

$$x_1 = x - X(t) \quad (13)$$

d) and finally the transformation

$$\Phi(\underline{\underline{r}}_1, t) = \exp \left\{ - \frac{i\mu}{\hbar} [\underline{\underline{r}}_1 \cdot \underline{\underline{\dot{R}}}] \right\} \bar{\bar{\Psi}} \quad (14)$$

with

$$\underline{\underline{r}}_1 = (x_1, y_1, z_1), \quad R(t) = [X(t), Y(t), Z(t)]$$

Eq (10) becomes

$$\left\{ - \frac{\hbar^2}{2\mu} \nabla_{x_1}^2 - i \frac{\hbar Q B}{\mu c} y_1 \nabla_{x_1} + \frac{1}{2} \frac{Q^2 B^2}{\mu c^2} y_1^2 - \frac{\hbar^2}{2\mu} (\nabla_{y_1}^2 + \nabla_{z_1}^2) + \right. \\ \left. - \frac{\mu}{2} \dot{R}^2 - Q \underline{\underline{\epsilon}} \cdot \underline{\underline{R}} - \frac{QB}{c} \dot{Y} X + x_1 \left[\mu \ddot{X} - Q \underline{\underline{\epsilon}}_x - \frac{QB}{c} \dot{Y} \right] \right\} \Phi = i \hbar \dot{\Phi}$$

$$\begin{aligned}
 & + y_1 \left[\mu \ddot{y} - Q \varepsilon_y + \frac{QB}{c} \dot{x} \right] + z_1 \left[\mu \ddot{z} - Q \varepsilon_z \right] \} \Phi(r, t) = \\
 & = i \hbar \dot{\Phi}(r, t)
 \end{aligned} \tag{15}$$

In eq. (15) we note that the term

$$L = \frac{\mu}{2} \dot{R}^2 + Q \underline{\varepsilon} \cdot R + \frac{QB}{c} \dot{y} x$$

gives the Lagrangian function for the classical system. Moreover the terms enclosed in squared brackets are equal to zero, because they represent the components of the classical equations of motion for a charged particle in presence of a magnetic and an electric field, given by

$$\begin{cases} \mu \ddot{x} - Q \varepsilon_x - \frac{QB}{c} \dot{y} = 0 \\ \mu \ddot{y} - Q \varepsilon_y + \frac{QB}{c} \dot{x} = 0 \\ \mu \ddot{z} - Q \varepsilon_z = 0 \end{cases} \tag{16}$$

The Lagrangian term in eq. (15) is removed by the transformation

$$\Phi = \exp \left[\frac{i}{\hbar} \int^t L dz \right] \chi \tag{17}$$

that gives rise to the new equation for χ

$$\left[-\frac{\hbar^2}{2\mu} \nabla_{r_1}^2 - \frac{i\hbar QB}{\mu c} y_1 \nabla_{x_1} + \frac{1}{2} \frac{Q^2 B^2}{\mu c^2} y_1 \right] \chi = i \hbar \dot{\chi} \tag{18}$$

Since the Hamiltonian of eq. (18) does not contain, explicitly, the coordinates x_1 and z_1 , its solution will be

$$\chi = \exp \left[\frac{i}{\hbar} (p_{x_1} x_1 + p_{z_1} z_1) \right] \eta(y_1, t) \tag{19}$$

where $\eta(y_1, t)$ is the solution of the equation

$$\left\{ -\frac{\hbar}{2\mu} \frac{\partial^2}{\partial y_1^2} + \frac{1}{2} \mu \omega_c^2 (y_1 - y_0)^2 + \frac{P_{z_1}^2}{2\mu} \right\} \eta = i\hbar \dot{\eta} \quad (20)$$

with $\omega_c = \frac{|Q|B}{\mu c}$ the cyclotron frequency and $y_0 = -\frac{P_{x_1} c}{QB}$.

Eq (20) is the Schrödinger equation for an harmonic oscillator oscillating along the y axis around the point y_0 with frequency ω_c . The solution of eq. (20) is given by ⁽²⁾

$$\eta(y_1, t) = C_m \exp\left[-\frac{i}{\hbar} E t\right] \exp\left[-\frac{\xi^2}{2}\right] H_m(\xi) \quad (21)$$

where

$$\xi = \left(\frac{\mu \omega_c}{\hbar}\right)^{1/2} (y_1 - y_0) \quad (22a)$$

and

$$E = \left(m + \frac{1}{2}\right) \hbar \omega_c + \frac{P_{z_1}^2}{2\mu} = E_m + \frac{P_{z_1}^2}{2\mu} \quad (22b)$$

$n=0,1,2,\dots$ is the quantum number characterising the n-th Landau level, $H_m(\xi)$ are Hermite polynomials and C_m is the normalisation constant.

We note that in eq. (20) the charge of the particle is present through y_0 so that the solutions for a positive and for a negative charged particle are different.

To obtain the solution of the original Schrödinger equation (equation (1)) we must go back from the wavefunction $\eta(y_1, t)$ to the wavefunction $\psi(\underline{r}, t)$. Using the unitary transformations given by the equations (19), (17), (14), (12) and (5), we get

$$\begin{aligned} \psi = & \tilde{C}_m \exp\left[-\frac{i}{\hbar} E_m t\right] \times \\ & \times \exp\left[\frac{i}{\hbar} f(\underline{p}, \underline{r})\right] \exp\left[-\frac{\xi^2}{2}\right] H_m(\xi) \end{aligned} \quad (23)$$

where

$$f(\underline{p}, \underline{z}) = \frac{Q}{c} \underline{A}_L \cdot \underline{z} - \frac{Q\beta}{c} Y \times + \\ + \mu \underline{z}_1 \cdot \dot{\underline{R}} + \int^t L d\tau + (P_{x_1} x_1 + P_{z_1} z_1) \quad (23a)$$

and

$$\tilde{C}_m = C \times C_m \quad (23b)$$

is the normalisation constant of the total wave function.

The quantities $X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$ in eq (23a) are the solutions of the equations of motion (eqs. 16) and can be easily found; in fact, adding the first equation of the system (16) to the second multiplied by i , and putting

$$\rho = \dot{X} + i \dot{Y}, \quad \dot{\rho} = \ddot{X} + i \ddot{Y}$$

we get

$$\dot{\rho} + i \frac{Q\beta}{c} \rho = -\frac{Q}{\mu c} (A_{Lx} + i \dot{A}_{Ly})$$

whose solution is

$$\rho = \dot{X} + i \dot{Y} = \\ = \frac{1}{\mu} \left[\pi_x - \frac{Q}{c} A_{Lx} \right] + \frac{i}{\mu} \left[\pi_y - \frac{Q}{c} A_{Ly} \right] \quad (24)$$

with

$$\pi_x + i \pi_y = -\frac{Q^2}{\mu c^2} B \int^t [A_{Ly} - i A_{Lx}] \times \\ \times \exp \left[-i \frac{Q\beta}{\mu c} (t - \tau) \right] d\tau \quad (25)$$

Moreover from eq. (24) we get the relations

$$\dot{\pi}_x = \mu \ddot{X} + \frac{Q}{c} \dot{A}_{Lx}, \quad \dot{\pi}_y = \mu \ddot{Y} + \frac{Q}{c} \dot{A}_{Ly}$$

that with the help of the first two equations of (16) give

$$\begin{cases} \dot{\pi}_x = \frac{QB}{c} \dot{y} & , & \dot{\pi}_y = -\frac{QB}{c} \dot{x} \\ X = -\frac{c}{QB} \pi_y & & Y = \frac{c}{QB} \pi_x \end{cases} \quad (26)$$

and also

$$X \pi_x = Y \pi_y$$

Finally integrating the last of the equations (16) and putting

$$\underline{P} = [P_x, \pi_y, P_z] \quad (27a)$$

and

$$\mathcal{P}^2 = \left| \underline{P} - \frac{Q}{c} A_L(t) \right|^2 - [P_x - \pi_x]^2 \quad (27b)$$

we get for the function $f(\underline{P}, \underline{z})$ of equation (23a) the more compact expression

$$f(\underline{P}, \underline{z}) = \underline{P} \cdot \underline{z} - \frac{1}{2\mu} \int^t \mathcal{P}^2 dz \quad (28)$$

The wavefunctions given in eqs. (23) - (28) are the appropriate ones for calculations of matrix elements and cross sections for scattering in ^{the} presence of two external fields, when the electron embedded in the magnetic and electric field is considered as ^{an} unperturbed system and the Coulomb potential is considered as ^a perturbation.

In order to clarify the physical meaning of the derived wavefunctions, it is useful to calculate the mean values of some physical quantities:

a) normalisation constant

The orthonormalisation condition gives

$$\begin{aligned} \langle \psi_a | \psi_b \rangle &= \int dx dy dz \psi_a^* \psi_b = \left(\frac{\mu \omega_c}{\hbar} \right)^{1/2} \int dx d\xi dz \psi_a^* \psi_b = \\ &= \tilde{C}_{ab}^2 \left(\frac{\mu \omega_c}{\hbar} \right)^{-1/2} (2\pi\hbar)^2 \pi^{1/2} (m_a! 2^{m_a})^{1/4} (m_b! 2^{m_b})^{1/4} \delta(p_{ax} - p_{bx}) \delta(p_{az} - p_{bz}) \delta_{m_a, m_b} \end{aligned}$$

so that when $m_a = m_b = m$ we have

$$\tilde{C}_m^2 = C \times C_m = \left(\frac{\mu \omega_c}{\hbar \pi} \right)^{1/4} \frac{1}{2\pi\hbar} [m! 2^m]^{-1/2} \quad (29)$$

b) energy

$$\begin{aligned} \langle E \rangle &= \langle \psi | i\hbar \frac{\partial}{\partial t} | \psi \rangle = E_m + \frac{1}{2\mu} \left[\left(\pi_x - \frac{Q}{c} A_{Lx} \right)^2 + \right. \\ &\quad \left. + \left(\pi_y - \frac{Q}{c} A_{Ly} \right)^2 + \left(p_z - \frac{Q}{c} A_{Lz} \right)^2 \right] \end{aligned} \quad (30)$$

the energy of the particle is then given by the energy of an harmonic oscillator plus a time dependent part due to the forcing caused by the laser field;

c) momentum

$$\begin{aligned} \langle \underline{p} \rangle &= \langle \psi | -i\hbar \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) | \psi \rangle = \\ &= p_x \hat{i} + \pi_y \hat{j} + p_z \hat{k} = \underline{p} \end{aligned} \quad (31)$$

the component of the momentum along y is not conserved;

d) coordinates

$$\begin{aligned} \langle \underline{r} \rangle &= \langle \psi | \hat{i} x + \hat{j} y + \hat{k} z | \psi \rangle = \\ &= \hat{i} x + \hat{j} \left(y_0 + \frac{c}{Q\beta} \pi_x \right) + \hat{k} z \end{aligned} \quad (32)$$

again the mean value along y is time dependent, moreover since

$$\xi = \left(\frac{\mu \omega_c}{\hbar} \right)^{1/2} (y - \langle y \rangle) \quad (33)$$

the mean squared deviation of ξ from its mean value is equal to zero

$$\langle \xi \rangle = 0$$

as is found for an harmonic oscillator.

From these mean values it is clear that the derived wavefunction describes the motion of a charged particle in a laser and in a magnetic field in terms of an harmonic oscillator along the y axis with center of oscillations given by

$$\langle y \rangle = y_0 - \frac{c}{QB} \pi_x(t) \quad (34)$$

The center of oscillation is then dependent on the particular polarisation of the laser field and oscillates about the point

$$y_0 = - \frac{P_x c}{QB} \quad \text{as in the absence of the laser.}$$

Along the z and x axes the charged particle moves under the influence of the electric field only, so the new effect is confined to the motion in the y direction.

Concluding this chapter we discuss, briefly, some particular cases of the solution (23).

a) Laser field equal to zero.

In this case $\pi_x = \pi_y = 0$ and consequently

$$\begin{cases} \underline{p} = [P_x, 0, P_z] \\ \mathcal{P}^2 = P_z^2 \\ f(\underline{p}, \underline{r}) = P_x x + P_z z - \frac{P_z^2}{2\mu} t \end{cases} \quad (35)$$

Moreover

$$\xi = \left(\frac{\mu \omega_c}{\hbar} \right)^{1/2} (y - y_0) \quad (36)$$

and the wavefunction (23) becomes

$$\begin{aligned} \psi(r, t) = & \tilde{C}_m \exp \left\{ -\frac{i}{\hbar} \left[E_m t + \frac{P_z^2}{2\mu} t \right] \right\} \times \\ & \times \exp \left\{ \frac{i}{\hbar} [P_x x + P_z z] \right\} \exp \left[-\frac{\xi^2}{2} \right] H_m(\xi) \end{aligned} \quad (37)$$

that is the wavefunction of a particle in presence of a magnetic field given by Landau and Lifchitz⁽³⁾.

We note that the solution (37) and the more general (23) have been derived in a cartesian geometry and that, in this case, the constant of motion are: the momenta along z and x, p_z and p_x , and the quantum number n. This is not the only geometry possible, in fact, as it has been shown in chapter 2, for collisions in presence of a magnetic field it is more convenient using cylindrical coordinates with constant of motion given by p_z , n, and m, where m is the component of the angular momentum along the direction of the magnetic field.

b) Magnetic field equal to zero.

In this case, since as $B \rightarrow 0$, $\xi \rightarrow \infty$ it is not possible to get the solution for $B=0$ as a limit of the wavefunction (23). The problem is analogous to the one already solved in chapter 3, for the case of particle scattering in presence of a magnetic field.

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CHAPTER 5

POTENTIAL SCATTERING IN THE PRESENCE OF A STATIC
MAGNETIC FIELD AND A RADIATION FIELD OF ARBITRARY
POLARIZATION

The free-free transitions ("bremsstrahlung") of a charged particle scattered by a potential, in a static uniform magnetic field, is of interest not only in itself, but for applications in astrophysics and plasma physics. We do not reference all the literature here but refer to, for example, Ferrante et al.⁽¹⁾

If a time dependent monochromatic electro-magnetic field (which could be thought of as a model of a laser) is also present, then it is possible to excite transitions between the quasi-Landau levels in both single and multi-photon modes, in both the direct and inverse bremsstrahlung processes.⁽⁵⁾ The problem has recently been addressed by Bergou et al.⁽²⁰⁾ These processes are closely related to laser photodetachment or laser photoionization in a magnetic field.^(2,3) Of interest is also the study of the line shape of the scattered radiation by electrons in a magnetic field.⁽⁴⁾

In this chapter we concentrate on the effects of polarisation of the radiation.

To simplify, we assume that the radiation field is homogeneous, single mode, and sufficiently low energy for the dipole approximation to be valid. The magnetic field is a static uniform field taken along the z-direction. The last part of the chapter discusses the modifications when the central potential $V(r)$ is allowed to perturb the

Landau energies, but not to distort their wavefunctions. This corresponds to the "distortion" approximation of Bates⁽⁵⁾ reintroduced recently by Rynfuku and Watanabe.⁽⁶⁾

We describe the scattering by choosing a gauge for the magnetic field such that its vector potential is

$$\underline{A}_M = (-By, 0, 0) \quad (1)$$

in Cartesian coordinates with respect to the scattering centre. The unperturbed wavefunction for a single scattering centre may be written

$$|n\rangle = \psi_n(\underline{r}, t) = C e^{-iE_n t/\hbar} e^{\frac{i}{\hbar} \underline{p} \cdot \underline{r}} \exp\left\{-\frac{i}{2m_0\hbar} \int^t p^2(t') dt'\right\} e^{-\frac{1}{2}\eta^2} H_n(\eta) \quad (2)$$

where E_n is the energy of the n^{th} Landau level

$$\underline{p} = (p_x, \pi_y(t), p_z) \quad (3)$$

and (n, p_x, p_z) are the quantum numbers of the electron. The quantity $\underline{\pi} = (\pi_x, \pi_y, 0)$ defines the time dependent motion of the electron at right angles to the magnetic field, and following Seely⁽⁷⁾

$$p^2(t) = \left| \underline{p}(t) - \frac{Q}{c} \underline{A}^L(t) \right|^2 - [p_x - \pi_x(t)]^2 \quad (4)$$

where Q is the charge of the particle of mass m_0 and \underline{A}^L the vector potential of the radiation field. We have

$$\eta = \left[y - y_0 - \frac{c}{2B} \pi_x \right] \rho_0^{-1} \quad (5)$$

where ρ_0 is the Larmor radius and $y_0 = -c p_x / QB$, is the centre of oscillation, in the absence of the radiation. The functions $H_n(x)$ are the usual Hermite polynomials.

With this choice of gauge the Landau quantum numbers are not (n, m_l, p_z) but are as specified above. We evaluate the Born approximation to the S-matrix

$$S_{f_i} = \int_{-\infty}^{\infty} \langle f|V|i \rangle dt \quad (6)$$

(where $|i\rangle = |n_i, p_{x_i}, p_{z_i}\rangle$ and similarly for $|f\rangle$), by averaging over all scattering centres in the plane normal to z . We are able to obtain a result for the total cross section σ_{if} from all states of principal quantum number n_i to all states of principal quantum number n_f , in which the electron momentum transfer is

$$q_x = p_{x_f} - p_{x_i}, \quad q_z = p_{z_f} - p_{z_i},$$

$$\sigma_{if} = \sum_{\nu = -\infty}^{\infty} \left(\frac{n_i!}{n_f!} \right) \left(\frac{2\pi}{\hbar} \right) \left(\frac{1}{2\pi\hbar} \right)^3 \frac{m_0^2}{p_{z_i} p_{z_f}} \int dq_y |V(q)|^2 |C_\nu(q)|^2 \rho^{n_f - n_i} e^{-\rho} |L_{n_i}^{n_f - n_i}(\rho)|^2 \quad (7)$$

where

$$\rho = \frac{1}{2\hbar^2} [q_x^2 + q_y^2] \rho_0^2 \quad (8)$$

and ν is the number of photons emitted or absorbed. The coefficient C_ν is the Fourier coefficient of

$$\begin{aligned} \mathcal{H} = & \exp\left\{ \frac{i}{m_0 \hbar} q_x \int^t dt' [\pi_x(t') + \frac{e}{c} A_x^L(t')] \right. \\ & \left. + \frac{i}{m_0 \hbar} q_z \frac{e}{c} \int^t dt' A_z^L(t') - \frac{i q_y}{\hbar^2} \rho_0^2 \pi_x(t') \right\}, \quad (9) \end{aligned}$$

where we have specialised to $Q = -e$. The $L_a^b(x)$ are Associated Laguerre polynomials,

$$L_a^b(x) = \frac{1}{a!} e^x x^{-b} \frac{\partial^a}{\partial t^a} (t^{b-a} e^{-t}), \quad (10)$$

and $V(q)$ is the Fourier transform of the potential. We sum and integrate over final states and over all degenerate initial states (which corresponds to an integral over p_{x_i} , p_{x_f} and a sum over n_f) to obtain

$$\sigma(n_i p_{z_i}, n_f p_{z_f}) = \sum_{v=-\infty}^{\infty} \frac{n_i!}{n_f!} \frac{2\pi}{\hbar} \frac{m_0^2}{p_{z_i} p_{z_f}} \left(\frac{1}{2\pi\hbar} \right)^3 \int dq_x \int dq_y |V(q)|^2 |C_v(q)|^2 \rho^{n_f - n_i} e^{-\rho} |L_{n_i}^{n_f - n_i}(\rho)|^2 \quad (11)$$

if $(n_f \geq n_i)$.

Energy conservation implies

$$n_f \hbar \omega_c + p_{z_f}^2 / 2m_0 = n_i \hbar \omega_c + p_{z_i}^2 / 2m_0 - v \hbar \omega_L \quad (12)$$

We point out that the cross sections (11) exhibit the special feature that the final electron momentum along the magnetic field appears in the denominator. This is the usual strong magnetic field result, which may give rise to quasi-resonances at the quasi-Landau state thresholds.

To proceed further, we need to specify the form of the scattering potential and determine $C_v(q)$ for a given radiation field vector potential. For the most general form (in dipole approximation) of $A^L(t) = [A_x^L \cos(\omega_L t + \phi_x), A_y^L \cos(\omega_L t + \phi_y), A_z^L \cos(\omega_L t + \phi_z)]$, we find

$$|C_v(q)|^2 = J_v^2(\lambda), \quad (13)$$

J_v being the Bessel function of integer order, while

$$\lambda = \lambda_0 (\alpha^2 + \beta^2)^{\frac{1}{2}}, \quad \lambda_0 = \frac{-e}{m_0 \hbar c} \quad (14)$$

We also have

$$\alpha = a \cos \phi_x - b \sin \phi_y + c \cos \phi_z + d \sin \phi_x + f \cos \phi_y$$

$$\beta = a \sin \phi_x + b \cos \phi_y + c \sin \phi_z - d \cos \phi_x + f \sin \phi_y,$$

$$a = A_x^L q_x \frac{\omega_L}{\omega_L^2 - \omega_c^2}; \quad b = A_y^L q_x \frac{\omega_c}{\omega_L^2 - \omega_c^2}; \quad c = \frac{A_z^L q_z}{\omega_L};$$

$$d = A_x^L q_y \frac{\omega_c}{\omega_L^2 - \omega_c^2}; \quad f = A_y^L q_y \frac{\omega_L}{\omega_L^2 - \omega_c^2} \quad (15)$$

As scattering potential we take a screened Coulomb potential $V(r) = (V_0/r) \exp(-\gamma r)$, yielding $V(q) = 2\pi V_0 \rho_0^2 / (\rho + \xi)^{-1}$, with $\xi = (\rho_0^2/2)(q_z^2/\hbar^2 + \gamma^2)$. With the appropriate procedures, corresponding results for Coulomb and the exponential potentials may be obtained.

To perform the required integrations in equation (11), it is useful to specify to some particular radiation field polarizations.

(1) Taking the radiation field linearly polarized along the z axis, $A^L(t) = A_z^L \cos \omega_L t$, we have $\lambda_1 = \lambda_0 \frac{q_z A_z^L}{\omega_L}$. In this case the squared Bessel function $J_\nu^2(\lambda_1)$ may be taken out of the integrals over q_x and q_y , and equation (11) becomes

$$\sigma_T = \sum_{\nu=-\infty}^{\infty} J_\nu^2(\lambda_1) \sigma_T^{(\nu)}, \quad (16)$$

where $\sigma_T^{(\nu)}$ is the total cross section in the absence of the radiation field with energy conservation still given by equation (12). Changing the integration variables, $\sigma_T^{(\nu)}$

may be written as

$$\sigma_T(\nu) = \sum_{n_f} \frac{2\pi m_o^2 \rho_o^2}{n^4} \frac{v_o^2}{k_{z_f} k_{z_i}} \left(\frac{n_i!}{n_f!} \right) B_{n_f, n_i}(\xi) \quad (17)$$

$(n_f \geq n_i)$, with

$$B_{n_f, n_i}(\xi) = \int_0^\infty (\rho + \xi)^{-2} \exp(-\rho) \rho^{n_f - n_i} |L_{n_i}^{n_f - n_i}(\rho)|^2 d\rho. \quad (18)$$

If in equation (18) we put $n_i = 0$ then $L_0^{n_f} = 1$, and the integration over ρ may be carried out to give

$$B_{n_f}(\xi) = (-\xi)^{n_f - 1} (n_f + \xi) e^\xi E_1(\xi) - (n_f - 1)! + \sum_{\ell=1}^{n_f - 1} (\ell - 1)! (n_f + \xi) (-\xi)^{n_f - 1 - \ell} \quad (19)$$

$(n_f \geq 2)$, with,

$$B_0 = \xi^{-1} - e^\xi E_1(\xi), \text{ and } B_1(\xi) = (1 + \xi) e^\xi E_1(\xi),$$

E_1 being the exponential integral.

This result (17) was first obtained, but in a different form, by Ventura⁽⁹⁾, while (17) and (18) were given by Pavlov and Panov⁽¹⁰⁾ but (19) is new. The different forms arise because of different Choices of coordinate system and gauge. Ventura's form for (17) is expressed in sums of products of squares of confluent hypergeometric functions of the second kind and associated Laguerre polynomials. We now demonstrate for some particular cases that his expression and ours are equivalent.

For $n_i = n_f = 0$ the result is trivial. For other cases we have been unable to show analytic identity, but find that the two expressions are identical if our quantity

$$A_1 = \frac{n_i!}{n_f!} B_{n_f, n_i}(\xi) \quad (20)$$

and Ventura's quantity

$$A_2 = n_i! n_f! [\psi(n_i + 1, n_i - n_f + 1, \xi)]^2 \sum_{s=0}^{n_i} \frac{s!}{(n_f - n_i + s)!} \times$$

$$\times [L_s^{n_f - n_i}(-\xi)]^2 + \frac{n_i!}{n_f!} [L_{n_i}^{n_f - n_i}(-\xi)]^2 \sum_{s=n_i+1}^{\infty} s!(s+n_f - n_i)! [\psi(s+1, n_i - n_f + 1, \xi)]^2 \quad (21)$$

are identical. We have investigated this for the case shown in Table 1 and find that this is always the case, allowing for the slow convergence of the sum in A_2 .

(2) For right hand circularly polarised light in the xy-plane, $\vec{A}^L(t) = (A_O^L \cos \omega_L t, A_O^L \sin \omega_L t, 0)$,

we find $\lambda_2 = \lambda_O A_O^L q_{\perp} / (\omega_L - \omega_C)$,

with $q_{\perp} = (q_x^2 + q_y^2)^{\frac{1}{2}}$. Using the relation connecting ρ to q_{\perp} (given by equation (8)), λ_2 is expressed as $\lambda_2 = t \rho^{\frac{1}{2}}$ with

$$t = e E_O^L \sqrt{2} / m_O \omega_L (\omega_L - \omega_C) \rho_O. \quad (22)$$

For this case, equation (11) becomes

$$\sigma_T = \sum_{\nu, n_f} \left(\frac{2\pi m_O^2 \rho_O^2}{\hbar^4} \right) \left(\frac{V_O^2}{k_{z_f} k_{z_i}} \right) \left(\frac{n_i!}{n_f!} \right) \times$$

$$\times \int_0^{\infty} d\rho (\rho + \xi)^{-2} \exp(-\rho) \rho^{n_f - n_i} |L_{n_i}^{n_f - n_i}(\rho)|^2 J_{\nu}^2(t \rho^{\frac{1}{2}}) \quad (23)$$

$(n_f \geq n_i)$,

Table 1

$\xi = 0.01$

n_i	n_f	A_1	A_2
0	5	4.966 (-2)	4.965 (-2)
0	100	1.009 (-4)	1.004 (-4)
5	10	1.304 (-1)	1.303 (-1)
10	20	3.102 (-2)	3.102 (-2)

$\xi = 1$

n_i	n_f	A_1	A_2
0	5	2.982 (-2)	2.981 (-2)
0	100	9.898 (-5)	9.843 (-5)
5	10	3.821 (-2)	3.821 (-2)
10	20	1.535 (-2)	1.535 (-2)

$\xi = 100$

n_i	n_f	A_1	A_2
0	5	8.914 (-5)	8.862 (-5)
0	100	2.494 (-5)	2.470 (-5)
5	10	7.628 (-5)	7.628 (-5)
10	20	6.295 (-5)	6.294 (-5)

and the integral is fairly readily evaluated numerically.

To understand the structure it is again convenient to restrict our attention to $n_i = 0$. Using the series representation of the squared Bessel function, the integration over ρ may then be carried out to give

$$\sigma_T = \sum_{\nu, n_f} \left(\frac{2\pi m_0^2 \rho_0^2}{\hbar^4} \right) \left(\frac{v_0^2}{k_{z_f} k_{z_i}} \right) \frac{1}{n_f!} \sum_{k=0}^{\infty} g(\nu, k, t) B_{n_f + \nu + k}(\xi), \quad (24)$$

with $B_{n_f + \nu + k}(\xi)$ defined by equation (19), and

$$g(\nu, k, t) = \frac{(-1)^k [2(\nu + k)]! (t/2)^{2(\nu + k)}}{[(\nu + k)!]^2 (2\nu + k)! k!},$$

$$\sum_k g(\nu, k, t) \equiv J_\nu^2(t).$$

for small t (weak radiation field, high frequencies, weak magnetic fields, and, of course, $\omega_L \neq \omega_C$), (24) reduces to

$$\sigma_T = \sum_{\nu=-\infty}^{\infty} J_\nu^2(t) \sigma_T^{(\nu)}, \quad (25)$$

where $\sigma_T^{(\nu)}$ is given by equation (17) with $n_i = 0$ and $B_{n_f + \nu}(\xi)$.

In general only the terms with $|\nu| \leq 1$ make significant contributions. In the general case $n_i \neq 0$ a similar result may be obtained but $B_{n_f + \nu + k}(\xi)$ is no longer given by (18). The approximation $E_{n+1} - E_n = \hbar\omega_C$ will be valid even in the presence of a Coulomb potential for large n_i, n_f (as discussed further below). Our result (22) then suggests that resonance effects become important

when $\omega_L \rightarrow \omega_C$. In this case the arguments of $J_\nu(t)$ become large, and their contribution to σ_T from multiphoton effects ($|\nu| > 1$) may dominate. On resonance a better approach may be to solve the problem of the potential $V(r)$ together with the magnetic field as exactly as possible. One would then identify a pair of levels whose separation was $\hbar\omega_L$, and use the normal Bloch model for the laser interaction with these two states.

- (3) (a) Left-handed circularly polarised radiation
(polarisation vector in the xy plane).

$$\vec{A}^L(t) = (A_O^L \cos\omega_L t, -A_O^L \sin\omega_L t, 0), \text{ gives}$$

$$\lambda_{3a} = \lambda_O A_O^L q_{\perp} / (\omega_L + \omega_C). \quad (26a)$$

- (b) Linear polarisation along x

$$\vec{A}^L(t) = (A_O^L \cos\omega_L t, 0, 0), \text{ gives}$$

$$\begin{aligned} \lambda_{3b} &= \lambda_O A_O^L [\omega_L^2 q_x^2 + \omega_C^2 q_y^2]^{\frac{1}{2}} / (\omega_L + \omega_C) (\omega_L - \omega_C) \\ &= \frac{1}{2} \lambda_O A_O^L q_{\perp} (\omega_L - \omega_C)^{-1} \end{aligned} \quad (26b)$$

as in case (2).

- (c) Right-handed circularly polarized light in the xz plane

$$\vec{A}^L(t) = (A_O^L \cos\omega_L t, 0, A_O^L \sin\omega_L t), \text{ gives}$$

$$\lambda_{3c} = \lambda_O A_O^L \left\{ \omega_L^2 q_x^2 + \frac{1}{2} [q_z (\omega_L^2 - \omega_C^2) + \omega_L \omega_C q_y]^2 \right\}^{\frac{1}{2}} / (\omega_L + \omega_C) (\omega_L - \omega_C)$$

again going into

$$\lambda_{3c} = \frac{\lambda_O A_O^L}{2} q_{\perp} (\omega_L - \omega_C)^{-1} \quad (26c)$$

again as in case (2). Similar results are obtained for other polarisations.

The results of case (3) may be summarized as follows:

- (i) Linear polarisation is neither necessary nor sufficient to produce the resonant denominator $(\omega_L - \omega_C)^{-1}$ in the Bessel function argument.
- (ii) Whatever the polarisation provided, with at least one component of the radiation field in the plane of the magnetically confined motion, a resonance denominator switches on multiphoton processes, when $\omega_L \rightarrow \omega_C$.
- (iii) When $\omega_L \rightarrow \omega_C$ the effect of the radiation field on the electrons motion along the magnetic field may be neglected.

We note that our results go over to those of other authors^(9,11,12,13) when the radiation field is switched off. As shown by Blumberg et al⁽¹⁴⁾ it is not straightforward to obtain the result in the case of no magnetic field from that for $B \neq 0$. However, the limit process has been carried out successfully for several case by e.g. Ohsaki⁽¹⁵⁾, Blumberg et al⁽¹⁴⁾ and Kara and McDowell⁽²⁾.

What are the possibilities of testing our result? It is easy to see that it can only apply for large n_i and n_f , because the potential causes both a level splitting and shift for low n_i, n_f . The level splitting is due to the symmetry breaking by the spherically symmetric potential; in effect the s degeneracy is removed, or in our terms the p_x degeneracy. Starace⁽¹⁶⁾ and Rau⁽¹⁷⁾ have shown that the energy levels are well approximated in the JWKB method, when the potential $r^{-1} = (\rho^2 + z^2)^{-\frac{1}{2}}$ is replaced by ρ^{-1} . that is the energies for a given m

and π are found as the solutions of

$$\int_{\rho_1}^{\rho_2} V(\epsilon_n, \rho)^{\frac{1}{2}} \rho d\rho = (n + \frac{1}{2})\pi$$

and for Coulomb potential

$$V(\epsilon_n, \rho) = \epsilon_n - \frac{m^2 - \frac{1}{4}}{\rho^2} - \frac{1}{4} \gamma^2 \rho^2 + \frac{2V_0}{\rho} - \frac{1}{4\rho^2} .$$

They show that for not too large n , the energy spacing $\omega_{n+1,n} = E_{n+1} - E_n + 3\gamma Ry$ whereas for very large n , $\omega_{n+1,n} + 2\gamma Ry$, which is the Landau spacing. Kara and McDowell⁽²⁾ confirmed this without use of the JWKB result. Furthermore this result is well verified experimentally^(3,21).

In considering free-free transitions, we have in mind an astrophysical plasma, such as might be associated with, say, a neutron star. We assume a field B of the order of 10^{13} G, thus would normally think that $\gamma \sim 10^4$ was the correct scaling parameter. However, the source of the electrons here is primarily fully stripped Fe, and the Coulomb field is $-\alpha_0/r$, with $\alpha_0 \sim 25$. Looking at $V(\rho)$ we see that

$$V(\rho, \alpha_0) = \alpha_0^2 V(\rho, 1)$$

and the effective scaling parameter is $\gamma' \sim \gamma(25)^{-2}$, so that as far as the continuum levels of the ion ("the quasi-Landau" levels) are concerned, we expect them to be similar to those calculated⁽¹⁸⁾ for atomic hydrogen at $\gamma' \sim 3$. Detailed energies for the low-lying states in Fe XXVI are given by Ruder et al⁽¹⁹⁾. At large n the spacing is

$$2\gamma Ry = 4 \times 10^3 Ry = 2 \times 10^5 eV = 2 \times 10^8 \text{ }^\circ\text{K}.$$

This is comparable with temperatures of interest in the

photosphere of neutron stars, i.e. $\Delta E \sim kT$, so the free-free process is reasonably rapid if the cross section is large.

Nevertheless the free-free transitions at $\Delta E \sim kT$ may be dominated by transitions between low lying levels. For example, for the (001) to (0-11) transition the energy difference⁽¹⁹⁾ at 10^{13} G is about 0.5 keV, whereas the Landau spacing is 4×10^3 Ry \sim 50 keV. Thus given a comparable cross section this contribution will be of order e^{10} times larger. It is thus important to calculate this and similar contributions to the free-free rate.

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CHAPTER 6

LASER-ASSISTED ELECTRON IMPACT IONISATION OF HELIUM

Laser assisted electron impact ionisation of atoms has been considered by several authors ^{(1), (2)}.

Very recently Cavaliere et al ⁽³⁾ and Banerji and Mittleman ⁽⁴⁾ have given explicit formulae for the triple differential cross section (TDC) in the First Born Approximation (FBA). Both sets of authors' results are equivalent, though the latter authors specify more exactly the approximations involved. Thus they apply to a low frequency electromagnetic field (so that the cross section may be regarded as slowly varying when the incident energy changes by $h\omega$ or, alternatively, that $h\omega$ is small compared with the energy of the slower of the outgoing electrons). Further, the field is treated as single mode, homogeneous, and in the dipole approximation, and is not in resonance with any atomic transition frequency ω_{ij} , while finally the laser field $|\underline{\epsilon}_L|$ at the atom should be small compared with the Coulomb field.

Cavaliere et al ⁽⁵⁾ have applied the theory to ionisation of atomic hydrogen, evaluating the scattering cross section in the FBA using a Coulomb wave to describe the slower final state electron. They predicted that in the case that \underline{E}_L was perpendicular to the momentum transfer \underline{K}_{if} of the faster final state electron, the normal dipole-like pattern of the TDC would split symmetrically. They also predict that when $\underline{\epsilon}_L$ is parallel to \underline{K}_{if} , the peaks do

not split but the ratio of the forward to the backward peak is considerably smaller than in the field free case.

Since measurements in atomic hydrogen, even in the field free case, are prohibitively difficult in the coplanar asymmetric (Ehrhardt) geometry, it is of interest to carry out calculations for helium. The present *chapter* presents such a calculation for helium at one energy, and one scattered electron angle in FBA. We are of course aware (Byron et al ⁽⁶⁾ Ehrhardt et al ^(7,8)) that in the field free case the FBA gives an inadequate description of the backward peak, the main correction being a Second Born Approximation effect. We hope to consider the Second Born Approximation in the laser assisted case at a later date.

We restrict ourselves here to one of the situations studied by Ehrhardt's group ⁽⁸⁾, namely an incident electron of energy $E_i = 256.5$ eV, wave vector \underline{k}_i , a fast outgoing electron of wave vector \underline{k}_f , at 4° to the incident direction, and a slow outgoing electron of energy 3 eV at angle θ_K (which varies) and wave vector \underline{K} .

Then the FBA to the TDC may be written

$$\begin{aligned} \frac{d^3\sigma(L)}{d\Omega_1 d\Omega_2 d\varepsilon} &= \sum_{n=-\infty}^{\infty} J_n^2(\lambda_n) \frac{d^3\sigma(0)}{d\Omega_1 d\Omega_2 d\varepsilon} \Big|_{k_i^2 + n\hbar\omega} \\ &= \sum_{n=-\infty}^{\infty} \sigma_n(L) \end{aligned} \quad (1)$$

where the left hand side is the TDC with the laser on, and the right hand side is a sum over the number of photons absorbed ($n < 0$) or emitted ($n > 0$) of the field free cross section at energy $E_n = k_i^2 + n\hbar\omega$, times a squared Bessel function of argument

$$\lambda_n = \frac{1}{\omega^2} (\underline{K}_{if} - \underline{K}) \cdot \underline{\epsilon}_L \cdot \quad (2)$$

We choose to describe the fast electrons ($\underline{k}_i, \underline{k}_f$) by plane waves, but to use a close-coupling expansion for the target state and the slow ejected electron. The relevant expression for the TDC in the absence of the field has been given by Robb et al ⁽⁹⁾ and need not be repeated here.

We choose to begin by studying the standard 1s - 2s - 2p close coupling model. The calculations were carried out using the IMPACT code, and a program GOSION (Jakubowicz and Moores ⁽¹⁰⁾) for the reduced matrix element. Our program for evaluating (1) above using any close coupling model for an arbitrary ($\underline{L}, \underline{S}$) coupled target will be submitted to Computer Physics Communications.

We tested our TDC code by comparing with the results of Robb et al ⁽⁹⁾ and of Jacobs ⁽¹¹⁾ and the comparison (Fig. 1), in which the experimental data (Ehrhardt et al ⁽⁸⁾) are normalised to Jacobs' result at the forward maximum, shows excellent agreement with the results of

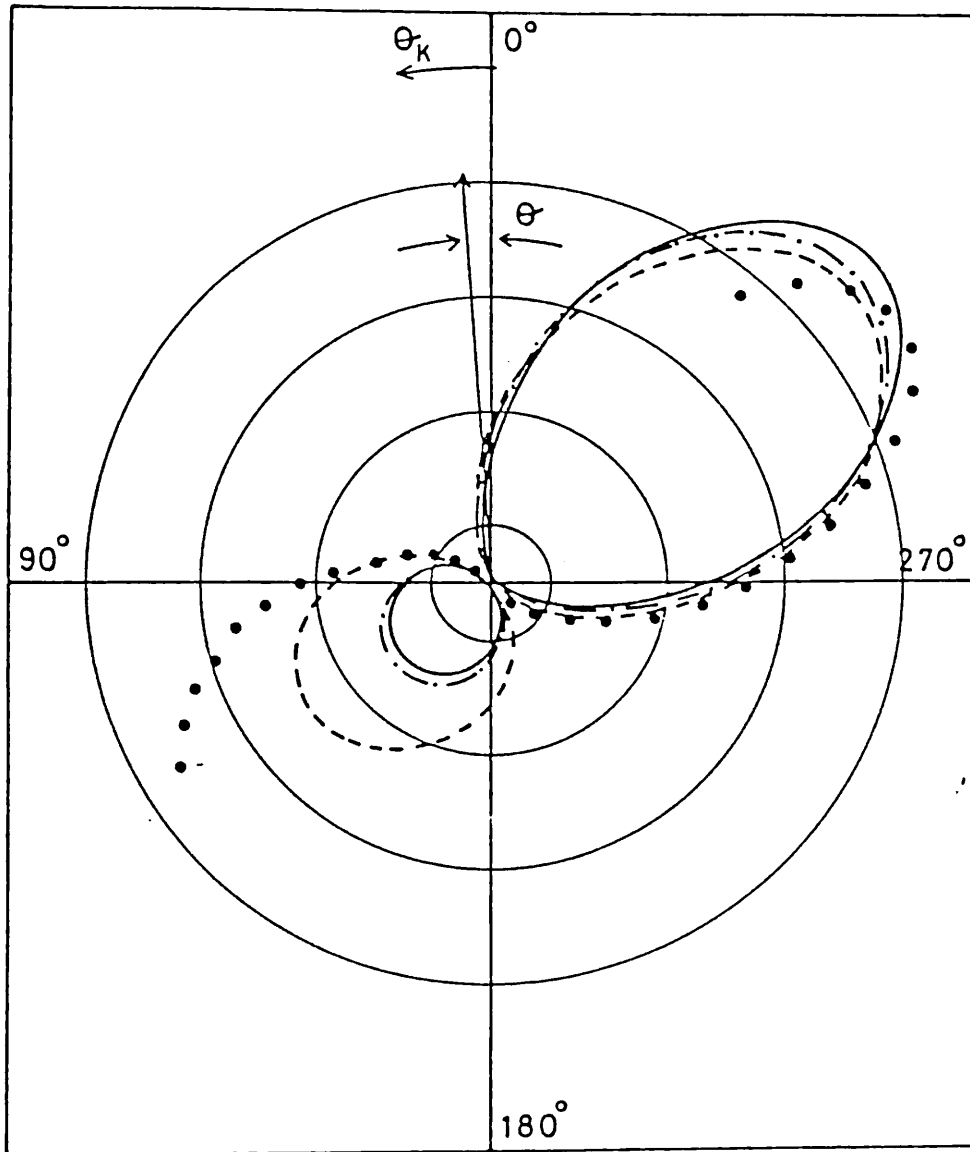


Fig. 1 TDCO for helium for a 256.5 eV incident electron energy, a scattering angle $\theta = 4^\circ$ and an ejected electron energy of 3.0 eV. Dashed line, Jacobs 1974; dashed point line, Robb et al 1975; solid curve, present work; dots, experimental results of Ehrhardt et al 1972.

Robb et al, but disagrees with Jacobs' results for the backward peak. We concur with the reasons advanced by Robb et al for their, and our, disagreement with Jacobs.

We then tested the low frequency approximation by calculating the field free cross section as a function of incident energy from 252 to 260 eV. The results for the forward and backward peaks (Fig. 2) show that the cross section is indeed slowly varying over $\Delta E = \hbar\omega$ for both frequencies considered. These were $\hbar\omega = 1.17$ eV, corresponding to a Neodymium-glass laser and $\hbar\omega = 0.12$ eV corresponding to a CO₂ laser.

We considered the following cases for each of the two frequencies, and a laser intensity of $5 \cdot 10^7$ Vcm⁻¹.

$$(a) \quad \underline{\epsilon}_L \perp \underline{K}_{if} ; \quad (b) \quad \underline{\epsilon}_L \parallel \underline{K}_{if} .$$

Case I $\hbar\omega = 1.17$ eV

(a) The results are shown in Fig. 3. They are symmetric about $\theta_k = 130^\circ$, the direction of \underline{K}_{if} . The upper most curve is the field-free case, while the others (labelled by n) are for $n = 0, 1, 2, 3$. The pattern is quite simple. The $n = 0$ contribution dominates at the forward and backward peak where it is very close to the field-free value (TDCO). The one-photon absorption contribution is small at the forward and backward peaks but has four maxima, with the two larger being close to the forward direction,

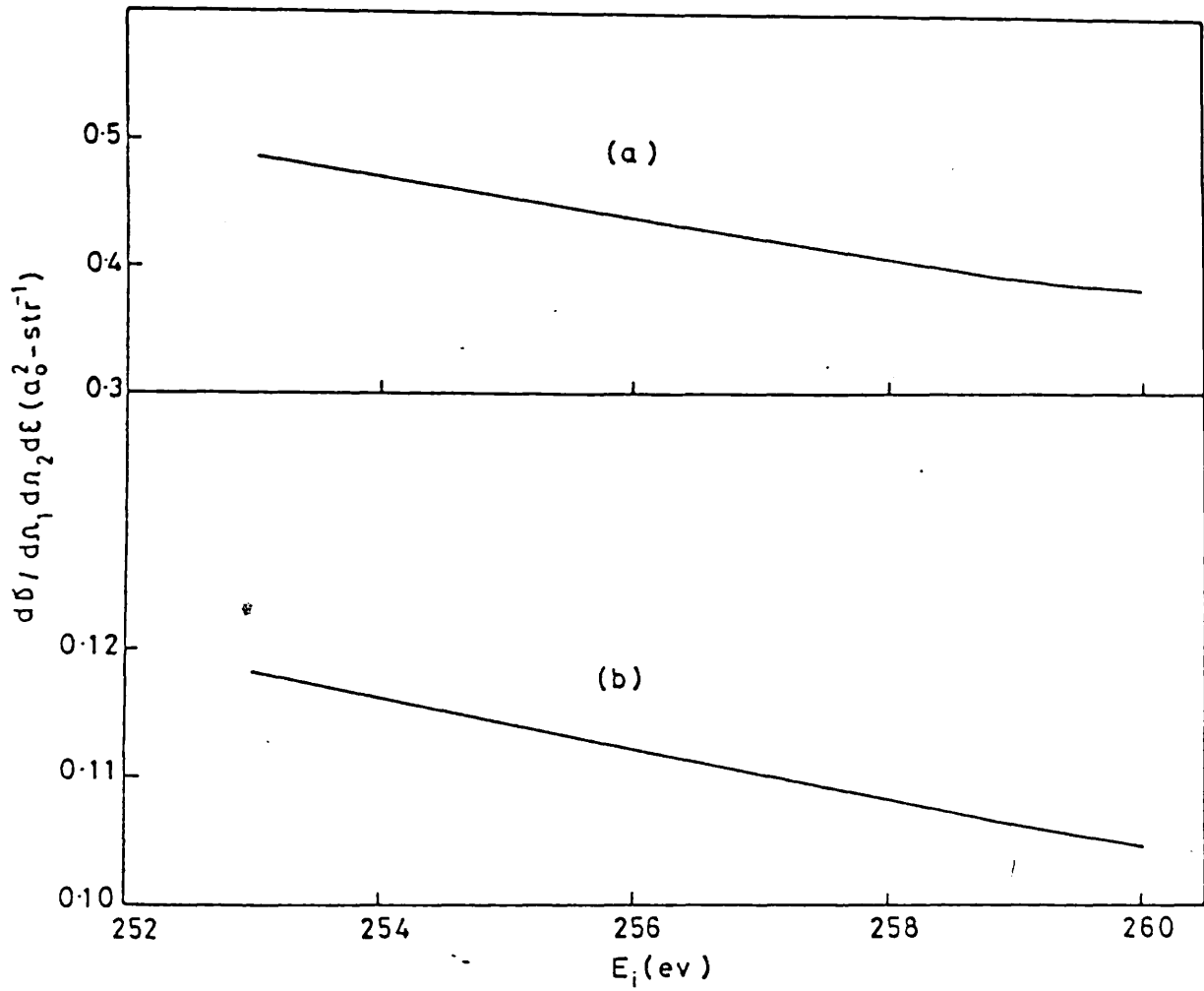


Fig. 2 TDCO value for the forward peak (a) and backward peak (b) as a function of the incident electron energy E_i ; other parameters as for Fig. 1.

with an intensity about one-fifth of the field-free value. There are two additional zeros due to zeros of $J_1(\lambda_1)$, which we discuss further below. The contributions for $n > 1$ are similar but decrease rapidly with increasing n .

Since, in this case, $\underline{\epsilon}_L \perp \underline{K}_{if}$, λ_n varies between 0.03 and 2.47; when λ_n is small $J_n^2(\lambda_n) \approx \lambda_n^{2n}$, so the partial cross sections decrease rapidly with increasing n near 120° and 310° . When λ_n is not small there is a zero of the field-free cross section.

The cross sections for emission ($n < 0$) are in general slightly lower (for fixed θ_K) than for absorption, since they are proportional to the field-free cross section at an energy $E_i + n\hbar\omega$ for emission and $E_i - n\hbar\omega$ for absorption, and Fig. 2 shows that the second is the larger. We note that the sum in eqn. 1 is to within our numerical accuracy equal to the TDCO.

(b) With $\underline{\epsilon}_L$ parallel to \underline{K}_{if} , the pattern (Fig. 4) is more complicated, but the complexity is due to the behaviour of the Bessel functions (Fig. 5). Thus for $n = 0$ $J_0(\lambda_0)$ has zeros near 50° and 210° close to the zeros of the TDCO and shows a similar pattern to the TDCO, though an order of magnitude smaller. However, for $n = 1$, $J_1(\lambda_1)$ has zeros near 90° , 170° , 270° and 360° , so the corresponding contribution to the TDC has these zeros in addition to those of the TDCO. Its maximum value occurs

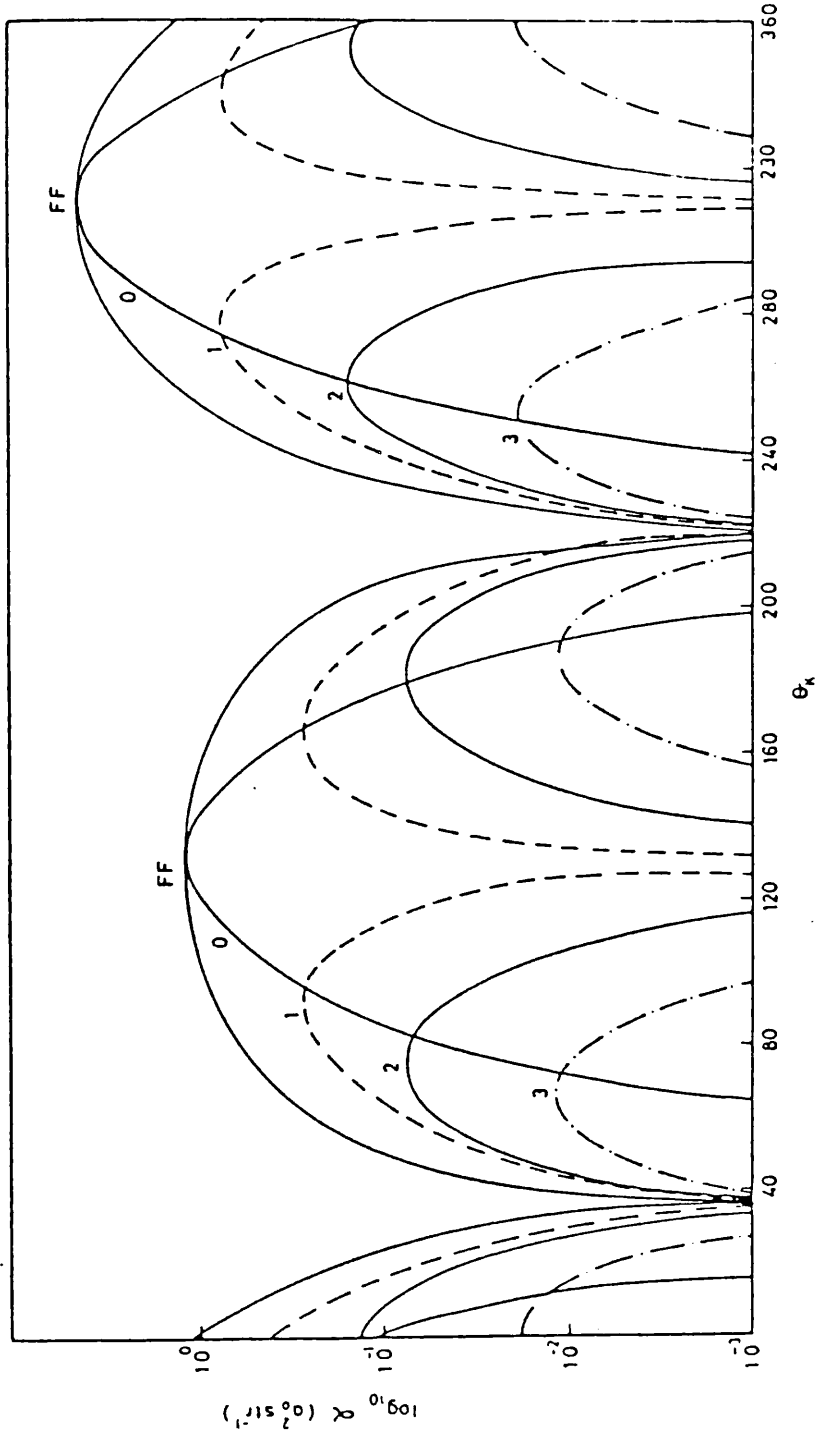


Fig. 3 TDC versus the ejected electron angle for $\underline{\epsilon}_L \perp \underline{K}_{if}$.

The FF curve is the TDCSO of Fig. 1; the numbers

on the curves indicate the number of photons

absorbed during the ionisation process;

$\hbar\omega = 1.17$ eV; other parameters as in Fig. 1.

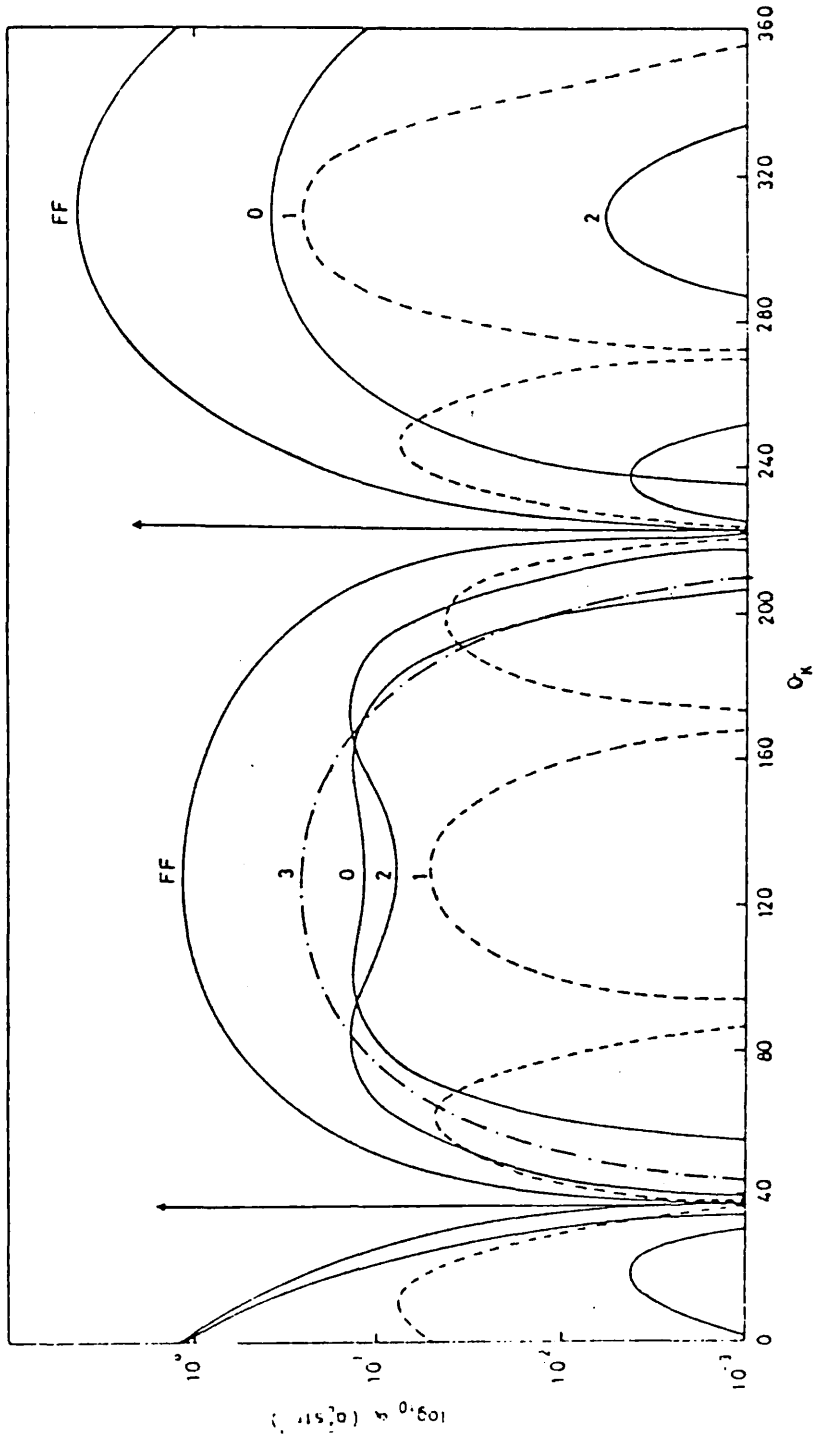


Fig. 4 As Fig. 3 but for $\underline{\epsilon}_L \parallel \underline{K}_{if}$.

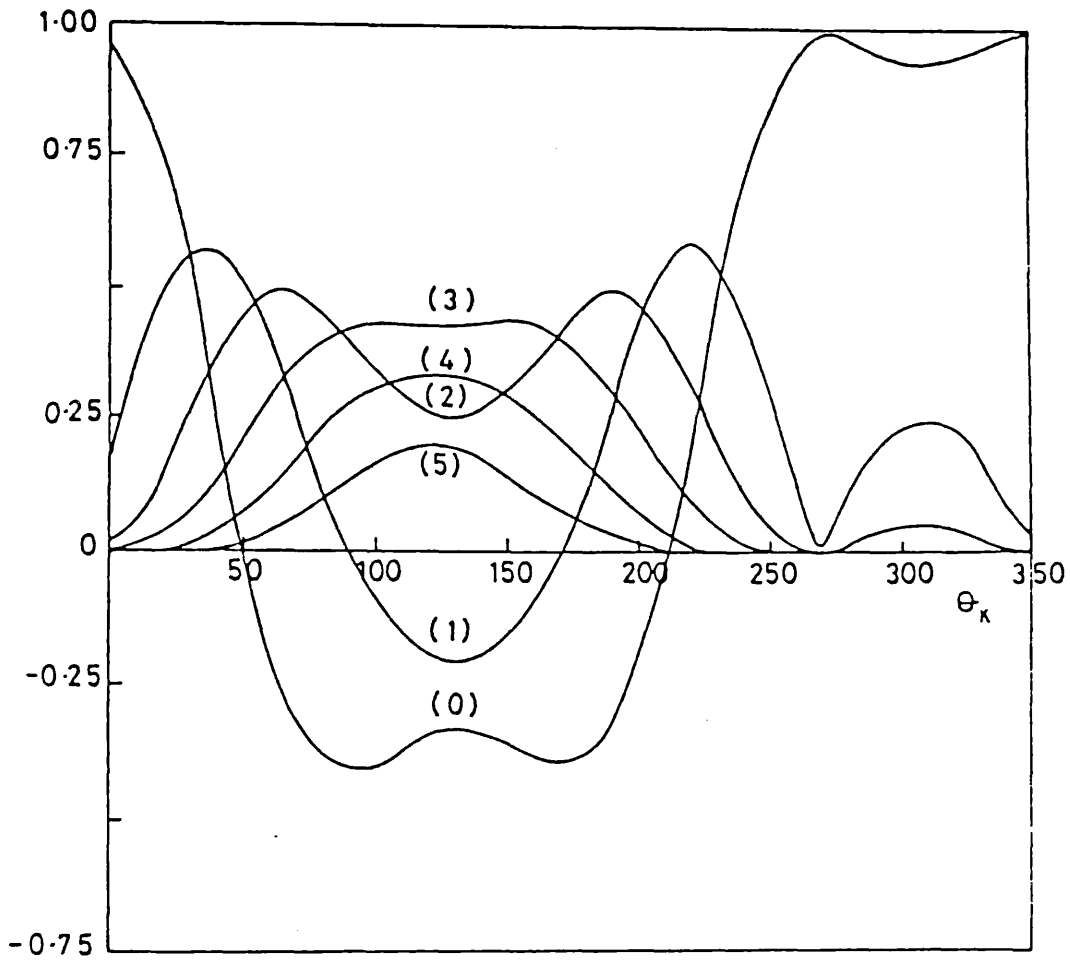


Fig. 5 Bessel functions $J_n(\lambda)$ as function of the ejected electron angle for the process described in Fig. 4. The numbers on the curves indicate the number of photons absorbed.

along $\hat{\mathbf{K}}_{if}$, where it is about 1/16 of the TDCO. Over a wide range of angles, far from a zero of any of the relevant Bessel functions, e.g. near 130° , three-photon absorption dominates.

Case II $\hbar\omega = 0.12$ eV

Here the arguments of the Bessel functions are large, indeed about a factor of one hundred larger than in the previous case, due to the factor ω^{-2} in the argument. Thus neither in case (a) nor in case (b) does any simple pattern emerge. Since $\hbar\omega \ll 1$ a.u., we may take the TDCO independent of energy, thus equation (1) becomes

$$\begin{aligned} \frac{d^3\sigma(L)}{d\Omega_1 d\Omega_2 d\varepsilon} &\approx \frac{d^3\sigma(0)}{d\Omega_1 d\Omega_2 d\varepsilon} \sum_{n=-\infty}^{\infty} J_n^2(\lambda_n) \\ &\approx \frac{d^3\sigma(0)}{d\Omega_1 d\Omega_2 d\varepsilon} \quad \text{since} \quad \sum_{n=-\infty}^{\infty} J_n^2(\lambda_n) \approx 1. \end{aligned}$$

Thus a measurement of the TDCS in the presence of a weak laser field is essentially a measurement of a squared Bessel function! Note, however, by choosing λ_0 such that $J_0(\lambda_0) = 0$, for example near 240° , with $\underline{\varepsilon}_L$ perpendicular to $\hat{\mathbf{K}}_{if}$; we see a situation dominated by single-photon processes, with $\sigma_1(L) \approx 2 \sigma_{-1}(L)$ i.e. by single-photon absorption, and this itself may be interesting experimentally.

Our conclusion that a measurement of the laser assisted cross section, knowing the field-free cross section, reduces

to a measurement of $J_n^2(x)$ is not restricted to ionisation, but is quite general, and applies equally to excitation. However, it does not hold if our assumptions of a weak field, low-frequency photons, single mode homogeneous laser, off-resonance, and slowly varying cross section, are not satisfied. Clearly theory and experiment should concentrate on investigating the phenomena when one or more of these assumptions do not hold.

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CHAPTER 7

CONDITIONS FOR THE OBSERVATION OF THE PREDICTED
SPLITTING OF THE LASER ASSISTED ELECTRON IMPACT
TRIPLE DIFFERENTIAL CROSS SECTION OF ATOMS

INTRODUCTION

The presence of a strong electromagnetic field ("laser") changes significantly the conditions of the electron-atom scattering processes. In fact the photons of the laser, exchanging energy and momentum can play the role of a third body, opening new channels and allowing the observation of electron-atom scattering parameters which would not otherwise be observable. Many of the processes studied, such as elastic and inelastic potential scattering,¹ electron impact ionisation,^{2,3} x-ray ionisation,^{4,5} Compton effect^{5,6} etc. lead to, in the first order treatment and when the laser is treated in dipole approximation, a cross section given by a product of a squared Bessel function (carrying all the information about the laser field) times the field free cross section calculated at a shifted final energy that takes into account the number of photons absorbed (see Eq.(1) below). The modulating effect of the oscillating squared Bessel function as a function of the scattering angles gives a significant modification of the angular distribution of the free particle in the final state. This effect is particularly prominent in electron impact ion-

isation of atoms. Recently Cavaliere et al² for hydrogen atoms and Zarccone et al³ for helium atoms have evaluated the triple differential cross section (TDC) in the first Born approximation (FBA) for different laser polarisations. In both cases the way that the shape of the angular distribution of the ejected electron is changed is shown to depend strongly both on the parameters characterising the laser (polarisation, intensity, frequency) and those of the electron-atom scattering process (incident and final electron energy, scattering angle). Moreover, the contribution to the cross section given by the process where no photons are absorbed exceeds almost everywhere the contribution given by process with $n \neq 0$. More recently Zangara et al⁷ have reinvestigated the ionisation problem using a more realistic laser model. For spatial or temporal inhomogeneity⁸ and for a multimode⁹ laser the cross section preserves the factorised structure of the homogeneous case. However, the squared Bessel function is replaced by a more complicated expression. For the cases considered they show that neither the inhomogeneities nor the multimode structure cancel or drastically alter the predictions made on the basis of the homogeneous, single mode laser. In this chapter we give a complete analysis of the dependence of the shape and the intensity of the TDC on the different parameters involved in the collision process for the case of a single mode homogeneous laser and also for the inhomogeneous and the multimode case. In particular, we will emphasize situations where the TDC for $n \neq 0$ photons dominate the zero photon one. This work is intended to give a useful guide to the choice of the collision and laser parameters involved in setting up a possible experiment.

HOMOGENEOUS, SINGLE MODE LASER

The theory of laser assisted electron impact ionisation with a single mode homogeneous laser has been discussed by different authors.^{10,11} Here we give only the main assumptions and the limitations assumed in the derivation of their result.

The laser field is treated classically as single mode and homogeneous. The field is taken in the dipole approximation. Further the effect of the laser on the atomic states is neglected, so that the laser electric field ϵ_L is assumed considerably smaller than the characteristic interatomic field $\epsilon_{at} = Z^3 e/a_0^2 = 5 \times 10^9$ V/cm, and in addition it is supposed off-resonance with any atomic transition frequency ω_{ij} . For the initial and final fast electron wavefunctions they assume laser modulated plane waves and for the slow ejected electron laser modulated continuum atomic states. The latter states are approximate solutions with positive energy of the Schroedinger equation of an atom in a laser field. This approximation is valid when the momentum provided to the free particle by the laser $p_L = \frac{e}{c} A(t)$ is smaller than the particle's own momentum $p = \hbar k$, i.e. when $\delta = \epsilon_L/\omega k \ll 1$.

Within these approximations, the FBA to the TDC may be written as

$$\frac{d^3\sigma(L)}{d\Omega_1 d\Omega_2 d\epsilon} = \sum_{n=-\infty}^{\infty} J_n^2(\lambda_n) \frac{d^3\sigma(0)}{d\Omega_1 d\Omega_2 d\epsilon} = \sum_{n=-\infty}^{\infty} \sigma_n(L) \quad (1)$$

where the left-hand side is the TDC with the laser on and the right-hand side is a sum over the number of photons absorbed ($n < 0$) or emitted ($n > 0$) of the field-free cross section (TDCO) at energy

$$k_f^2 = k_i^2 - \kappa^2 + E + n\hbar\omega, \quad (2)$$

times a squared Bessel function of argument

$$\lambda_n = \frac{1}{\omega^2} (\underline{k}_{if} - \underline{\kappa}) \cdot \underline{\epsilon}_L. \quad (3)$$

In Eq. (2)-(3) $\underline{k}_{if} = \underline{k}_i - \underline{k}_f$ is the momentum transfer, $\underline{\kappa}$ is the slow ejected electron's momentum, E is the atomic ionisation potential, and ϵ_L is the laser electric field intensity. As shown in our previous paper³ the TDCO is a slowly varying function of the final energy. For processes involving a small number of photons, it can be regarded as a constant over a wide range of n . All the features of the cross section due to the laser field are buried in the Bessel functions. Moreover, for H and He, the TDCO presents the well known two-peak structure³ showing that the maximum ionisation probability is given when the ejected electron leaves the atom along the directions $\pm \underline{k}_{if}$ and is zero in the direction perpendicular to it. Due to the factorised structure of the cross section, the TDC will also be zero in the direction perpendicular to \underline{k}_{if} ; consequently any change in the shape of the TDC can occur only in a small angular range around the directions $\pm \underline{k}_{if}$. For this reason we can restrict our analysis by fixing the ejected

electron angle at $\theta_k = \hat{k} \cdot \hat{k}_{if} = 0$. The Bessel function is an oscillating function with the different low-order zeros ordered in the following way⁽¹²⁾

$$\lambda_{n \neq 0}^I = 0, \quad \lambda_0^I = 2.405, \quad \lambda_1^{II} = 3.832$$

(4)

$$\lambda_2^{II} = 5.136 \quad \lambda_0^{II} = 5.520 \quad \lambda_3^{II} = 6.382$$

etc., where the upper index is the order of the zero and the lower is n .

The change in the structure of the TDC and the relative importance of contributions from different numbers of photons n depends strongly on the particular range of variation of the Bessel function argument λ_n Eq.(3). For very small values of λ ($\lambda \approx 0$) the main contribution to the TDC comes, of course, from the $n = 0$ process, since $\lambda_{n \neq 0}^I = 0$. Increasing λ the $n = 0$ process loses importance, so that when $\lambda \approx \lambda_0^I$, the $n = 1$ process is more important for by $\lambda \approx 1.45$ we have

$$J_1(1.45) \approx J_0(1.45). \quad (5)$$

For values of $\lambda \gg 1$ the situation becomes more complicated, and since the zeros of the Bessel function become much closer we find a more complex and modulated structure for the TDC.

The variation of λ as a function of the electron ejected angle depends strongly on the collision parameters and on the laser parameters. For example, in our previous calculations for Helium⁽³⁾ with the following parameters:

$$E_i = 256.5 \text{ eV}, \quad E_{eJ} = 3 \text{ eV}, \quad \theta_s = 4^\circ$$

$$\hbar \omega = 1.17 \text{ eV} \quad \text{and} \quad \epsilon_L = 5 \cdot 10^7 \text{ V/cm} \quad \text{we have}$$

we have

$$|\lambda| \approx (0.4, 4.4) \quad \text{for} \quad \underline{\epsilon}_L \parallel \underline{k}_{if}$$

and

(5)

$$|\lambda| \approx (0.0, 2.4) \quad \text{for} \quad \underline{\epsilon}_L \perp \underline{k}_{if}$$

while, in the case considered by Cavaliere et al² for hydrogen with $E_i = 150 \text{ eV}$, $E_{eJ} = 5 \text{ eV}$, $\theta_s = 5^\circ$, $\omega = 1.17$ and $\epsilon_L = 10^6 \text{ V/cm}$ we have

$$|\lambda| \approx (0.008, 0.1) \quad \text{for} \quad \underline{\epsilon}_L \parallel \underline{k}_{if}$$

and

(6a')

$$|\lambda| \approx (0.0, 0.064) \quad \text{for} \quad \underline{\epsilon}_L \perp \underline{k}_{if} .$$

The difference in the range of λ in Eq.(6) and (6a) gives a complete explanation of the different shape of the TDC for the two polarisations and/or the two examples considered. In fact in Cavaliere et al's² example $\lambda \ll 1$ and the only Bessel function zero contributing to the change in the structure of the TDC is the $\lambda = 0$ zero, the $n = 1$ photon process being much smaller than the $n = 0$ one.

One interesting experimental situation is when processes with $n \neq 0$ dominate the $n = 0$ process. In this context it is useful to investigate for which values of the laser parameters (intensity, frequency and polarisation) we can have $J_0(\lambda) = 0$ and some $J_{n \neq 0}(\lambda) \neq 0$. For reasons discussed above, we confine our analysis to the region around $\theta_k = 0$, and we use the collision parameters given above (Eq.5) for electron-Helium ionisation.

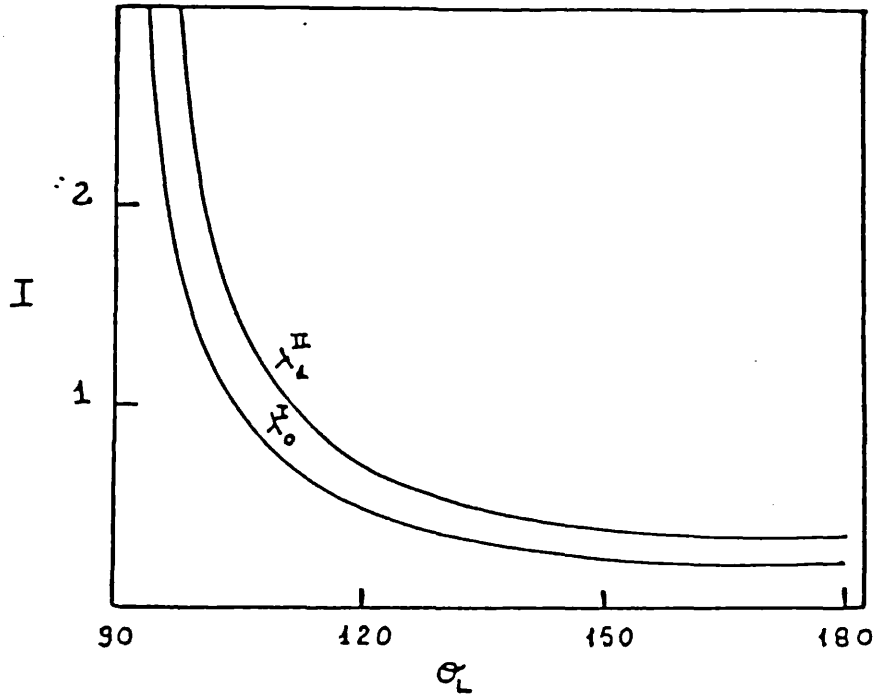


FIGURE 1. The curves relate the values of the laser electric field intensity I [10^9 V cm $^{-1}$] and laser polarisation θ_L (measured from the momentum transfer \vec{k}_L for which $\lambda = \lambda_0^I$ the first zero of J_1 or $\lambda = \lambda_1^{II}$ the second zero of J_1 , in the simple laser model, with the geometry specified in the text).

In Figure 1 we plot the value of the electric field laser intensity and the laser polarisation angle $\theta_L = \hat{\underline{e}}_L \cdot \hat{\underline{k}}_{if}$ which give the zeros $\lambda_0^I = 2.405$ and $\lambda_1^{II} = 3.832$ respectively. We see that for not too large a given laser intensity it is possible to select a laser polarisation corresponding to the $\lambda = \lambda_0^I$ so that the probability of a zero photon process with the slow electron ejected along \vec{k}_{if} is zero. Accordingly, the probability for the $n = 1$ process will be $J_1^2(\lambda_0^I) \approx 0.27$, which implies a large cross section.

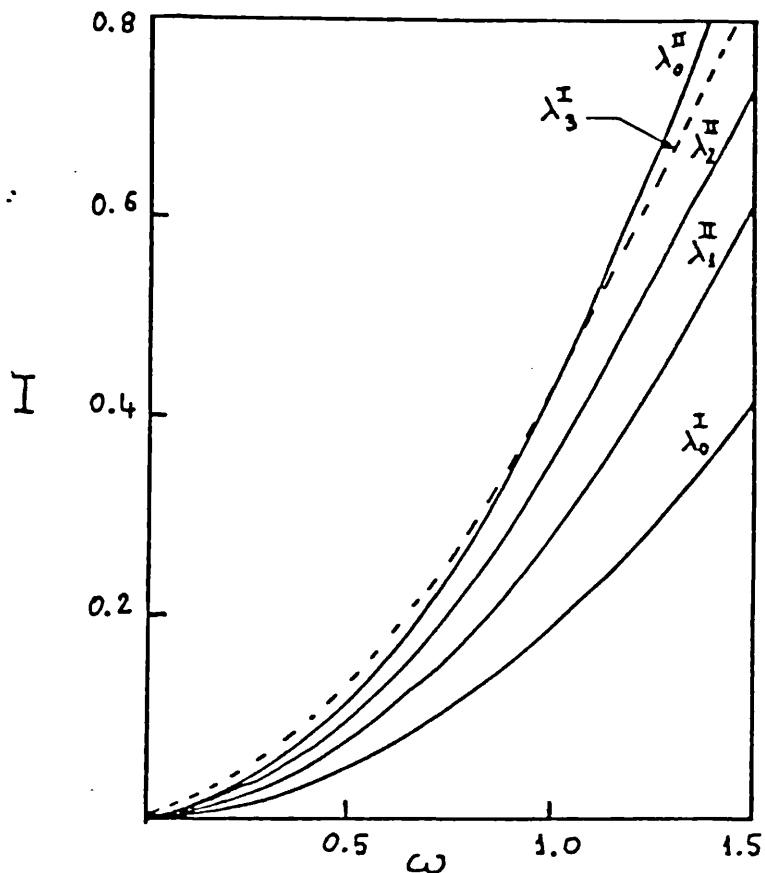


FIGURE 2. As Figure 1, but now relating the intensity to the frequency ω (eV) for λ_0^I , λ_1^{II} , λ_2^{II} , λ_3^{II} , λ_0^{II} for $\theta_L = 0$.

It is important to note that, due to the small difference between the two curves, this effect can be observed only with a laser with a field intensity determined to better than 10%. We see also that the most favourable situation for this geometry is at low intensity for $\theta_L = 0$, while for $\theta_L = 90^\circ$ the intensity must be very large, and the model is inapplicable.

In Figure 2 we instead show for the more convenient polarisation $\theta_L = 0$, the laser intensity and frequency for several values of λ . Again, for a given value of frequency

we can choose a value of the intensity corresponding to the curve with λ_0^I so that $J_0^I = 0$. We note that for low frequency, or high frequency and high intensity we have a situation where it is not possible to select a particular zero of $J_0(\lambda)$, because this would require a laser with an unrealistic homogeneity (for the low frequency case) or an unrealistic resolution for the high frequency and intensity case.

In the next sections we will examine in more detail the influences of the laser properties (inhomogeneity and multimode structure) on the intensity and shape of the TDC.

INHOMOGENEOUS SINGLE MODE LASER

A real laser normally has a strong macroscopic inhomogeneity associated with the decision to focus the radiation in order to increase the field in the collision chamber. Consequently different target atoms will be found in regions of different field strength and the measured differential cross sections may be affected by such an inhomogeneity.

The theory of particle-atom collisions in the presence of an inhomogeneous laser has been developed in recent years.^{7,8} The inhomogeneity may be both temporal and spatial, and in both cases the factorised structure of the cross section of Eq.(1) is still valid. However, the squared Bessel function is now replaced by a more complicated function $F^2(n,\lambda)$.^{7,8}

The basic assumptions of the theory are (i) the laser is supposed single mode, with an amplitude depending on the spatial or temporal coordinates; (ii) the scale of variation of the field amplitude is assumed very small compared with the typical lengths defined by the collision

event. In this scheme the cross section is calculated following the same procedures used for the single mode homogeneous laser with constant amplitude, and then an average is performed over all the relevant space-time points where or when the particular collision occurs.

The resulting averaged cross section depends on a parameter γ given by the ratio of the electron beam dimension s to the laser beam dimension α , for a spatially inhomogeneous laser, and to the ratio of the detector response time τ_0 to the duration of the laser pulse τ_L for a temporarily inhomogeneous laser. For the spatially inhomogeneous case with $\gamma \gg 1$ we are in a situation where few of the scattering events are in the laser field. With temporal inhomogeneity where the detector is not able to respond separately to two collisions within τ_0 , in which time ($\tau_0 \gg \tau_i$) the laser intensity may have changed appreciably. In the usual Gaussian model of inhomogeneity, the average electric field involved in the collision is smaller than that for a homogeneous laser, so many-photon events should be less important.

When $\gamma \ll 1$ the laser field intensity for any collision in the scattering chamber may be considered constant during the collision, and the inhomogeneity of the laser does not affect the process.

The most difficult case to discuss (as always) is when the scales are comparable, i.e. $\gamma \sim 1$, and we examine this below, first quoting the results of the earlier papers.^{7,8}

For a spatially inhomogeneous laser with a laser field intensity distribution characterised by a Gaussian

$$\phi(x,y) = \exp\{-1/2 \alpha(x^2 + y^2)\} \quad (7)$$

the squared Bessel function J^2 is replaced in the factorised TDC, Eq.(1) by⁷

$$F_s^2(n, \lambda) = \sum_{k=0}^{\infty} G_v(\lambda_n, k) \frac{1}{(v+k)} \{1 - \exp[-(v+k)]\} \quad (8)$$

where $v = |n|$ and

$$G_v(\lambda_n, k) = \frac{(-1)^k (2v+2k)! (1/2\lambda_n)^{2(v+k)}}{k! [(v+k)!]^2 (2v+k)!} \quad (9)$$

For a temporal inhomogeneity with a field intensity given by a Gaussian function

$$f(t) = \exp\{-1/2 \frac{\pi}{\tau_L} t^2\} \quad (10)$$

We have similarly⁷

$$F_t^2(n, \lambda) = \sum_{k=0}^{\infty} G_v(\lambda_n, k) \left\{ \frac{1}{(v+k)^{1/2}} \operatorname{erf}\{0.5[\pi(v+k)]^{1/2}\} \right\} \quad (11)$$

in terms of the error function.

The behaviour of these complex expressions is far from obvious, but the behaviour is readily computable for λ_n not too large ($\lambda_n \leq 10$).

In Figures 3 and 4 we compare the behaviour of $F_s^2(n, \lambda)$ and $F_t^2(n, \lambda)$ with J_n^2 as a function of λ for the cases $n = 0$ and $n = 1$ respectively. As expected the inhomogeneity decreases the contribution from the $n = 1$ process in favour of the $n = 0$ process, especially for small values of λ . For both the temporal and spatial inhomogeneity the curves keep the oscillatory behaviour of the Bessel function, even though they are smoother and the zeros are dumped out. Moreover, the distance between successive minima is larger than the corresponding distance between two successive zeros of the homogeneous case. From this fact we

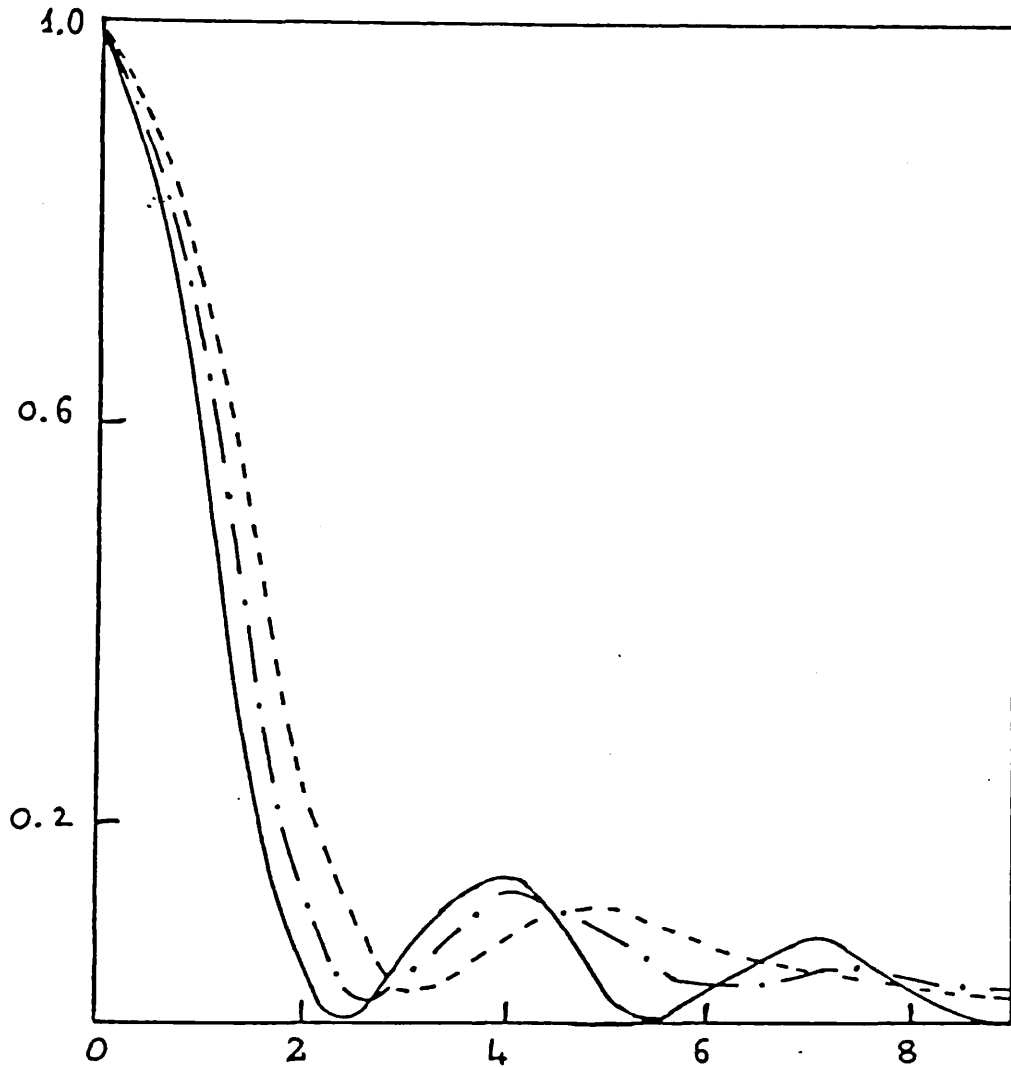


FIGURE 3. — $J_n^2(\lambda)$; --- $F_s^2(n, \lambda)$; -.- $F_t^2(n, \lambda)$
compared for $0 \leq \lambda \leq 8$ and $n = 0$.

can deduce that the TDC will present a laser modulated behaviour, but with a less rich and pronounced structure. Under conditions when a splitting of the forward and backward peak is predicted ($n = 1$ process) the resulting lobes will be rather further apart than in the homogeneous case.

In the homogeneous case it is always possible to choose parameters of the scattering process and/or of the laser, so

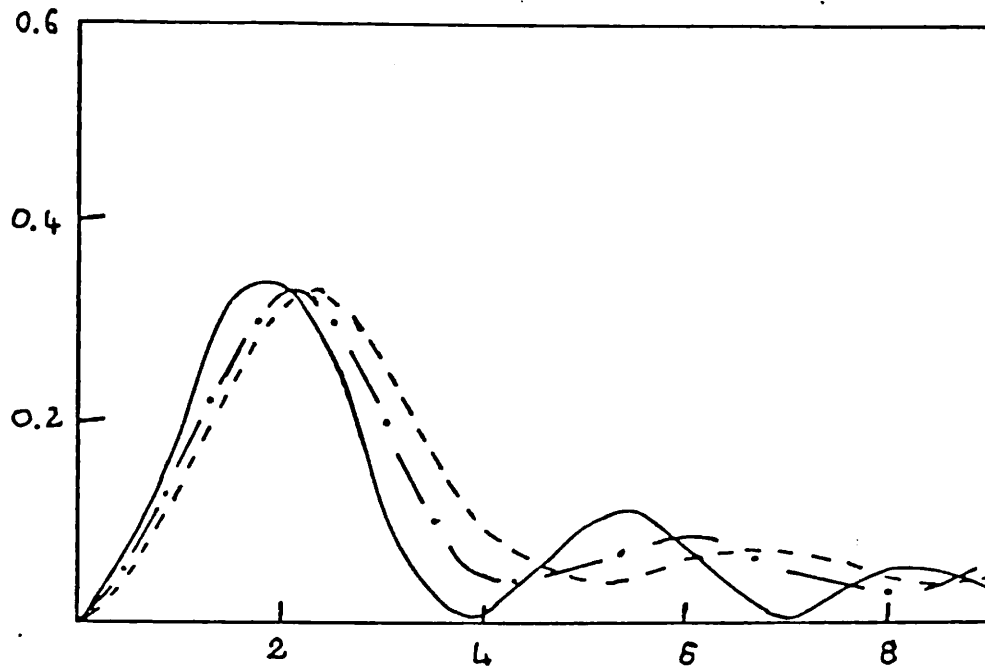


FIGURE 4. As Figure 3 but $n = 1$.

that for a particular ejected electron angle the $n \neq 0$ photon processes are more important than the $n = 0$ process. In the present case this will occur for slightly larger values of λ than in the homogeneous case. In conclusion we can say that for small values of λ the behaviour of the inhomogeneous functions F_s^2 and $F_t^2(n)$ are very similar to that of the squared Bessel function. For increasing λ F_s^2 and F_t^2 rapidly get out of phase with J_n^2 , showing a phase advance and strong damping. Thus the behaviour of the system with inhomogeneities is increasingly different from that of the homogeneous system. However, these inhomogeneities will only be important at low frequencies and/or high intensities as we have seen in section 2.

MULTIMODE LASER

In this section we will examine the modifications expected

in the laser assisted electron impact ionisation TDC if the laser is multimode. Following Zoller,⁹ the radiation of a multimode laser with a large number of uncorrelated modes may be represented by a chaotic field. Using the Gaussian properties and the first-order correlation function, and allowing the laser frequency band-width $\Delta\omega$ to become vanishingly small, it is possible again to get a factorised TDC of the same form as the homogeneous single mode case, with

$$F_{MM}^2(n,\lambda) = \exp\{-1/2 \lambda_n^2\} I_n(1/2 \lambda_n^2) \quad (12)$$

where I_n is the modified Bessel function (Abramowitz and Stegun, ref.(12) Eq.9.6.3). A comparison of the function $F_{MM}^2(n,\lambda)$ with the square Bessel function is made in Figures 5 and 6 for $n = 0$ and $n = 1$ respectively. The function $F_{MM}^2(n,\lambda)$ is monotonic, decreasing with increasing λ and shows no oscillations or zeros. Consequently, we expect the same behaviour as in the single mode homogeneous case for very small values of λ , $\lambda \ll 1$. Moreover, in this case it is not possible to have a situation where the $n = 1$ contribution to the cross section is larger than the $n = 0$ one. The only zero present is for $\lambda = 0$ and $n \neq 0$, so again for a laser polarisation angle $\theta_L = 90$ we have a splitting of the forward and backward peak, but it is not possible to have such a splitting for any other polarisation.

CONCLUSION

More realistic laser models than used in the derivation of the Kroll-Watson result, which allow for spatial or temporal inhomogeneity or multimode effects, do not change the shape

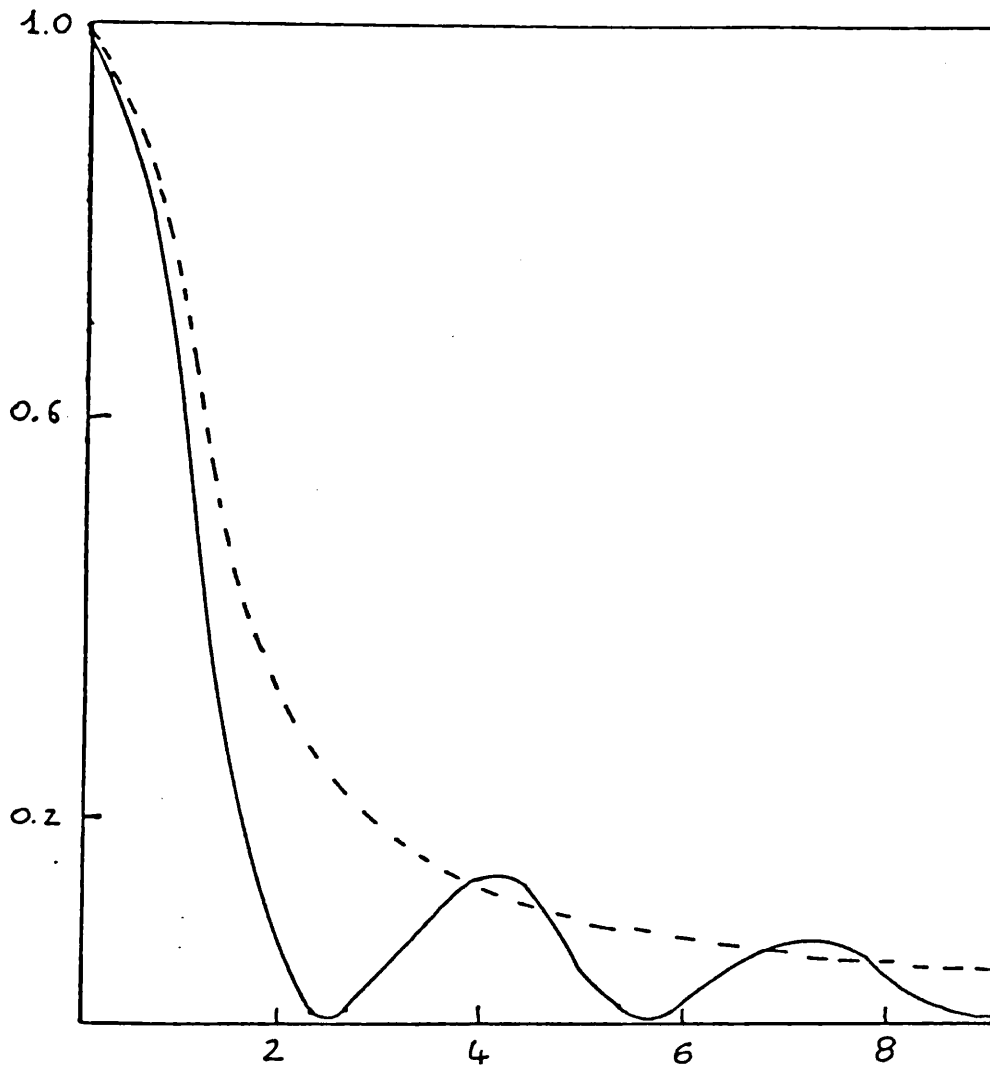


FIGURE 5. — $J_n^2(\lambda)$; --- $F_{MM}^2(n, \lambda)$ for $n = 0$.

nor strongly affect the magnitude of the predicted TDC obtained using a simple single mode homogeneous laser model, provided the laser parameters (frequency, intensity and polarisation) and the collision parameters (incident energy scattering angle, final energy and target atom used) are chosen in order that the value of the argument of the relevant Bessel function is restricted to $\lambda \ll 1$.

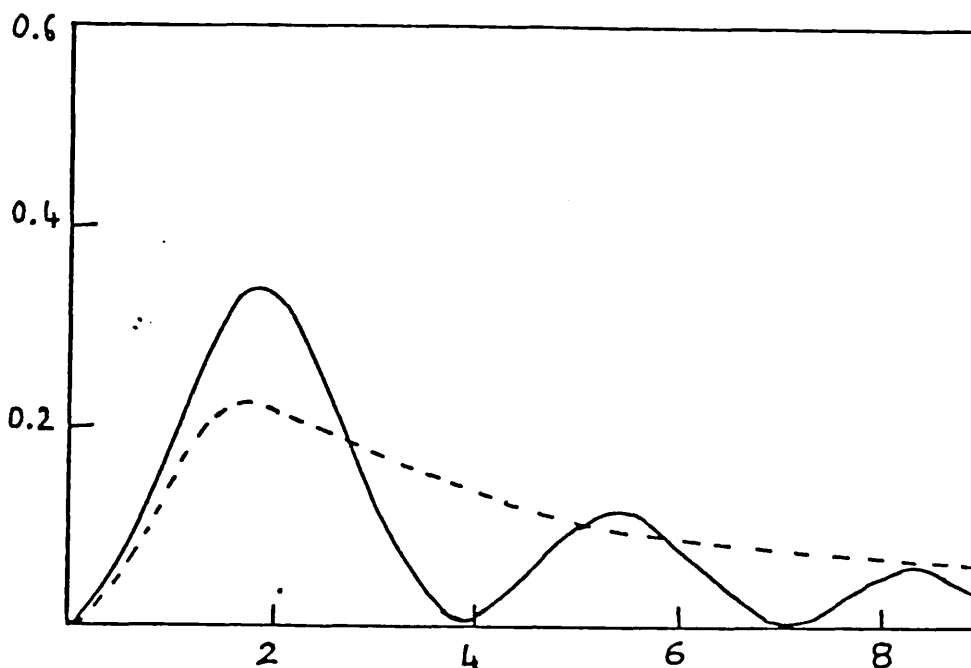


FIGURE 6. As Figure 5 but $n = 1$.

For the more interesting experimental situation when for $\lambda \gg 1$ processes with $n = 1$ are predicted to be more likely than that with $n = 0$ in the simple model, the more realistic laser models predict an attenuation of the effect, and in some models the predicted splitting is wiped out, except in the case of when the laser polarisation is perpendicular to the momentum transfer.

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