A PHASE SHIFT METHOD

OF

MEASURING MICROWAVE ABSORPTION IN LIQUIDS

WITH EXPERIMENTAL RESULTS

A Thesis submitted to the University of London
for the Degree of Ph.D. in Physics
by
Adeline M. Southon

ProQuest Number: 10097990

ProQuest.

A PHASE SHIFT METHOD

OF

MEASURING MICROWAVE ABSORPTION IN LIQUIDS
WITH EXPERIMENTAL RESULTS

ABSTRACT

The method evolved from an investigation of
transmission line methods of measuring the absorption
of electromagnetic radiations in liquids.   An equation
was derived relating the phase angle of the voltage
reflection coefficient to the length of the liquid
filled line beyond a short circuit.

$$\tan \phi = \frac{2\beta_{o}(\alpha \sinh 2\alpha l + \beta \sin 2\beta l)}{(\beta_{o}^{2} - \alpha^{2} - \beta^{2})\cosh 2\alpha l - (\beta_{o}^{2} + \alpha^{2} + \beta^{2})\cos 2\beta l}$$

The experimental results show that this equation
describes exactly the behaviour of the phase angle as
the depth of liquid filling the transmission line is
varied.   The equation has been shown to be true for
a wide range of liquids:-

(a)   Non-absorbing liquids, i.e. Benzole.

(b)   Liquids with a low absorption, i.e.
        Chlorobenzene and Cyclohexanol.

(c)   Dilute solutions of polar substances in
        non-polar solvents, i.e. Chlorobenzene in
        Benzole.

(d)   Liquids with high absorptions, i.e. Water
        and Alcohol.

(e)   Electrolytic solutions, i.e. Sodium Chloride
        in water.

Values of the phase constants and attenuation coefficients
for these liquids have been deduced from the graphs
showing phase angle plotted against liquid depth, with
an accuracy that compares favourably with other established
methods.

The method has been applied to an investigation of
the absorption of microwaves (λ = 9.1 cms) by ox blood
and serum at room temperature and the inner body
temperature of 37°C, and these results have been com-
pared with those obtained for distilled water at these
two temperatures.

The variation of the phase constant and attenuation
coefficient of sodium chloride solutions with concentration
has also been investigated, from 0.15N (physiological
saline) up to 2N.

This method, which depends upon measurement of phase shift alone for the determination of α and β, requires a very much simpler form of apparatus than other methods.

The method is best suited to liquids with low values of α and high values of β, and a modification to the apparatus and experimental method has been suggested which should enable values to be obtained more rapidly. This modification has not so far been made, since the accuracy obtainable with this phase shift method is inferior to that of the null techniques recently introduced into dielectric measurements.

CONTENTS

Chapter 2

ACKNOWLEDGMENTS

List of symbols

Symbols with the suffix o have been used to refer to quantities measured in air.

α The attenuation coefficient measured in neper cms^{-1}.

β The phase constant measured in radians cms^{-1}.

P   The propagation constant.

Z_o The characteristic impedance of the air filled line.

Z_o' The characteristic impedance of the dielectric filled portion of the line.

k   The voltage reflection coefficient of the termination.

K   The ratio of the amplitude of the reflected to that of the incident voltage.

ϕ The phase change that occurs on reflection.

R   The resistance per unit length of the line.

L   The inductance per unit length of the line.

G   The conductance per unit length of the line.

C   The capacitance per unit length of the line.

w   The angular frequency.

l   The liquid depth.

Δ The phase shift.

L   The length of the transmission line.

λ The wavelength.

$\bar{\varepsilon}$ The complex dielectric constant.

ε' The real part of the dielectric constant.

ε'' The imaginary part of the dielectric constant.

ε_s The static dielectric constant.

ε_∞ The value ε' approaches at the highest
        frequencies.

INTRODUCTION

A general description of dielectric absorption
in the microwave region.

When a dielectric material is subjected to an
alternating field E, there is, in general a phase
difference between the dielectric displacement vector
D and the electric field vector E.   This phase
difference δ can be described by the introduction of
a complex dielectric constant $\bar{\varepsilon}$, where

$$\bar{\varepsilon} = \varepsilon' - j\varepsilon'',$$

δ is known as the loss angle of the dielectric and

$$\tan\delta = \frac{\varepsilon''}{\varepsilon'}$$

where $\tan\delta$ is the dissipation factor.   The energy
dissipated in the dielectric is directly proportional
to ε'', so that ε'' is known as the loss factor.

The behaviour of a dielectric material depends
upon the frequency of the alternating field, the
chemical composition of the dielectric and its
temperature.   There are two distinct types of
dielectric behaviour depending upon whether the phase

difference between D and E is a measurable quantity
or not.   If a phase difference exists between D and
E there is a dissipation of energy in the dielectric
and an absorption of electro-magnetic waves.   If
however, the phase difference is zero, $\varepsilon'' = 0$, and
the static relationship,

$$D = \varepsilon E$$

is applicable to an alternating field, under these
conditions no energy is absorbed from the electro-
magnetic field.

The phase difference may be due to one or more
of three causes.

(a)   Electrical conductivity.

(b)   Relaxation effects, due to the permanent
dipoles in the dielectric being unable to
follow the rapid changes in the direction
of the field.

(c)   Resonance effects, due to the vibrations
of atoms, ions or electrons.

Maxwell's electromagnetic equations brought out
the close relationship between the static dielectric
constant and the refractive index of the medium.
The refractive index n at a certain frequency is
defined by

$$v = \frac{c}{n}$$

where v is the phase velocity of the electromagnetic
wave at that frequency and c the velocity in vacuum.
If no moving charges are present, Maxwell showed that

$$v = \frac{c}{\sqrt{\varepsilon\mu}}$$

where μ is the magnetic permeability of the medium,
and does not differ appreciably from unity for the
materials under consideration.   Hence

$$\varepsilon = n^2$$

This equation is also valid when the movements of the
charges are slow compared with the velocity c, so that
it applies when low frequency electromagnetic waves
are propagated in the dielectric.   This relationship
is confirmed experimentally, for it is found that for
wavelengths larger than a certain critical value, the
dielectric constant is independent of frequency and
equal to the static value of the dielectric constant ε_s,
which is equal to the square of the refractive index.
This critical wavelength depends upon the chemical
composition of the dielectric and on its temperature.
For example, for liquids possessing permanent dipoles
such as water and alcohol it is about 15 and 40 cms.
respectively at 20°C, whereas for non-polar dielectrics

it is generally in the infra-red when dielectric loss
due to resonance effects begins.

In frequency regions where the complex dielectric
constant differs from the static value, the refractive
index is also found to be dependent on the frequency.
This is expressed by the introduction of a complex
refractive index $\bar{n}$ where

$$\bar{n} = n' - jn''$$

which, since $n = n'$, is generally written in the form,

$$\bar{n} = n(I - jk)$$

where k is the absorption index.
ε' and ε'' are related to n and k by the equations:-

$$\varepsilon' = n^2(I - k^2)$$

and $$\varepsilon'' = 2n^2k$$

The present work deals with measurements at a
frequency of 3.3×10^9 sec^{-1}, so that the effect of
the phase difference in other regions of the electro-
magnetic spectrum is not discussed. This frequency
lies in the microwave region which includes frequencies
from 10^7 to 10^{11} sec^{-1}. In the microwave region the
dielectric loss is mainly due to the relaxation effect

of the permanent dipoles, whereas at optical frequencies
it is due to resonance effects.   (The distinction
between these two effects is rather arbitrary, but they
are usually separated.)

The relaxation effect, when the permanent dipoles
of the dielectric are unable to follow the very rapid
alterations in the direction of the field E, can be
described by the introduction of the dielectric
relaxation time τ.   The time which permanent dipoles
require to reach an equilibrium distribution after the
application of a static external field E is somewhere
between 10^{-6} and 10^{-13} secs.   The time depends upon the
temperature and structure of the dielectric.   It can
be shown that the complex dielectric constant is
related to τ by the relation,

$$\overline{\varepsilon} \;=\; \varepsilon_{\infty} \;+\; \frac{\varepsilon_s - \varepsilon_{\infty}}{1 + jw\tau}$$

where ε_s is the static dielectric constant,

   ε_{∞} is the value ε' approaches at the highest
        frequencies,

and $\overline{\varepsilon}$ is the complex dielectric constant at the
        angular frequency w.

Dividing $\overline{\varepsilon}$ into its real and imaginary parts,

$$\varepsilon' \;=\; \varepsilon_{\infty} \;+\; \frac{\varepsilon_s - \varepsilon_{\infty}}{1 + w^2\tau^2}$$
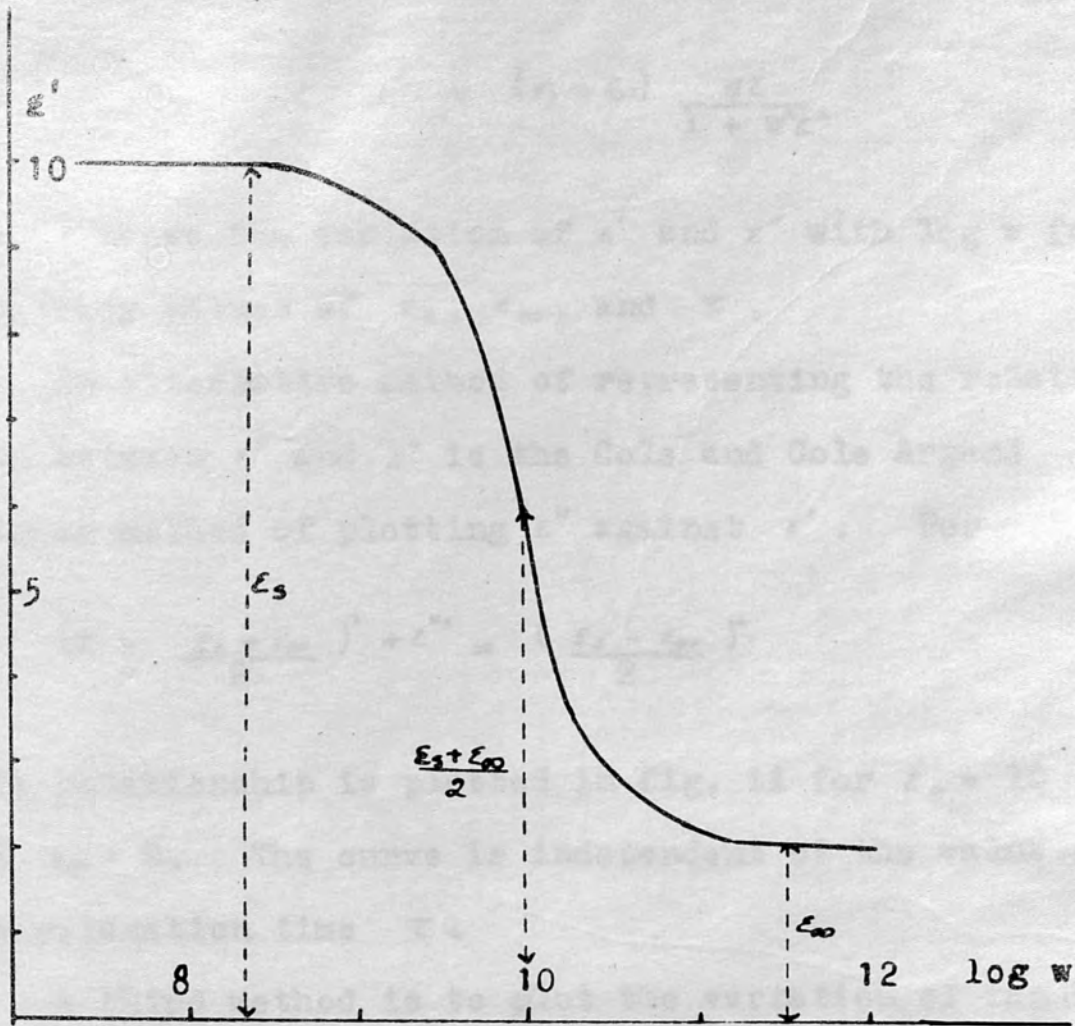
$$\varepsilon_s = 10, \quad \varepsilon_\infty = 2, \quad \tau = 10^{-10} \text{ sec}$$
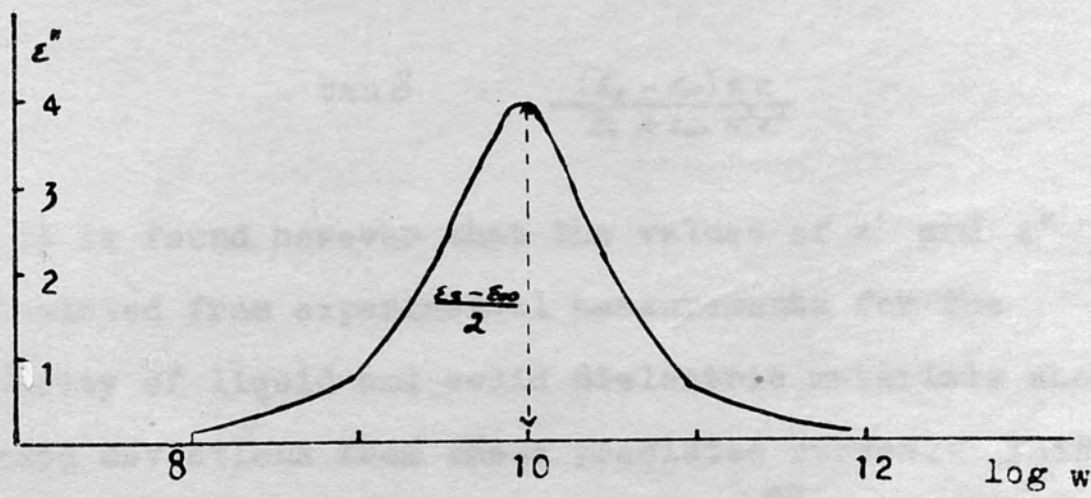
Fig. i   The variation of ε' and ε'' with $\log_{10} w$.

and
$$\varepsilon'' = (\varepsilon_s - \varepsilon_\infty) \frac{w\tau}{1 + w^2\tau^2}$$

Fig. i shows the variation of ε' and ε'' with log w for arbitrary values of ε_s, ε_∞, and τ.

An alternative method of representing the relationship between ε' and ε'' is the Cole and Cole Argand diagram method of plotting ε'' against ε'. For

$$\left(\varepsilon' - \frac{\varepsilon_s + \varepsilon_\infty}{2}\right)^2 + \varepsilon''^2 = \left(\frac{\varepsilon_s - \varepsilon_\infty}{2}\right)^2$$

This relationship is plotted in fig. ii for $\varepsilon_s = 10$ and $\varepsilon_\infty = 2$. The curve is independent of the value of the relaxation time τ.

A third method is to plot the variation of $\tan\delta$ against log w as is shown in fig. iii for the same arbitrary values of ε_s, ε_∞ and τ.

$$\tan\delta = \frac{(\varepsilon_s - \varepsilon_\infty) w\tau}{\varepsilon_s + \varepsilon_\infty w^2\tau^2}$$

It is found however that the values of ε' and ε'' calculated from experimental measurements for the majority of liquid and solid dielectric materials show marked deviations from these predicted curves. This indicates that the model used by Debye[*2] in deriving these equations is too simple for general validity.

* Debye uses a relaxation time τ^* related to the τ used above by $\tau = \tau^*(\varepsilon_s + 2)/(\varepsilon_\infty + 2)$
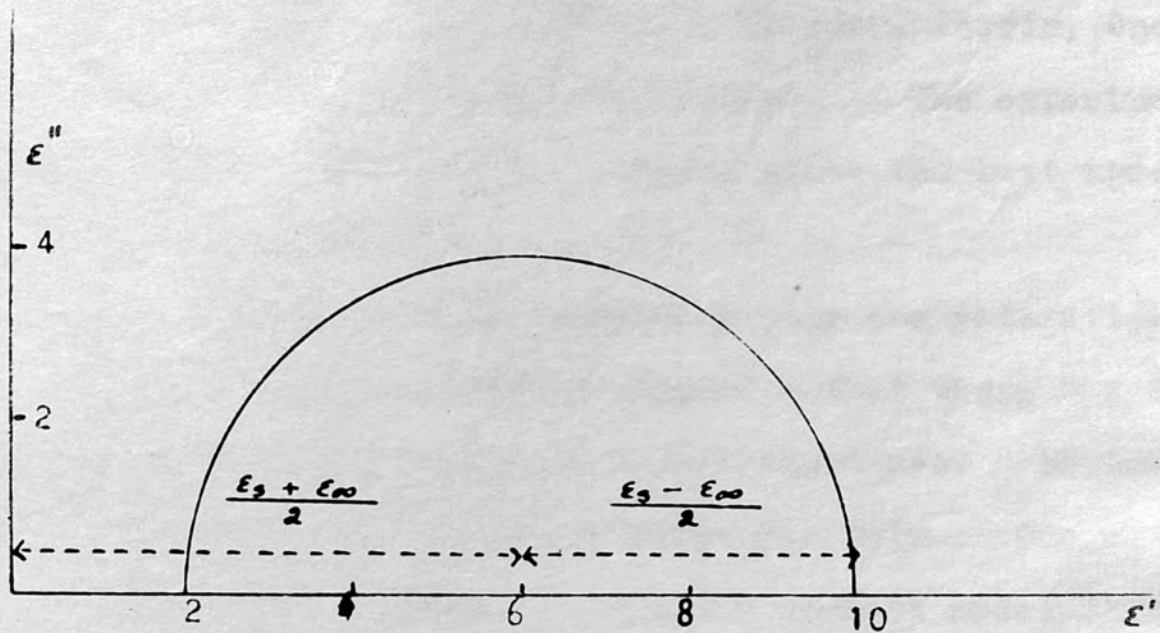
Fig. ii   Cole and Cole diagram.

$$\varepsilon_s = 10, \qquad \varepsilon_\infty = 2, \qquad \tau = 10^{-10} \text{ sec}$$
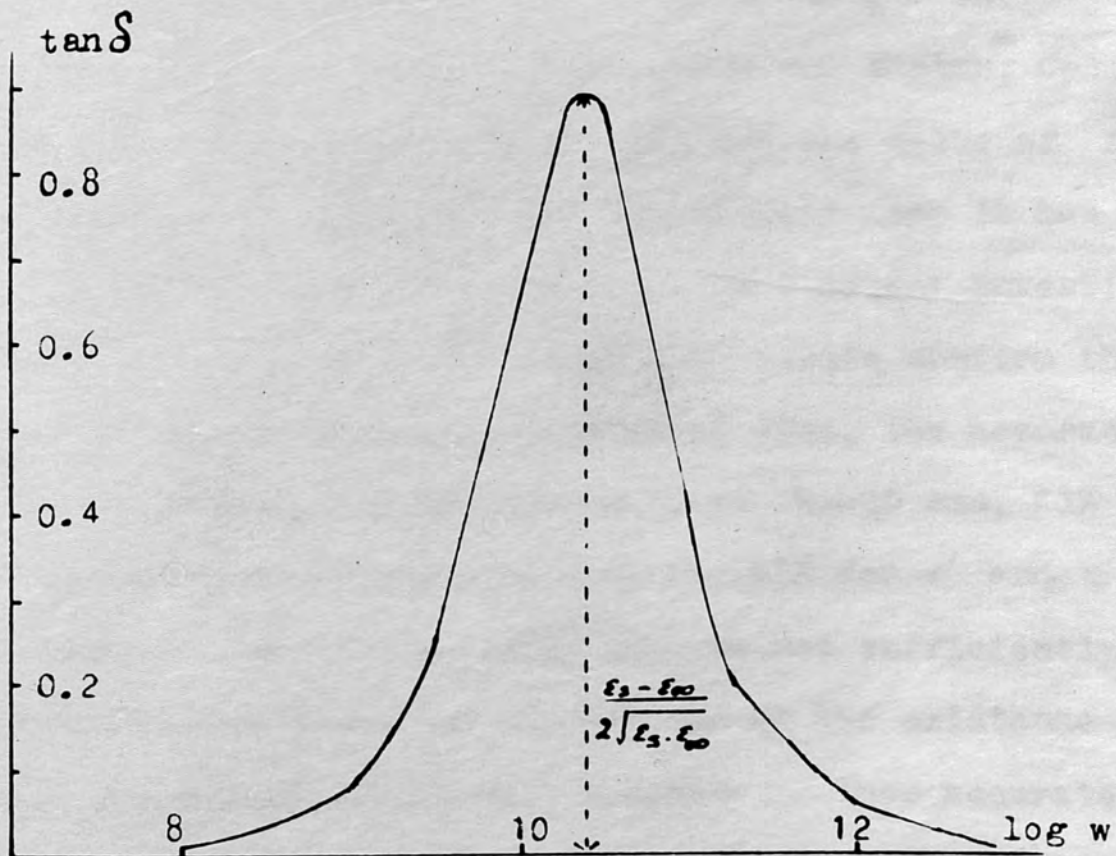
Fig. iii   The variation of $\tan \delta$ with $\log_{10} w$.

Various alternative models and extensions to the general theory have been put forward by Onsager,[3] Perrin,[4] Oncley,[5] Cole and Cole,[1] Kirkwood[6] and others.[7,8] The experimental data is used to see which theory gives the best agreement with experiment.

The simple theory considers only one relaxation time. Experiment however suggests that there may in fact be a distribution of relaxation times. Böttcher,[9] page 363, discusses the evidence for this. One of the difficulties in deciding upon the correct model is that reliable values of ε' and ε'' in this frequency range are difficult to obtain. For instance, water, which has been investigated by many authors using a variety of methods, is claimed by some (Lane and Saxton,[10] Collie, Ritson and Hasted)[11] to possess but one value of τ, while others (Younker and von Hippel)[12] say it has a wide variety of relaxation times. In a recent investigation of water by Cook,[13] although his results confirm the existence of a single relaxation time, the accuracy of his results ($\pm 2\%$ for ε' and ε'' at $\lambda_o = 10$ cms, $\pm 1\%$ for ε', $\pm 2.5\%$ for ε'' at $\lambda_o = 6.48$ cms, and $\pm 1\%$ for ε' and ε'' at $\lambda_o = 3.195$ cms and $\lambda_o = 1.262$ cms) is not sufficiently high to rule out the possibility of the existence of a narrow relaxation time spectrum. Thus accurate measurement of quantities from which ε' and ε'' can be

calculated is of great importance in giving information as to the structure of dielectric materials.

In the microwave region, ε' and ε'' can be found indirectly by transmission line or optical methods. In the transmission line methods $\bar{\varepsilon}$ is derived from the impedance Z or the propagation constant P of the dielectric medium, while in the optical methods it is calculated from measurements of n and k. Standing or travelling waves may be used for measuring any of these quantities. Travelling waves are used for the general optical method. The intensities of the reflected and transmitted beam are measured and n and k calculated from Fresnel's equation. Baz[14] and Kebbel[15] used this method, but in the microwave region boundary problems arise owing to the wavelength being comparable to the dimensions of the sample and standing waves are set up between the transmitter and the receiver causing intensity fluctuations. So it is seen that the optical method is not satisfactory when carried into the microwave region.

The longer wavelength of the microwave region as opposed to optical wavelengths is an advantage if standing waves are used. Drude[16] was the first to use this technique using Lecher lines and his two methods have since been adapted in many variations. Knerr[17] measured n and k for water using both an optical free wave

method and a standing wave method with Lecher lines, covering a range of wavelengths from 20.44 to 6.48 cms. He found the Lecher line method slightly superior to the optical one. It was not however until the electromagnetic field was enclosed in a hollow pipe or coaxial line, so eliminating the effects of stray fields and boundary effects, that the transmission line method superceded the optical one.

Oscillators capable of producing undamped oscillations at these wavelengths were only available by about 1936, and the previous dielectric measurements, with damped electric waves from spark sources, are unreliable. The development of radar during the 1939-45 war brought great advances not only in the design of microwave oscillators, such as the magnetron and the klystron, but in all circuit elements and the techniques required at these frequencies.

The Roberts and von Hippel[18] method of measuring ε' and $\tan\delta$ from measurements on the standing wave pattern set up in a short circuited transmission line or waveguide was developed during the war; and the use of resonant cavities was introduced by Willis Jackson[19] and others for measurements on materials with small loss angles. Redheffer[20] reviews the large number of methods used for dielectric measurements at microwave frequencies

and shows how they are all developed from the same basic equations.

Advances in the design of piston attenuators and phase changers at microwave frequencies have made possible null methods of measurement within the medium. (Buchanan[21] and Branin and Smyth.[22])

In addition to providing evidence as to the structure of dielectric materials, measurements of the absorption of electromagnetic waves by body fluids and tissues are of great importance medically so that the effects on the human body of irradiation by these wavelengths can be known. Recent work on this problem has been carried out by England and Sharples[23,24] and Cook.[13]

Chapter 1

THE THEORY OF THE PHASE ANGLE METHOD

OF ABSORPTION MEASUREMENTS

1.1   Introduction.

This method is based on an equation relating
the phase angle ϕ , due to the presence of a thickness
l of dielectric above a short circuit in a transmission
line, to l .   The variation of ϕ with ℓ is described
and the effect of different liquids on this variation
discussed.

1.2   Derivation of the phase angle equation.

Consider a transmission line filled to a distance
ℓ with a dielectric with an appreciable value of $\propto$.

$$
\begin{array}{|c|c|c|}
\hline
Z_o' & Z_o & \begin{array}{c} V_o \\ I_o \end{array} \\
\hline
\end{array}
$$
$\longleftarrow \ell \longrightarrow$
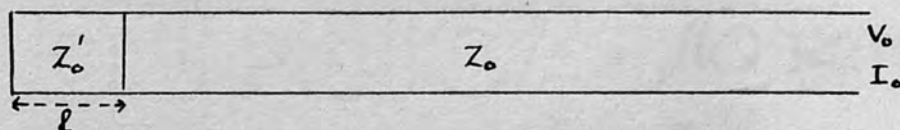
Fig. 1.1

From standard transmission line theory, the equations
describing the voltage and current wave configurations
in the filled part of the line are,

$$ V' = V_o' (e^{Pl} + Ke^{j\phi} e^{-Pl}) $$

and
$$I' = \frac{V_o'}{Z_o'}(e^{Pl} - Ke^{j\phi}e^{-Pl})$$

where P is the propagation constant and $k = Ke^{j\phi}$ is the voltage reflection coefficient of the dielectric. Thus the terminating impedance of the unfilled line, Z_T, or the input impedance of the filled line, Z' input, is given by the expression

$$Z_T = Z' \text{ input} = Z_o'\frac{(e^{Pl} + Ke^{j\phi}e^{-Pl})}{(e^{Pl} - Ke^{j\phi}e^{-Pl})}$$

If the filled line is terminated in a short circuit, $K = 1$ and $\phi = \pi$, and

$$Z' \text{ input} = Z_o'\frac{(e^{Pl} - e^{-Pl})}{(e^{Pl} + e^{-Pl})}$$

$$= Z_o'\tanh Pl \quad \dots\dots\dots\dots\dots\dots(1.1)$$

For the airfilled portion of the line:-

The propagation constant $P_o = j\beta_o$

$$= \sqrt{(R_o + jwL_o)(G_o + jwC_o)}$$

and at V.H.F. $P_o = jw\sqrt{L_oC_o} \quad \dots\dots\dots\dots(1.2)$

The characteristic impedance $Z_o = \sqrt{\frac{R_o + jwL_o}{G_o + jwC_o}}$

and at V.H.F. $Z_o = \sqrt{\frac{L_o}{C_o}} \quad \dots\dots\dots\dots\dots(1.3)$

While for the dielectric filled portion:-

The propagation constant $\quad\quad P = \alpha + j\beta$ ....(1.4)

$$= \sqrt{(R + jwL)(G + jwC)}$$

and at V.H.F. $\quad\quad\quad\quad\quad\quad P = \sqrt{jwL(G + jwC)}$ ..(1.5)

The characteristic impedance $\quad Z'_o = \sqrt{\dfrac{R + jwL}{G + jwC}}$

and at V.H.F. $\quad\quad\quad\quad\quad\quad Z'_o = \sqrt{\dfrac{jwL}{G + jwC}}$ ......(1.6)

Thus from equations (1.2) and (1.5)

$$\frac{P_o}{P} = \frac{jw\sqrt{L_oC_o}}{[jwL(G + jwC)]^{\frac{1}{2}}} \quad ..(1.7)$$

$$= \sqrt{\frac{jwL_oC_o}{L(G + jwC)}} \quad ...(1.7)$$

and from equations (1.3) and (1.6)

$$\frac{Z'_o}{Z_o} = \sqrt{\frac{jwL}{(G + jwC)}\frac{C_o}{L_o}} \quad ...(1.8)$$

$$= \frac{L}{L_o}\sqrt{\frac{jwC_o}{(G + jwC)}\frac{L_o}{L}} \quad ..(1.8)$$

and from equation (1.7) $\quad\quad \dfrac{Z'_o}{Z_o} = \dfrac{LP_o}{L_oP}$ ....(1.8)

Hence if $L_o = L$,

$$\frac{Z'_o}{Z_o} = \frac{P_o}{P} = \frac{j\beta_o}{\alpha + j\beta} \quad ....(1.9)$$

Thus equation (1.1) may be written, using equation (1.9),

$$\frac{Z_T}{Z_o} = \frac{j\beta_o}{\alpha + j\beta}\tanh(\alpha + j\beta)l \quad ....(1.10)$$

The voltage reflection coefficient

$$k = K e^{j\phi}$$

is related to the terminating impedance of the line, Z_T, by the expression

$$\frac{Z_T}{Z_o} = \frac{1 + k}{1 - k} \quad \dots\dots\dots\dots\dots (1.11)$$

or $\quad k = \dfrac{Z/Z_o - 1}{Z/Z_o + 1} \quad \dots\dots\dots\dots (1.11)$

Substituting for Z_T/Z_o in equation (1.11) from (1.10)

$$k = \frac{j\beta_o \sinh(\alpha + j\beta)1 - (\alpha + j\beta)\cosh(\alpha + j\beta)1}{j\beta_o \sinh(\alpha + j\beta)1 + (\alpha + j\beta)\cosh(\alpha + j\beta)1} \quad \dots (1.12)$$

Rationalising equation (1.12), putting

$$M = \alpha + j\beta$$

and $\quad N = \alpha - j\beta$

$$k = \frac{\beta_o^2 shM1shN1 - (\alpha^2 + \beta^2)chM1chN1 + j\beta MchM1shN1 + j\beta NshM1chN1}{\beta_o^2 shM1shN1 + (\alpha^2 + \beta^2)chM1chN1 - j\beta_o MchM1shN1 + j\beta_o NshM1chN1}$$

$$\dots\dots\dots (1.12)$$

$$= \frac{\beta_o^2(ch2\alpha 1 - ch j2\beta 1)\tfrac{1}{2} - (\alpha^2 + \beta^2)(ch2\alpha 1 + ch j2\beta 1)\tfrac{1}{2}}{D}$$

$$+ j\left\{\frac{\beta M(sh2\alpha 1 - sh j2\beta 1)\tfrac{1}{2} + \beta_o N(sh2\alpha 1 + sh j2\beta 1)\tfrac{1}{2}}{D}\right\} \quad \dots (1.12)$$

where D is

$$\beta_o^2(ch2\alpha 1 - ch j2\beta 1)\tfrac{1}{2} + (\alpha^2 + \beta^2)(ch2\alpha 1 + ch j2\beta 1)\tfrac{1}{2}$$

$$+ j\beta_o\left\{M(sh2\alpha 1 - sh j2\beta 1)\tfrac{1}{2} - N(sh2\alpha 1 + sh j2\beta 1)\tfrac{1}{2}\right\}$$

Whence writing

$$a = \beta_o^2 - \alpha^2 - \beta^2 \quad \text{and} \quad b = \beta_o^2 + \alpha^2 + \beta^2$$

$$k = \frac{a\,ch2\alpha l - b\cos2\beta l + j2\beta_{o}(\alpha\,sh2\alpha l + \beta\sin2\beta l)}{b\,ch2\alpha l - a\cos2\beta l + 2\beta_{o}\beta\,sh2\alpha l - 2\beta_{o}\alpha\sin2\beta l}$$

Therefore

$$\tan\phi = \frac{2\beta_{o}(\alpha\,sh2\alpha l + \beta\sin2\beta l)}{(\beta_{o}^{2} - \alpha^{2} - \beta^{2})ch2\alpha l - (\beta_{o}^{2} + \alpha^{2} + \beta^{2})\cos2\beta l} \qquad \dots\dots\dots(1.13)$$

1.3   An alternative method of deriving the phase angle equation.

An alternative method of deriving equation (1.13) is as follows.

Let

$$\frac{Z_{T}}{Z_{o}} = u + jv \qquad \dots\dots\dots\dots(1.14)$$

Then using equation (1.11) the reflection coefficient may be expressed,

$$k = \frac{u + jv - 1}{u + jv + 1} \qquad \dots\dots\dots\dots(1.15)$$

which when rationalised becomes,

$$k = \frac{u^{2} + v^{2} + 2jv - 1}{u^{2} + v^{2} + 1 + 2u} \qquad \dots\dots\dots\dots(1.15)$$

and since

$$k = Ke^{j\phi}$$

$$\tan\phi = \frac{2v}{u^{2} + v^{2} - 1} \qquad \dots\dots\dots\dots(1.16)$$

From equation (1.10)

$$\frac{Z_{T}}{Z_{o}} = \frac{j\beta_{o}}{\alpha + j\beta}\tanh(\alpha + j\beta)l = u + jv$$

Thus

$$u + jv = \frac{j\beta_o(\alpha - j\beta)}{\alpha^2 + \beta^2} \cdot \frac{\sinh(\alpha + j\beta)l\cosh(\alpha - j\beta)l}{\cosh(\alpha + j\beta)l\cosh(\alpha - j\beta)l}$$

$$= \left(\frac{\beta_o\beta - j\beta_o\alpha}{\alpha^2 + \beta^2}\right)\left\{\frac{\sinh 2\alpha l + j\sin 2\beta l}{\cosh 2\alpha l + \cos 2\beta l}\right\}$$

Therefore equating real and imaginary parts:-

$$u = \frac{\beta_o\beta\sinh 2\alpha l - \beta_o\alpha\sin 2\beta l}{(\alpha^2 + \beta^2)(\cosh 2\alpha l + \cos 2\beta l)}$$

and

$$v = \frac{\beta_o\alpha\sinh 2\alpha l + \beta_o\beta\sin 2\beta l}{(\alpha^2 + \beta^2)(\cosh 2\alpha l + \cos 2\beta l)}$$

Substituting these values for u and v in equation (1.16)

$$\tan\phi$$

$$= \frac{2\beta_o(\alpha \, \text{sh}2\alpha l + \beta\sin 2\beta l)(\alpha^2 + \beta^2)(\text{ch}2\alpha l + \cos 2\beta l)}{\beta_o^2(\beta^2\text{sh}2\alpha l + \alpha^2\sin 2\beta l + \alpha^2\text{sh}2\alpha l + \beta^2\sin 2\beta l) - (\alpha^2 + \beta^2)^2(\text{ch}2\alpha l + \cos 2\beta l)^2}$$

$$= \frac{2\beta_o(\alpha \, \text{sh}2\alpha l + \beta\sin 2\beta l)(\alpha^2 + \beta^2)(\text{ch}2\alpha l + \cos 2\beta l)}{(\alpha^2 + \beta^2)\{\beta_o^2(\text{ch}2\alpha l - 1 + 1 - \cos 2\beta l) - (\alpha^2 + \beta^2)(\text{ch}2\alpha l + \cos 2\beta l)\}}$$

$$= \frac{2\beta_o(\alpha \, \text{sh}2\alpha l + \beta\sin 2\beta l)(\text{ch}2\alpha l + \cos 2\beta l)}{\{\beta_o^2(\text{ch}2\alpha l - \cos 2\beta l) - (\alpha^2 + \beta^2)(\text{ch}2\alpha l + \cos 2\beta l)\} \times (\text{ch}2\alpha l + \cos 2\beta l)}$$

$$= \frac{2\beta_o(\alpha\sinh 2\alpha l + \beta\sin 2\beta l)}{(\beta_o^2 - \alpha^2 - \beta^2)\cosh 2\alpha l - (\beta_o^2 + \alpha^2 + \beta^2)\cos 2\beta l}$$

as obtained in equation (1.13).

1.4 <u>The relationship between the phase angle ϕ and</u>
<u>the observed shift Δ of the standing wave pattern</u>
<u>as the depth of the dielectric filling the line</u>
<u>varies.</u>

In testing the validity of equation (1.13) experimentally, the shift Δ of the standing wave pattern as the depth of dielectric filling the transmission line was observed instead of the phase angle ϕ .

Fig. (1.2) represents a length L of transmission line terminated in an impedance Z_T.
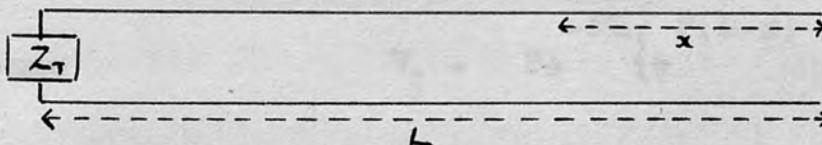
Fig. 1.2

The expressions for the voltage and current at any point x are given by

$$V_x = Ae^{Px} + Be^{-Px}$$

and

$$I_x = \frac{Be^{-Px} - Ae^{Px}}{Z_o}$$

When x = L,

$$V_L = Ae^{PL} + Be^{-PL}$$

and

$$I_L = \frac{Be^{-PL} - Ae^{PL}}{Z_o}$$

But

$$Z_T = \frac{V_L}{I_L} = Z_o \frac{(Ae^{PL} + Be^{-PL})}{Be^{-PL} - Ae^{PL}}$$

or $\quad Ae^{PL}(Z_T + Z_o) = Be^{-PL}(Z_T - Z_o)$

From equation (1.11)

$$k = \frac{Z_T - Z_o}{Z_T + Z_o}$$

Therefore $\quad A = kBe^{-2PL}$

Thus the voltage at any point x may be written,

$$V_x = B(ke^{-2PL}e^{Px} + e^{-Px})$$

k is complex, $\quad k = Ke^{j\phi}$

Hence $\quad V_x = Be^{-PL}\{e^{P(L-x)} + Ke^{j\phi}e^{-P(L-x)}\}$

For a transmission line in air, $\alpha_o = 0$, and so $P = j\beta_o$.

With a crystal detector, which obeys a square law, the observed signal is proportional to $|V_x|^2$. Hence multiplying by the complex conjugate,

$$|V_x|^2 = B^2 e^{-2PL}\{1 + K^2 + K(e^{j2\beta_o(L-x)}e^{-j\phi} + e^{-j2\beta_o(L-x)}e^{j\phi})\} \quad\ldots\ldots(1.17)$$

$$= B^2 e^{-2PL}\{1 + K^2 + K2\cosh j[2\beta_o(L-x) - \phi]\} \quad\ldots(1.17)$$

$$= B^2 e^{-2PL}\{1 + K^2 + 2K\cos 2[\beta_o(L-x) - \tfrac{\phi}{2}]\} \quad\ldots(1.17)$$

The maximum and minimum values of $|V_x|^2$ occur when the cosine term of equation (1.17) is 1 or -1 respectively.

Hence, writing $y = L-x$, the conditions for maximum and minimum signals are:-

$$2\left(\beta_o y - \frac{\phi}{2}\right) = 2n\pi$$

and

$$2\left(\beta_o y - \frac{\phi}{2}\right) = (2n+1)\pi \quad \text{respectively,}$$

where

$$n = 0, 1, 2, 3, \ldots\ldots$$

Thus maximum signals occur when

$$\beta_o y = n\pi + \frac{\phi}{2}$$

so that the phase angle ϕ of the voltage on reflection from the termination Z_T is determined when the positions of the maxima have been found.

If the terminal impedance is a short circuit, i.e. $Z_T = 0$, $k = -1$ and $\phi = \pi$, the position of the maxima y_o is given by,

$$y_o = \frac{\pi}{\beta_o}\left(n + \frac{1}{2}\right)$$

When the terminating impedance is a short circuited length of line containing liquid, the phase angle becomes $\pi + \phi$, and the new position of the maxima y' is given by,

$$y' = \frac{1}{\beta_o}\left(n\pi + \frac{\pi}{2} + \frac{\phi}{2}\right)$$

So that the shift of the maxima as liquid enters above the short circuit is $y' - y_o$, i.e.,

$$y' - y_o = \frac{\phi}{2\beta_o} = \frac{\phi \lambda_o}{4\pi} = \Delta \qquad \ldots\ldots\ldots\ldots(1.18)$$

This shift Δ of the maxima (or minima) can be found experimentally, and so the phase angle of equation (1.13) due to the introduction of a depth l of liquid dielectric in a short circuited transmission line, can be calculated by multiplying the observed shift Δ by $\frac{4\pi}{\lambda_o}$, for from equation (1.18)

$$\phi = \frac{4\pi}{\lambda_o} \Delta \qquad \ldots\ldots\ldots\ldots(1.19)$$

Since a shift Δ of $\lambda_o/4$ may equally well be interpreted as a shift of minus $\lambda_o/4$, the phase angle ϕ may be considered to lie always in the range plus $2\beta_o\frac{\lambda_o}{4}$ to minus $2\beta_o\frac{\lambda_o}{4}$, that is from plus π to minus π.

Equation (1.13) may be expressed in terms of the phase shift using equation (1.19):-

$$\Delta = \frac{\lambda_o}{4\pi} \tan^{-1}\left\{ \frac{2\beta_o(\alpha\,\text{sh}2\alpha l + \beta\sin2\beta l)}{(\beta_o^2 - \alpha^2 - \beta^2)\,\text{ch}2\alpha l - (\beta_o^2 + \alpha^2 + \beta^2)\cos2\beta l} \right.$$

$$\ldots\ldots\ldots\ldots(1.20)$$

This equation was verified experimentally by observing the shift Δ caused by the introduction of

different depths l of liquids into a short circuited
coaxial transmission line. The form of the graph of $\triangle$
against l depends upon the attenuation coefficient
and phase constant of the liquid used, and it has been
found possible to deduce values for α and β from
these graphs. The effect of the liquid dielectric is
discussed in the next paragraph.

1.5   Theoretical discussion of the variation of phase
          angle with liquid depth for different liquids.

Equation (1.13) may be written:-

$$\tan \phi = -\frac{2\beta_0\beta}{(\beta^2+\alpha^2+\beta_0^2)}\left\{\frac{\sin 2\beta l + \frac{\alpha}{\beta}\sinh 2\alpha l}{\cos 2\beta l + \frac{\beta^2+\alpha^2-\beta_0^2}{\beta^2+\alpha^2+\beta_0^2}\cosh 2\alpha l}\right\}$$

$$\dots\dots\dots(1.21)$$

1.5.1   Non-absorbing liquids.

For non-absorbing liquids $\alpha = 0$, and equation
(1.21) becomes:-

$$\tan \phi = -\frac{2\beta_0\beta}{\beta^2+\beta_0^2}\left\{\frac{\sin 2\beta l}{\cos 2\beta l + \frac{\beta^2-\beta_0^2}{\beta^2+\beta_0^2}}\right\}$$

Thus, neglecting the constant term

$$\frac{2\beta_0\beta}{\beta^2+\beta_0^2},$$

cos2βl

sin2βl

L
1

Fig. 1.3   β₀ = 0.6,   α = 0,   β = 2.5

tan φ
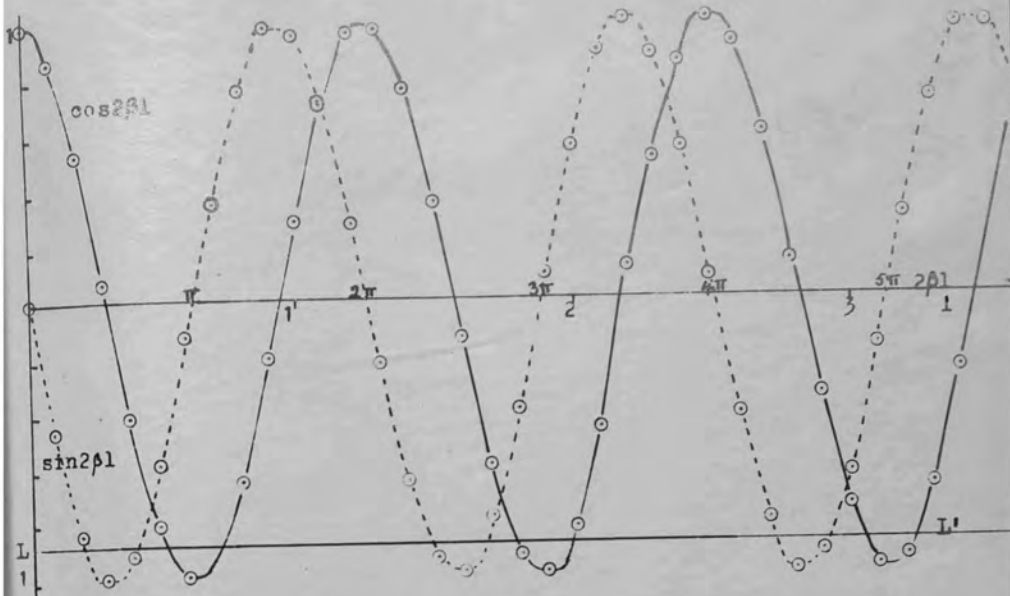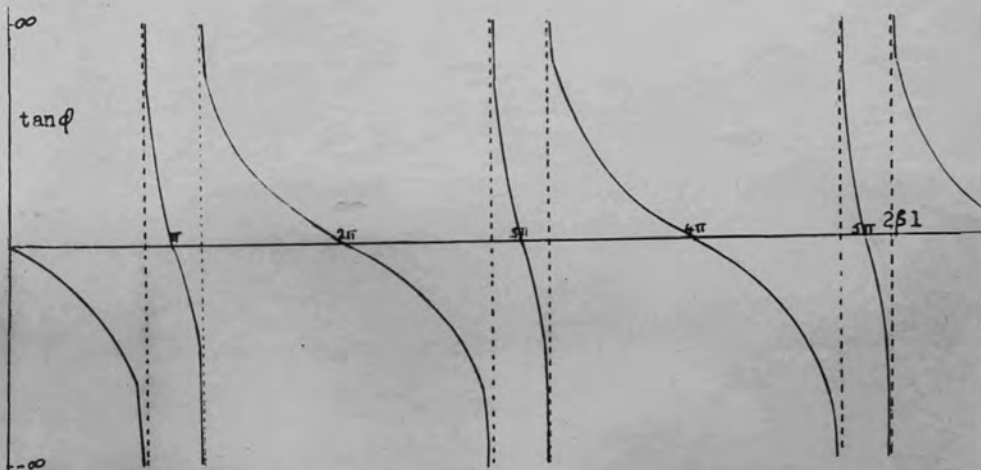
Fig. 1.4   The variation of tan φ with l.

φ
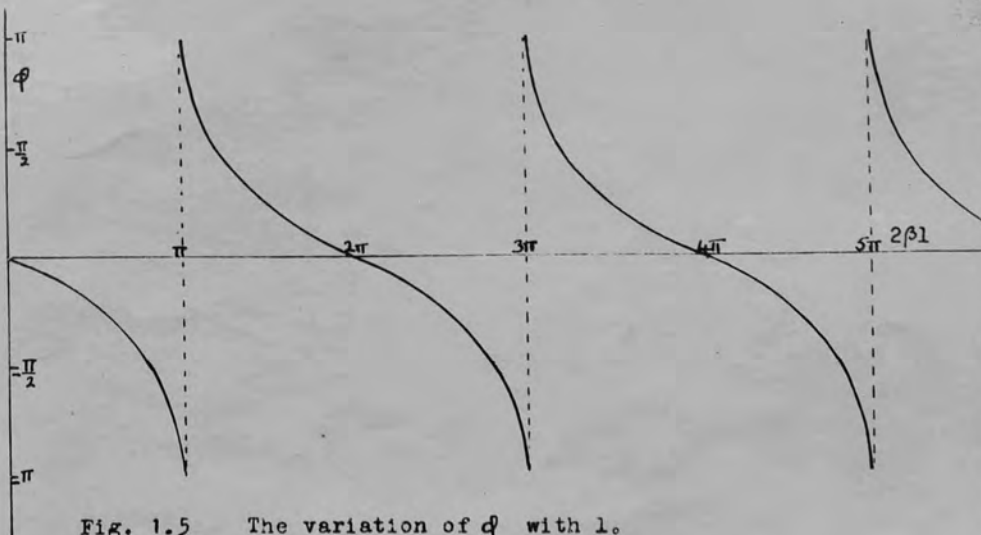
Fig. 1.5   The variation of φ with l.

the numerator of $\tan\phi$ is given by $\sin2\beta l$, and the
denominator by the height of the $\cos2\beta l$ curve above
the line

$$y = \frac{\beta^2 - \beta_0^2}{\beta^2 + \beta_0^2}$$

(LL' in fig. 1.3). Thus $\tan\phi$ is zero when $\sin2\beta l$
is zero, and approaches infinity when $\cos2\beta l$ and the
line LL' intersect, (denominator zero). The curves for
$\tan\phi$ and ϕ (ϕ being considered to lie always between
plus and minus π) against $2\beta l$ are shown in figs. (1.4)
and (1.5). For figs. (1.3), (1.4) and (1.5) the
arbitrary values chosen for β_0 and β were 0.6 and 2.5
respectively, giving for LL'

$$y = 0.89$$

and corresponding to a liquid with a dielectric constant
of 17.

If $\beta = \beta_0$, i.e. an air filled line, LL' coincides
with the $2\beta l$ axis and $\tan\phi$ becomes infinite at $\pi/2$.
(See fig. 1.6)

As the ratio of β/β_0 increases, LL' moves downwards
and its points of intersection with $\cos2\beta l$ close up in
pairs so that the gradients of $\tan\phi$ and ϕ become
relatively less steep in the range

$$\phi = +\frac{\pi}{2} \text{ to } -\frac{\pi}{2}$$

Fig. 1.6 $\beta_o = \beta$ $\left(\frac{1}{\ }\right)$
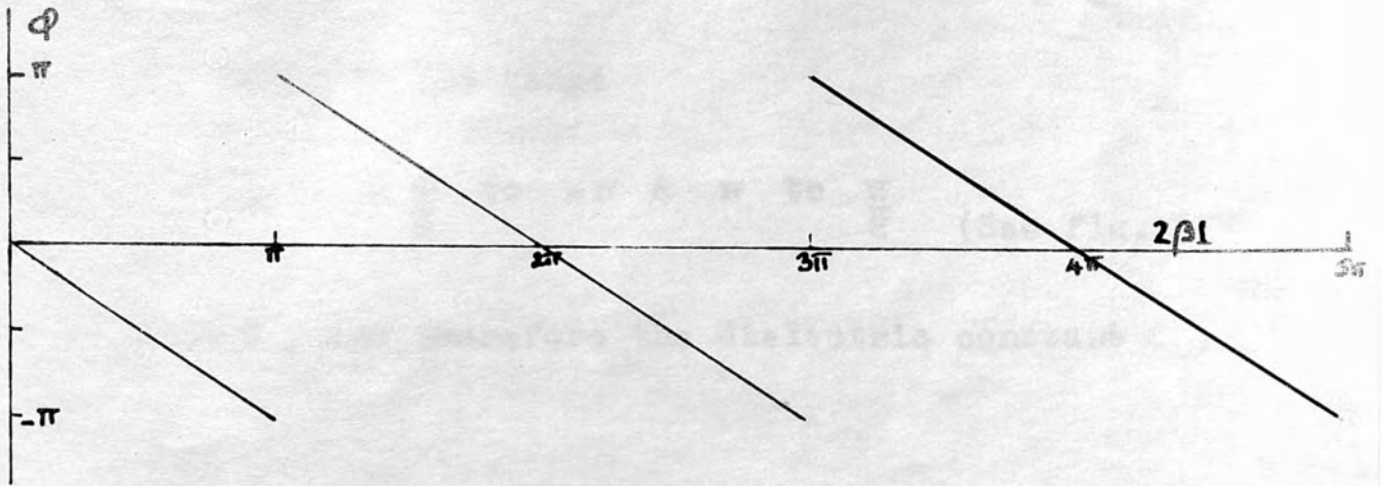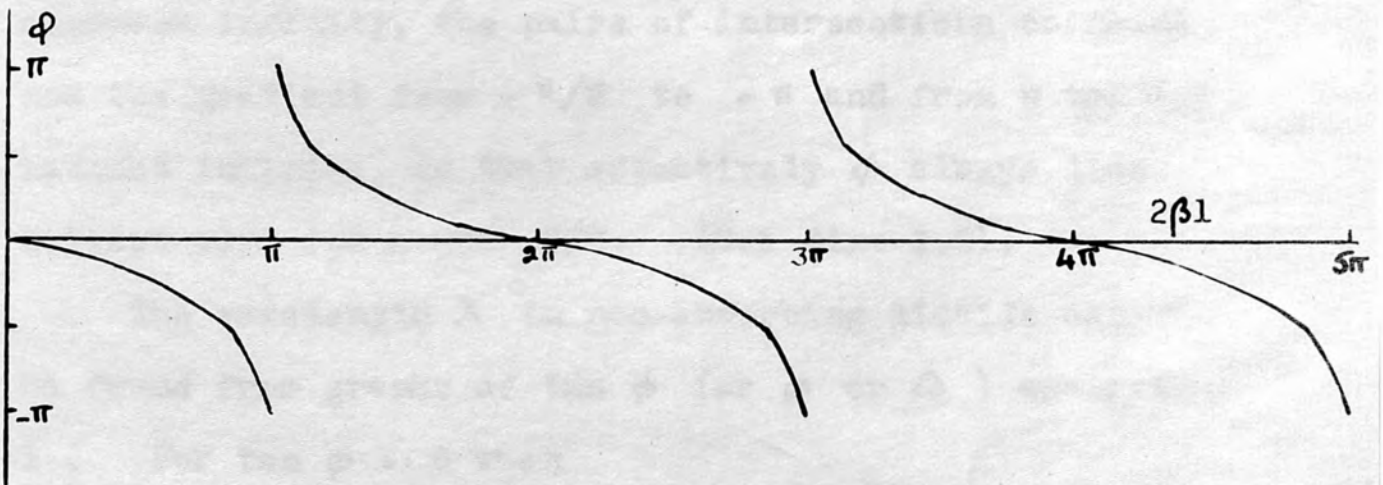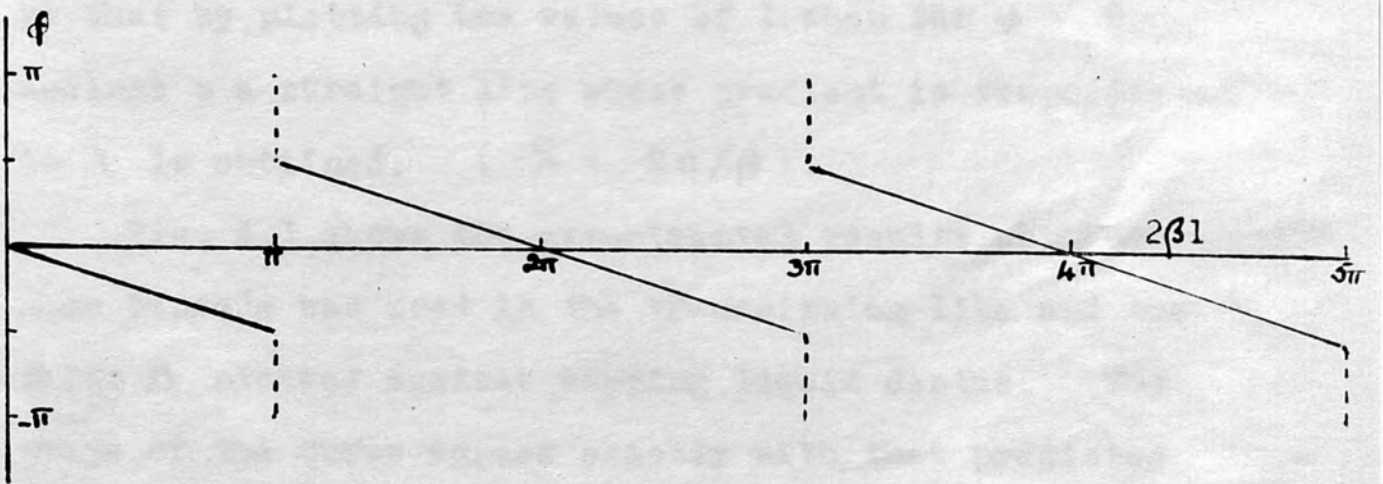
Fig. 1.7

Fig. 1.8 $\beta \to \infty$

and more steep in the range

$$\phi = -\frac{\pi}{2} \text{ to } -\pi \ \& \ \pi \text{ to } \frac{\pi}{2}$$ (See fig. 1.7)

When β , and therefore the dielectric constant ε , for

$$\varepsilon = \left(\frac{\lambda_o}{\lambda}\right)^2 = \left(\frac{\beta}{\beta_o}\right)^2$$

approach infinity, the pairs of intersections coincide and the gradient from $-\pi/2$ to $-\pi$ and from π to $\pi/2$ becomes infinite, so that effectively ϕ always lies between plus and minus $\pi/2$. (See fig. 1.8).

The wavelength λ in non-absorbing liquids can be found from graphs of tan ϕ (or ϕ or Δ) against l . For tan $\phi = 0$ when

$$2\beta l = n\pi$$

so that by plotting the values of l when tan $\phi = 0$ against n a straight line whose gradient is proportional to λ is obtained. ($\lambda = 2\pi/\beta$).

Fig. 4.1 shows the experimental results obtained when benzole was used in the transmission line and the shift Δ plotted against varying liquid depths. The shape of the curve agrees exactly with that predicted by the above theory.

1.5.2 Absorbing liquids.

For absorbing liquids $\alpha > 0$, and equation (1.21) applies.

$$\tan \phi = -\frac{2\beta_o\beta}{(\beta^2+\alpha^2+\beta_o^2)}\left\{\frac{\sin2\beta1 + \frac{\alpha\sinh2\alpha1}{\beta}}{\cos2\beta1 + \left(\frac{\beta^2+\alpha^2-\beta_o^2}{\beta^2+\alpha^2+\beta_o^2}\right)\cosh2\alpha1}\right\} \quad ...(1.21)$$

Thus neglecting the constant term,

$$\frac{2\beta_o\beta}{\beta^2+\alpha^2+\beta_o^2}$$

the numerator of $\tan\phi$ is given by the height of the curve

$$y = -\sin2\beta1 \quad\quad \text{above} \quad\quad y = \frac{\alpha\sinh2\alpha1}{\beta}$$

and the denominator by the height of

$$y = \cos2\beta1 \quad\quad \text{above} \quad\quad y = \frac{\beta^2+\alpha^2-\beta_o^2}{\beta^2+\alpha^2+\beta_o^2}\cosh2\alpha1$$

This is illustrated in fig. 1.9 with the arbitrary values,

$$\beta_o = 0.6 \quad\quad\quad \beta = 2.5 \quad\quad\quad \alpha = 0.1$$

The effect of the term containing α in the numerator is to shift the zeros of $\tan\phi$ (and of ϕ) slightly, alternate zeros being shifted in opposite directions.

The effect of the term containing α in the denominator is to replace the line LL' by an inverted

$\beta_0 = 0.6, \quad \alpha = 0.1, \quad \beta = 2.5$
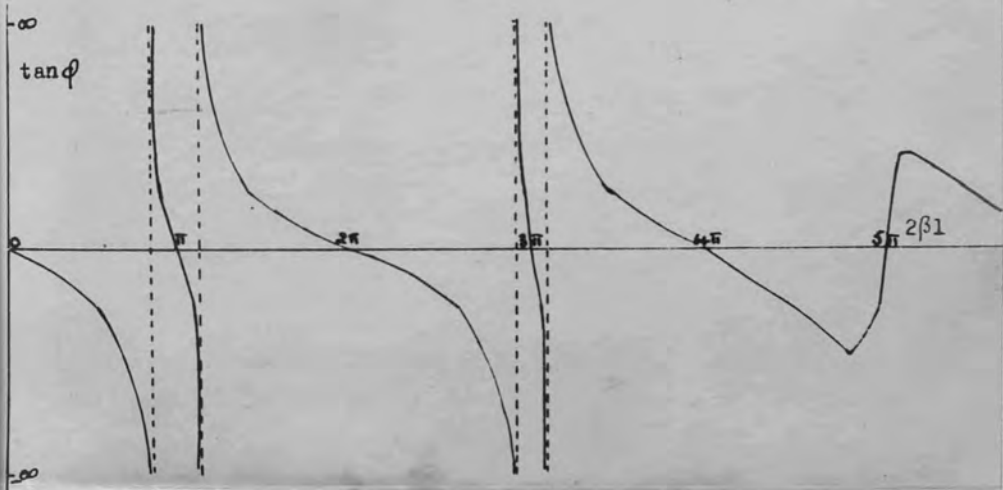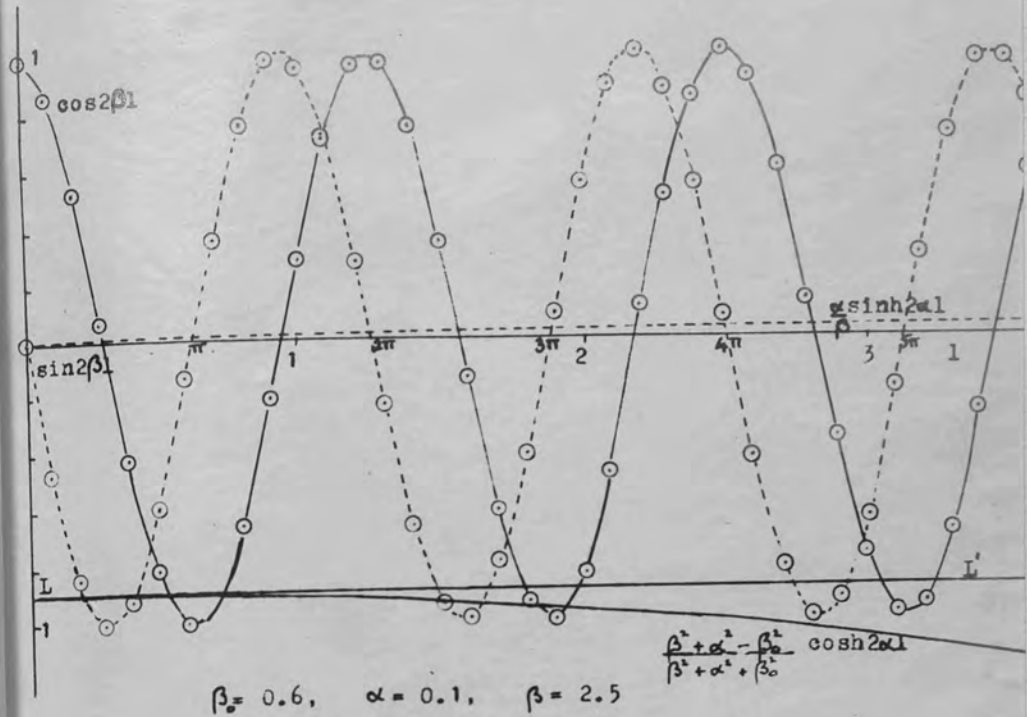
The variation of $\tan \phi$ with l.
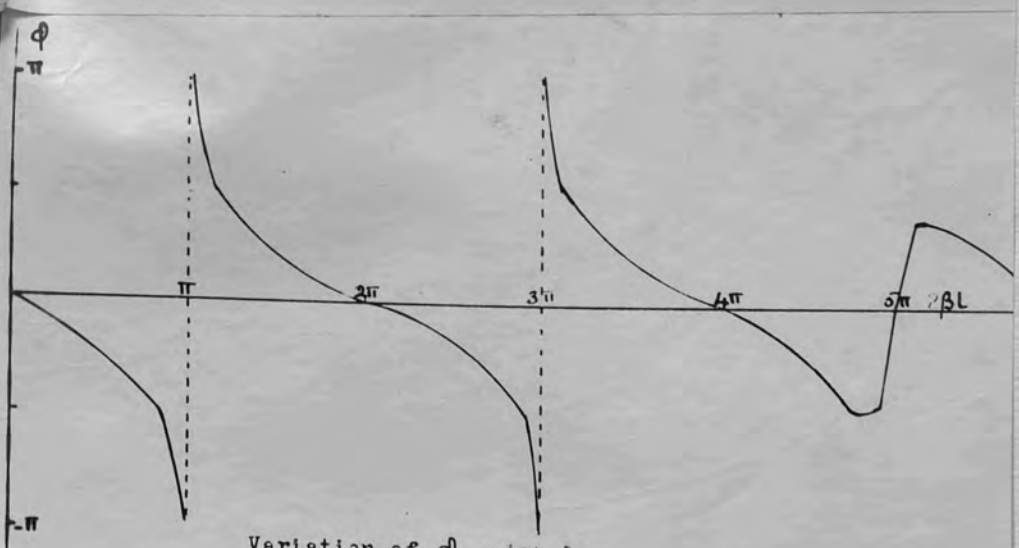
Variation of ϕ with l.

Fig. 1.9

cosh curve, so that the points of intersection of the
cosine and cosh curves, which give the positions at
which tan ϕ approaches infinity, are closer the greater
the depth of the liquid. Eventually the two curves do
not intersect, and tan ϕ no longer reaches infinity.
ϕ then has minima and maxima which are respectively
greater than $-\pi/2$ and less than $\pi/2$.

As l approaches infinity,

$$\cosh 2\alpha l \longrightarrow \sinh 2\alpha l$$

and both functions approach infinity. Equation (1.13)
may be written,

$$\tan \phi = -2\beta_0 \frac{\left\{ \dfrac{\beta \sin 2\beta l}{\cosh 2\alpha l} + \alpha \tanh 2\alpha l \right\}}{(\beta^2 + \alpha^2 + \beta_0^2) \dfrac{\cos 2\beta l}{\cosh 2\alpha l} + (\beta^2 + \alpha^2 - \beta_0^2)} \qquad \ldots \ldots (1.22)$$

Tanh2αl differs from 1 by less than 1% if 2αl is
greater than 2.65, and sinh2αl and cosh2αl approach
infinity for 2αl greater than about 5. So that under
these conditions,

$$\tan \phi \longrightarrow - \frac{2\beta_0 \alpha}{\beta^2 + \alpha^2 - \beta_0^2} \qquad \ldots \ldots \ldots (1.23)$$

Since $\beta \geqslant \beta_0$, this expression must be negative and
therefore tan ϕ (and ϕ and Δ) will eventually
settle down to a steady negative value.

1.5.3 <u>The effect of α and β on the variation of</u>
      <u>$\tan\phi$ with l .</u>

At a wavelength of 10 cms, liquids at room
temperature show a variation in α from 0 to about 0.8
nepers cms^{-1}, and in β from slightly greater than 1 to
about 6 radians cms^{-1}.

1.5.3.1 <u>Liquids with low values of both α and β .</u>

The effect of a liquid with a value of β only
slightly greater than one, which corresponds to a
wavelength λ in the liquid of the order of 6 cms, is
to expand the sine and cosine curves of fig. 1.9
laterally. With a small value for α , of the order
of 0.01, the term in $\sinh 2\alpha l$ is going to have very
little effect, while the inverted cosh term will start
at about - 0.5 and will increase only slowly in value
intersecting the cosine curve several times before
reaching a value greater than 1. The $\tan\phi$ curve
then no longer goes off to infinity but has turning
points, the value of $\tan\phi$ at these turning points
gradually decreasing as l increases until the liquid
depth is about 200 cms when $\tan\phi$ will have a constant
negative value.

Figs. 1.10 and 1.11 show graphs of the variation of
Δ and $\tan\phi$ with l calculated from equations (1.13)
and (1.19) for $\beta_0 = 0.69$, $\alpha = 0.11$, $\beta = 1.6$.
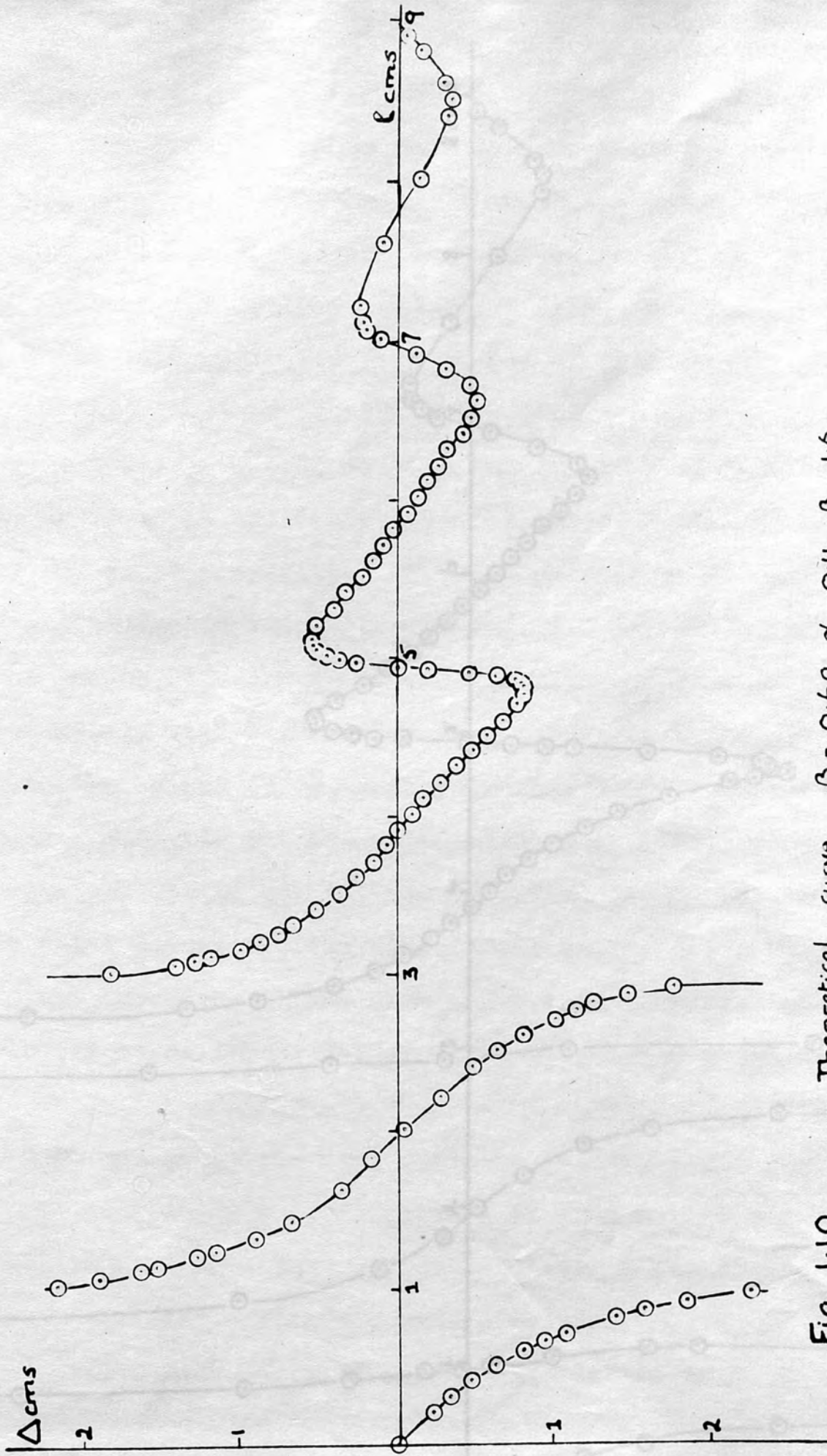
Fig. 1.10   Theoretical curve $\beta_o = 0.69$, $\alpha = 0.11$, $\beta = 1.6$
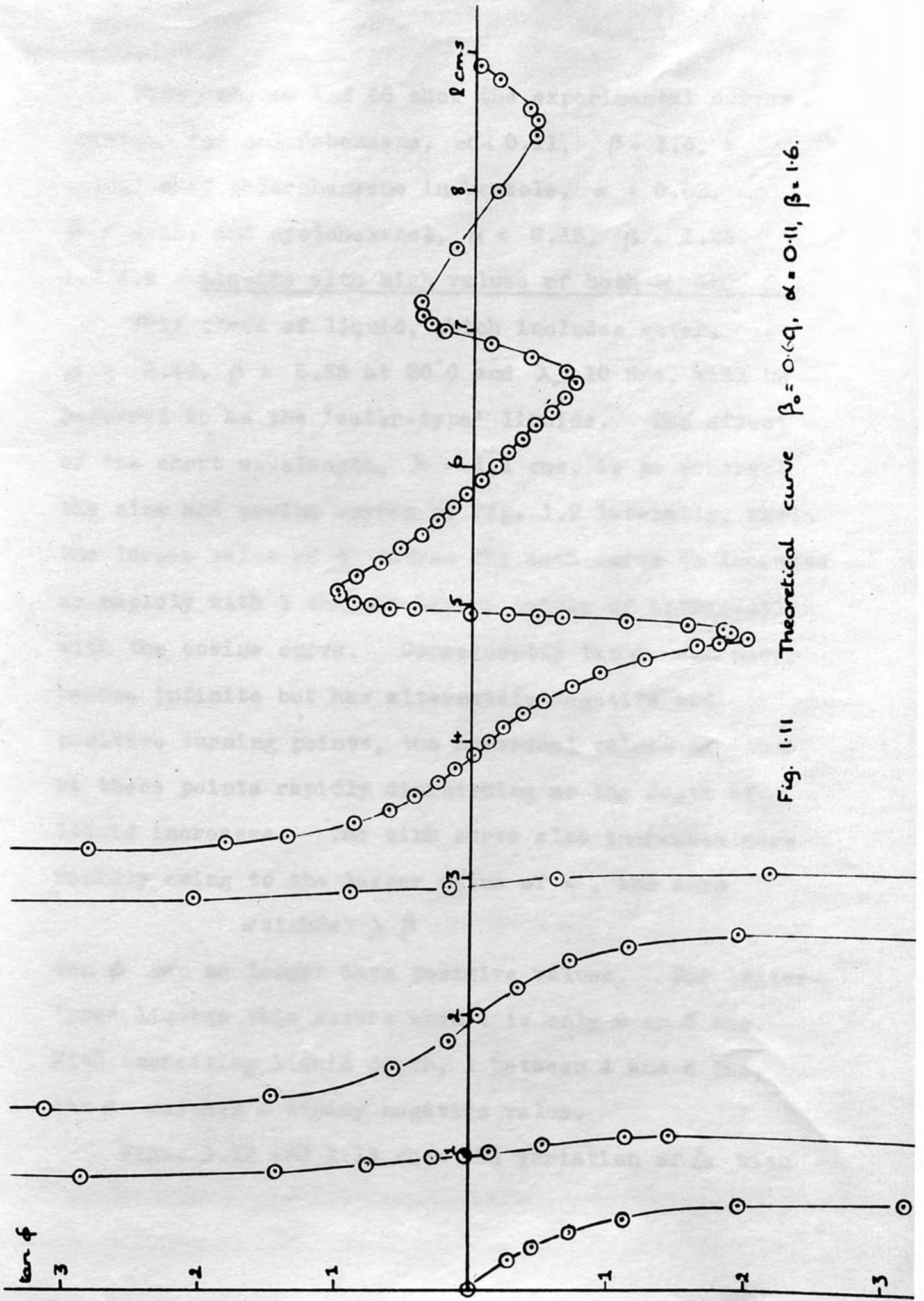
Fig. 1.11 Theoretical curve $\beta_0 = 0.69$, $\alpha = 0.11$, $\beta = 1.6$.

Figs. 43, 44 and 45 show the experimental curves obtained for chlorobenzene, $\alpha = 0.11$, $\beta = 1.6$, a solution of chlorobenzene in benzole, $\alpha = 0.08$, $\beta = 1.45$; and cyclohexanol, $\alpha = 0.15$, $\beta = 1.23$.

1.5.3.2 <u>Liquids with high values of both α and β.</u>

This class of liquid, which includes water, $\alpha = 0.46$, $\beta = 5.55$ at $20°C$ and $\lambda_o = 10$ cms, will be referred to as the 'water-type' liquids. The effect of the short wavelength, $\lambda = 1.1$ cms, is to contract the sine and cosine curves of fig. 1.9 laterally, while the larger value of α causes the cosh curve to increase so rapidly with l that it has no points of intersection with the cosine curve. Consequently $\tan \phi$ can never become infinite but has alternately negative and positive turning points, the numerical values of $\tan \phi$ at these points rapidly diminishing as the depth of liquid increases. The sinh curve also increases more rapidly owing to the larger value of α, and when

$$\alpha \sinh 2\alpha l > \beta$$

$\tan \phi$ can no longer have positive values. For 'water-type' liquids this occurs when l is only 2 or 3 cms. With increasing liquid depth, l between 4 and 6 cms, $\tan \phi$ assumes a steady negative value.

Figs. 1.12 and 1.13 show the variation of Δ with
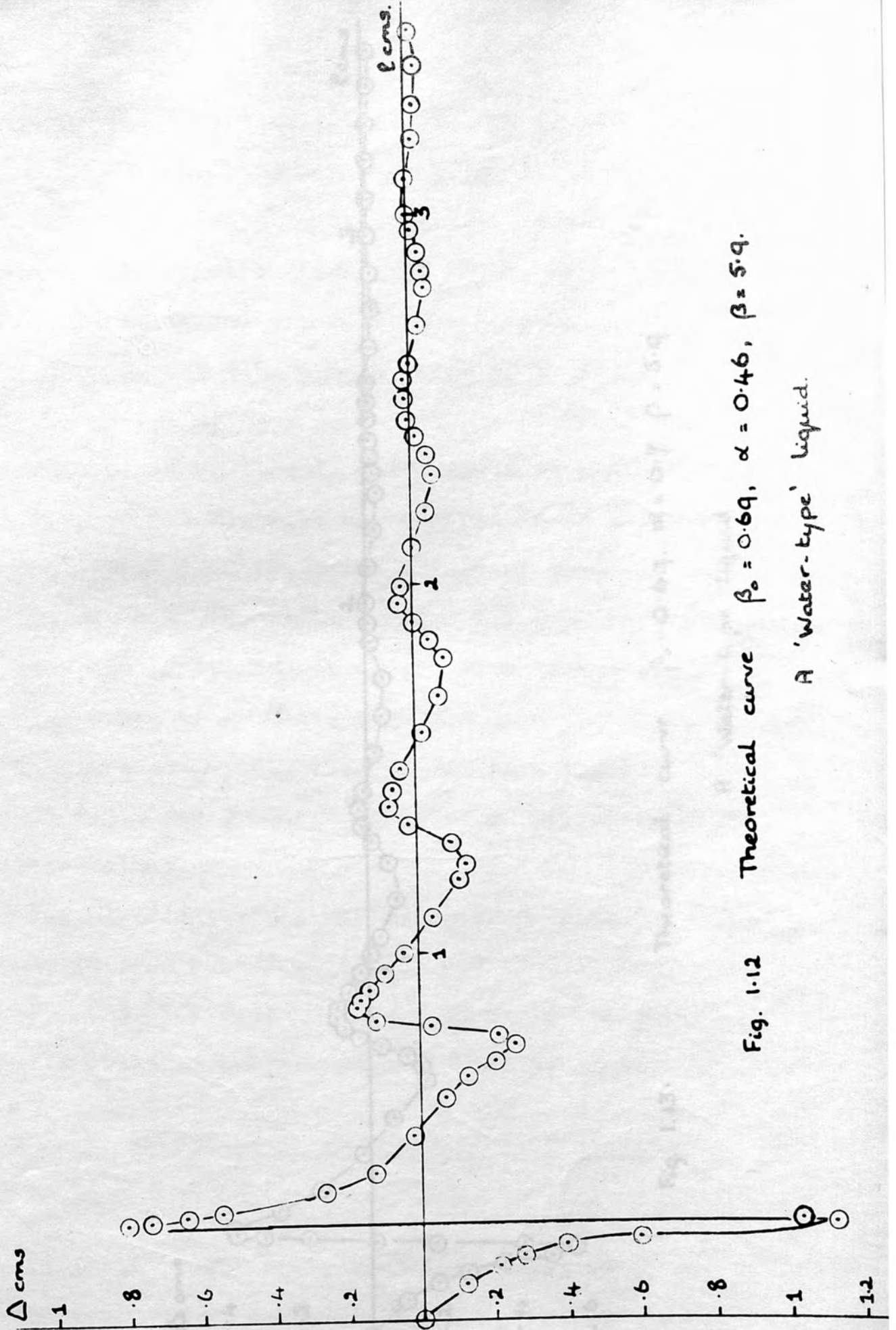
Fig. 1.12   Theoretical curve $\beta_0 = 0.69$, $\alpha = 0.46$, $\beta = 5.9$.

A 'water-type' liquid.

... equations (1.48) and (1.19) ...

... and $\alpha = 0.49$ (Fig. 1.12) ...

... Figs. (4.19), (4.51) ...

... curve obtained for water, ...

... as shown, $\alpha = 0.49$, $\beta = 5.71$.

... liquids with a high value of α and a ...

value of β ...

This class of liquid, which includes ...

$\alpha = 0.7$, $\beta = 1.96$, will be referred to as ...

'alcohol-type' liquids. The effect of the ...

length, $\lambda = 3.6$ Å, as ... ... 1.3.1.1 ... ...

the side and centre virial ... Fig. 1.9 ... ...

The high value of α ... ... the peak and the ...

series increase rapidly with ..., and ... ...

a small value for β ensures that it ... may ... ...

positive values.

Fig. 1.14 shows the variation of Δ with ...

calculated from equations (1.18) and (1.19) ...

$\alpha = 0.7$ and $\beta = 1.96$. ... Fig. 4.00 shows the ...

of curve obtained for absolute alcohol at 15 °C.

... The effect of β on the variation of Δ ...

with ℓ ...

Equation (1.21) shows that ... value of β, ...

the value of ... is relatively independent ...

Δ cms

ℓ cms

Theoretical curve $\beta_0 = 0.69$, $\alpha = 0.7$, $\beta = 5.9$

A 'water-type' liquid.

Fig. 1.13

l calculated from equations (1.13) and (1.19) for
β_o = 0.69, β = 5.9 and α = 0.46 (fig. 1.12) and
α = 0.7 (fig. 1.13). Figs. (4.19), (4.33) show the
experimental curves obtained for water, α = 0.57,
β = 5.99, ox serum, α = 0.68, β = 5.71.

1.5.3.3 <u>Liquids with a high value of α and a low</u>

<u>value of β</u> .

This class of liquid, which includes alcohol,
α = 0.7, β = 1.86, will be referred to as the
'alcohol-type' liquids. The effect of the long wave-
length, λ = 3.4, is, as in section 1.5.3.1, to expand
the sine and cosine curves of fig. 1.9 laterally.
The high value of α makes both the cosh and sinh
curves increase rapidly with l, and this combined with
a small value for β means that tan ϕ may never attain
positive values.

Fig. 1.14 shows the variation of Δ with l
calculated from equations (1.13) and (1.19) for β_o = 0.69,
α = 0.7 and β = 1.86. Fig. 4.80 shows the experiment-
al curve obtained for absolute alcohol at 18°C.

1.5.4 <u>The effect of β_o on the variation of tan ϕ</u>

<u>with l</u> .

Equation (1.21) shows that the effect of β_o on
the value of tan ϕ is relatively unimportant compared
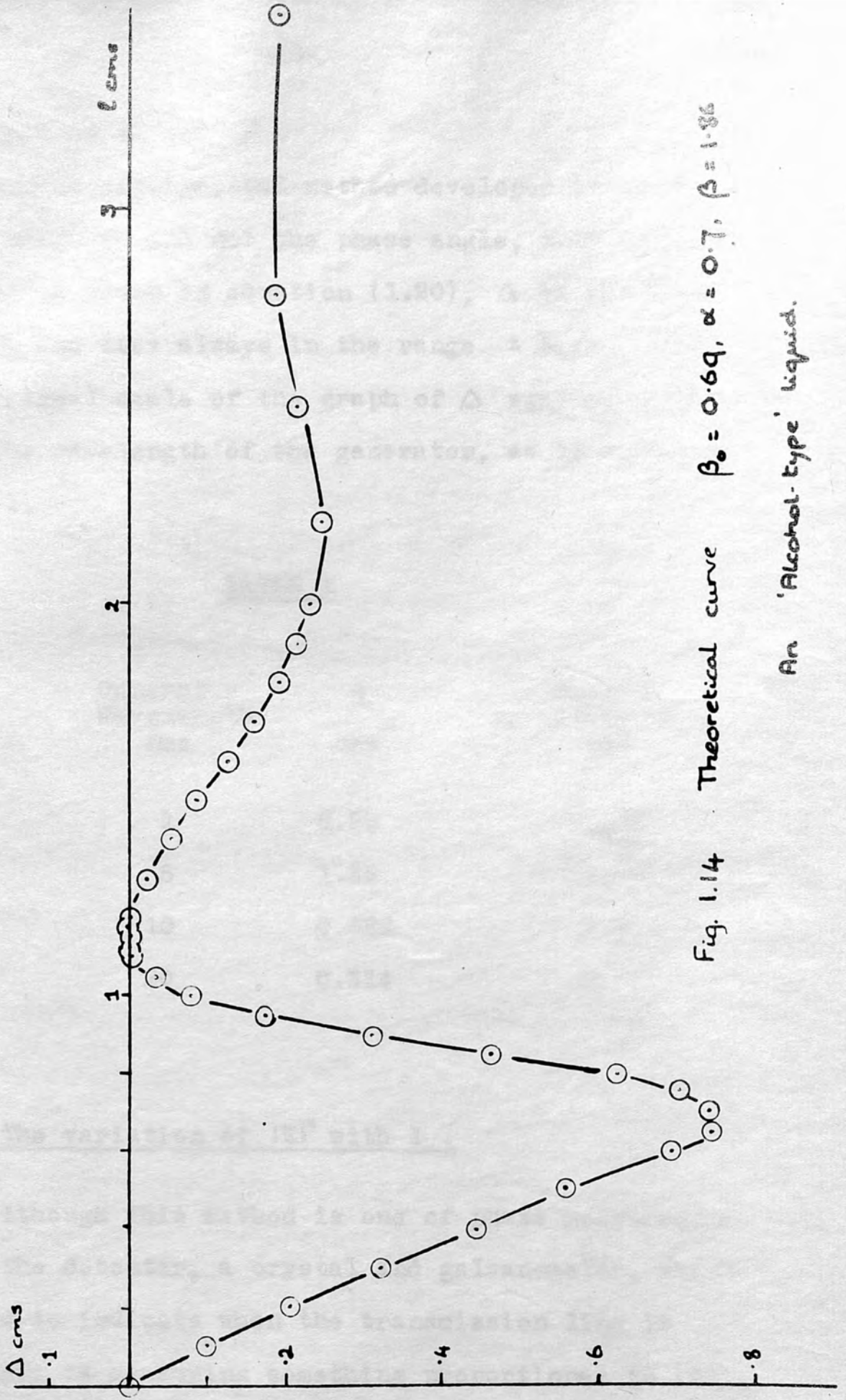
Fig. 1.14. Theoretical curve $\beta_0 = 0.69$, $\alpha = 0.7$, $\beta = 1.86$

An 'Alcohol-type' liquid.

with that of α and β .

In the experimental method developed it is the phase shift Δ and not the phase angle, that is measured, and, as is shown in equation (1.20), Δ varies directly with λ_o and lies always in the range $\pm \lambda_o/4$. Hence the vertical scale of the graph of Δ against l depends upon the wavelength of the generator, as is shown in Table 1.

TABLE 1

Generator Wavelength cms	β_o cms^{-1}	Maximum variation of Δ cms
1	6.28	0.25
5	1.25	1.25
10	0.628	2.5
20	0.314	5

1.6 The variation of $|K|^2$ with l .

Although this method is one of phase measurement only, the detector, a crystal and galvanometer, which is used to indicate when the transmission line is resonant, is measuring something proportional to $|K|^2$,

where K, the ratio of the amplitude of the reflected
to the incident voltage, is given by

$$k = Ke^{j\phi}$$

Hence it is of interest to investigate the variation
of $|K|^2$ with l .

From equation (1.12)

$$k = \frac{j\beta_o\sinh(\alpha + j\beta)l - (\alpha + j\beta)\cosh(\alpha + j\beta)l}{j\beta_o\sinh(\alpha + j\beta)l + (\alpha + j\beta)\cosh(\alpha + j\beta)l} \quad ....(1.12)$$

Multiplying by the complex conjugate, and putting

$$M = \alpha + j\beta$$

and $\quad N = \alpha - j\beta$

$$|K|^2 = \frac{\beta_o^2 shMlshNl + MNchMlchNl + j(\beta_o MchMlshNl - \beta_o NshMlchNl)}{\beta_o^2 shMlshNl + MNchMlchNl - j(\beta_o MchMlshNl - \beta_o NshMlchNl)}$$

$$= \frac{\frac{1}{2}\beta_o(ch2\alpha l - chj2\beta l) + \frac{1}{2}(\alpha^2 + \beta^2)(ch2\alpha l + chj2\beta l)}{D}$$

$$+ \frac{-j\{\frac{1}{2}\beta_o M(sh2\alpha l - shj2\beta l) - \frac{1}{2}\beta_o N(sh2\alpha l + shj2\beta l)\}}{D} \quad ..(1.24)$$

where D is

$$\frac{1}{2}\beta_o^2(ch2\alpha l - \cos2\beta l) + \frac{1}{2}(\alpha^2 + \beta^2)(ch2\alpha l + \cos2\beta l)$$

$$- j\{\frac{1}{2}\beta_o M(sh2\alpha l - j\sin2\beta l) - \frac{1}{2}\beta_o N(sh2\alpha l + j\sin2\beta l)\}$$

Whence writing

$$a = \beta_o^2 - \alpha^2 - \beta^2 \quad \text{and} \quad b = \beta_o^2 + \alpha^2 + \beta^2$$

$$|K|^2 = \frac{b ch2\alpha l - a cos2\beta l - 2\beta(\beta sh2\alpha l - \alpha sin2\beta l)}{b ch2\alpha l - a cos2\beta l + 2\beta(\beta sh2\alpha l - \alpha sin2\beta l)} \quad \ldots\ldots(1.24)$$

1.7 An alternative method of deriving the $|K|^2$ equation.

Following the treatment of section 1.5, where

$$\frac{Z_T}{Z_o} = u + jv \quad \ldots\ldots(1.14)$$

the reflection coefficient may be expressed,

$$k = \frac{u + jv - 1}{u + jv + 1}$$

Multiplying by the complex conjugate,

$$|K|^2 = \frac{u^2 - 2u + v^2 + 1}{u^2 + 2u + v^2 + 1} \quad \ldots\ldots\ldots(1.25)$$

From section 1.5

$$u = \frac{\beta_o \beta sinh2\alpha l - \beta_o \alpha sin2\beta l}{(\alpha^2 + \beta^2)(cosh2\alpha l + cos2\beta l)}$$

and

$$v = \frac{\beta_o \alpha sinh2\alpha l + \beta_o \beta sin2\beta l}{(\alpha^2 + \beta^2)(cosh2\alpha l + cos2\beta l)}$$

Let $|K|^2 = \dfrac{N}{D} = \dfrac{u^2 - 2u + v^2 + 1}{u^2 + 2u + v^2 + 1} \quad \ldots\ldots\ldots(1.25)$

and $u = \dfrac{X}{Z}, \quad v = \dfrac{Y}{Z}$

Hence $|K|^2 = \dfrac{X^2 + Y^2 + Z(Z-2X)}{X^2 + Y^2 + Z(Z+2X)} = \dfrac{N}{D}$

and substituting the values of u and v given above

$$N = \beta_o^2(\alpha^2 + \beta^2)\,\mathrm{sh}^2 2\alpha l + \beta_o^2(\alpha^2 + \beta^2)\,\sin^2 2\beta l + (\alpha^2 + \beta^2)^2(\mathrm{ch}2\alpha l + \cos 2\beta l)^2$$
$$- 2\beta_o(\beta\,\mathrm{sh}2\alpha l - \alpha\sin 2\beta l)(\alpha^2 + \beta^2)(\mathrm{ch}2\alpha l + \cos 2\beta l)$$

$$= (\alpha^2 + \beta^2)\Big\{ \beta_o^2(\mathrm{ch}^2 2\alpha l - \cos^2 2\beta l) + (\alpha^2 + \beta^2)(\mathrm{ch}2\alpha l + \cos 2\beta l)^2$$
$$- 2\beta_o(\beta\,\mathrm{sh}2\alpha l - \alpha\sin 2\beta l)(\mathrm{ch}2\alpha l + \cos 2\beta l)\Big\}$$

$$= (\alpha^2 + \beta^2)(\mathrm{ch}2\alpha l + \cos 2\beta l)\Big\{ (\beta_o^2 + \alpha^2 + \beta^2)\,\mathrm{ch}2\alpha l$$
$$+ (\alpha^2 + \beta^2 - \beta_o^2)\cos 2\beta l - 2\beta_o(\beta\,\mathrm{sh}2\alpha l - \alpha\sin 2\beta l)\Big\}$$

Similarly,

$$D = (\alpha^2 + \beta^2)(\mathrm{ch}2\alpha l + \cos 2\beta l)\Big\{ (\beta_o^2 + \alpha^2 + \beta^2)\,\mathrm{ch}2\alpha l$$
$$+ (\alpha^2 + \beta^2 - \beta_o^2)\cos 2\beta l + 2\beta_o(\beta\,\mathrm{sh}2\alpha l - \alpha\sin 2\beta l)\Big\}$$

Therefore,

$$\frac{N}{D} = \frac{(\beta_o^2 + \alpha^2 + \beta^2)\,\mathrm{ch}2\alpha l + (\alpha^2 + \beta^2 - \beta_o^2)\cos 2\beta l - 2\beta_o(\beta\,\mathrm{sh}2\alpha l - \alpha\sin 2\beta l)}{(\beta_o^2 + \alpha^2 + \beta^2)\,\mathrm{ch}2\alpha l + (\alpha^2 + \beta^2 - \beta_o^2)\cos 2\beta l + 2\beta_o(\beta\,\mathrm{sh}2\alpha l - \alpha\sin 2\beta l)}$$

or putting $\quad a = \beta_o^2 - \alpha^2 - \beta^2 \quad$ and $\quad b = \beta_o^2 + \alpha^2 + \beta^2$

$$|K|^2 = \frac{b\,\mathrm{ch}2\alpha l - a\cos 2\beta l - 2\beta_o(\beta\,\mathrm{sh}2\alpha l - \alpha\sin 2\beta l)}{b\,\mathrm{ch}2\alpha l - a\cos 2\beta l + 2\beta_o(\beta\,\mathrm{sh}2\alpha l - \alpha\sin 2\beta l)}$$

as in equation (1.24) of section 1.6.

Fig. 1.15 shows the variation of $|K|^2$ with l calculated from equation (1.24) for $\beta_o = 0.69$, $\alpha = 0.11$, and $\beta = 1.6$ representing the behaviour for a liquid with low absorption, and fig. 1.16 shows the behaviour

for a 'water-type' liquid calculated for $\alpha = 0.7$ and $\beta = 5.9$, and fig. 1.17 for an 'alcohol-type' liquid calculated for $\alpha = 0.7$ and $\beta = 1.86$.

Fig. 1.15 Theoretical curve of the variation of $|K|^2$ with ℓ

$\beta_0 = 0.69, \ \alpha = 0.11, \ \beta = 1.6$
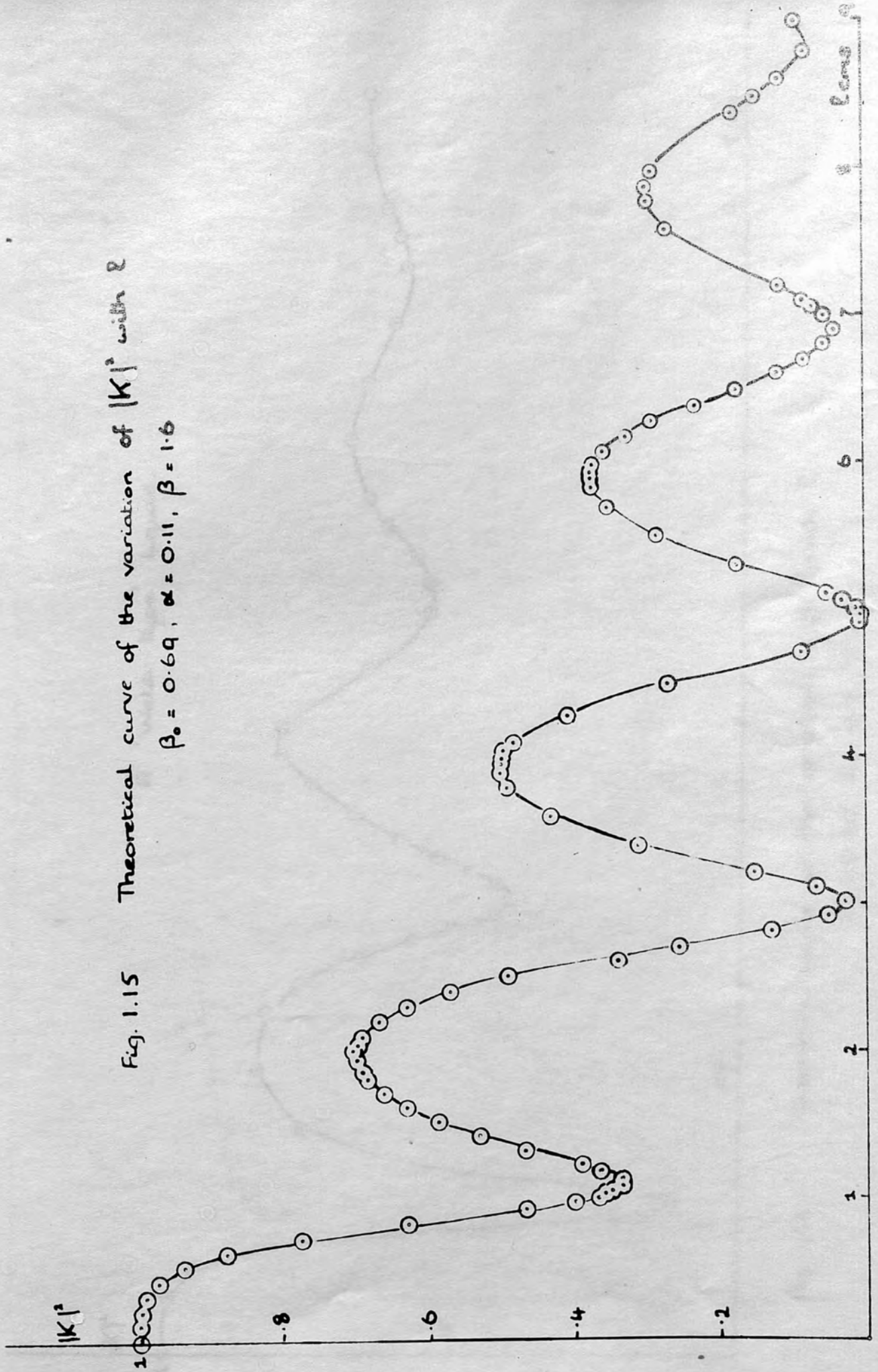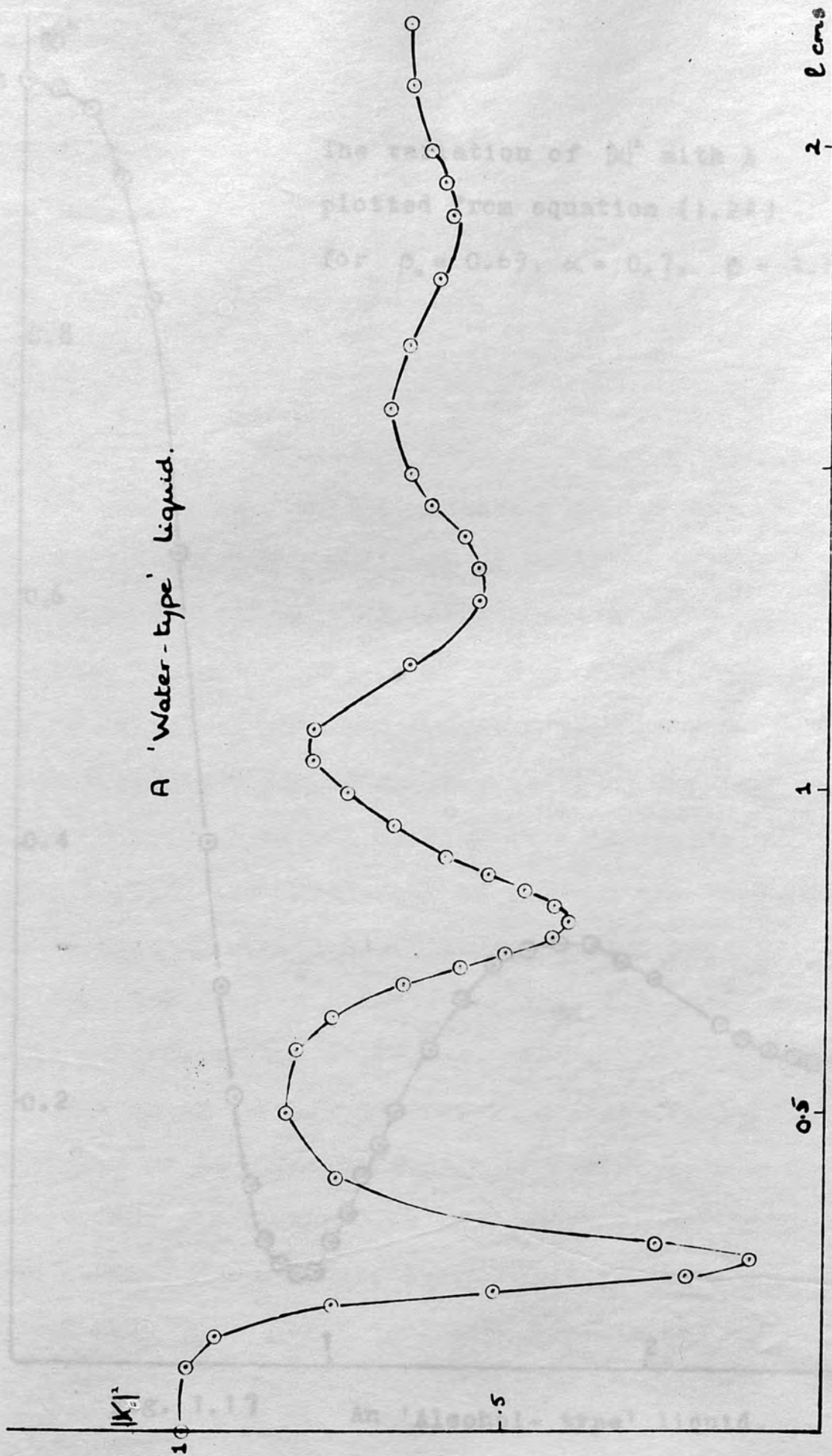
A 'Water-type' liquid.

Fig. 1.16   Theoretical curve of the variation of $|K|^2$ with ℓ.

$\beta_0 = 0.69$, $\alpha = 0.7$, $\beta = 5.9$

The variation of $|k|^2$ with l
plotted from equation (1.24)
for $\rho_o = 0.69$, $\alpha = 0.7$, $\beta = 1.86$
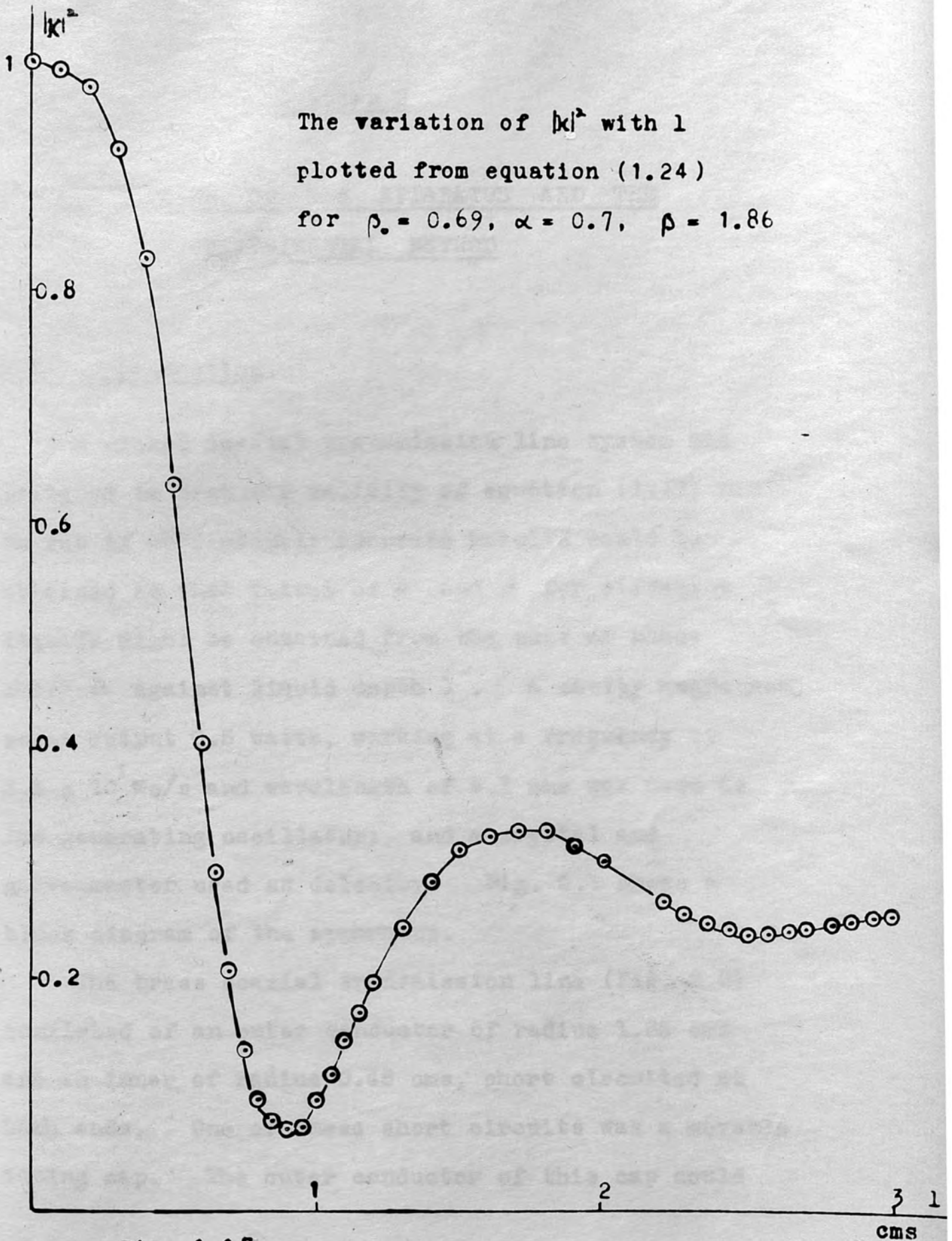
Fig. 1.17   An 'Alcohol- type' liquid.

Chapter 2

DESCRIPTION OF THE APPARATUS AND THE EXPERIMENTAL METHOD

2.1 Introduction.

A closed coaxial transmission line system was designed to test the validity of equation (1.13) and to see if sufficiently accurate results could be obtained so that values of α and β for different liquids might be obtained from the plot of phase shift Δ against liquid depth l. A cavity magnetron, power output 0.5 watts, working at a frequency of 3.3×10^3 Mc/s and wavelength of 9.1 cms was used as the generating oscillator; and a crystal and galvanometer used as detector. Fig. 2.1 shows a block diagram of the apparatus.

The brass coaxial transmission line (Fig. 2.2) consisted of an outer conductor of radius 1.85 cms and an inner of radius 0.45 cms, short circuited at both ends. One of these short circuits was a movable tuning cap. The outer conductor of this cap could
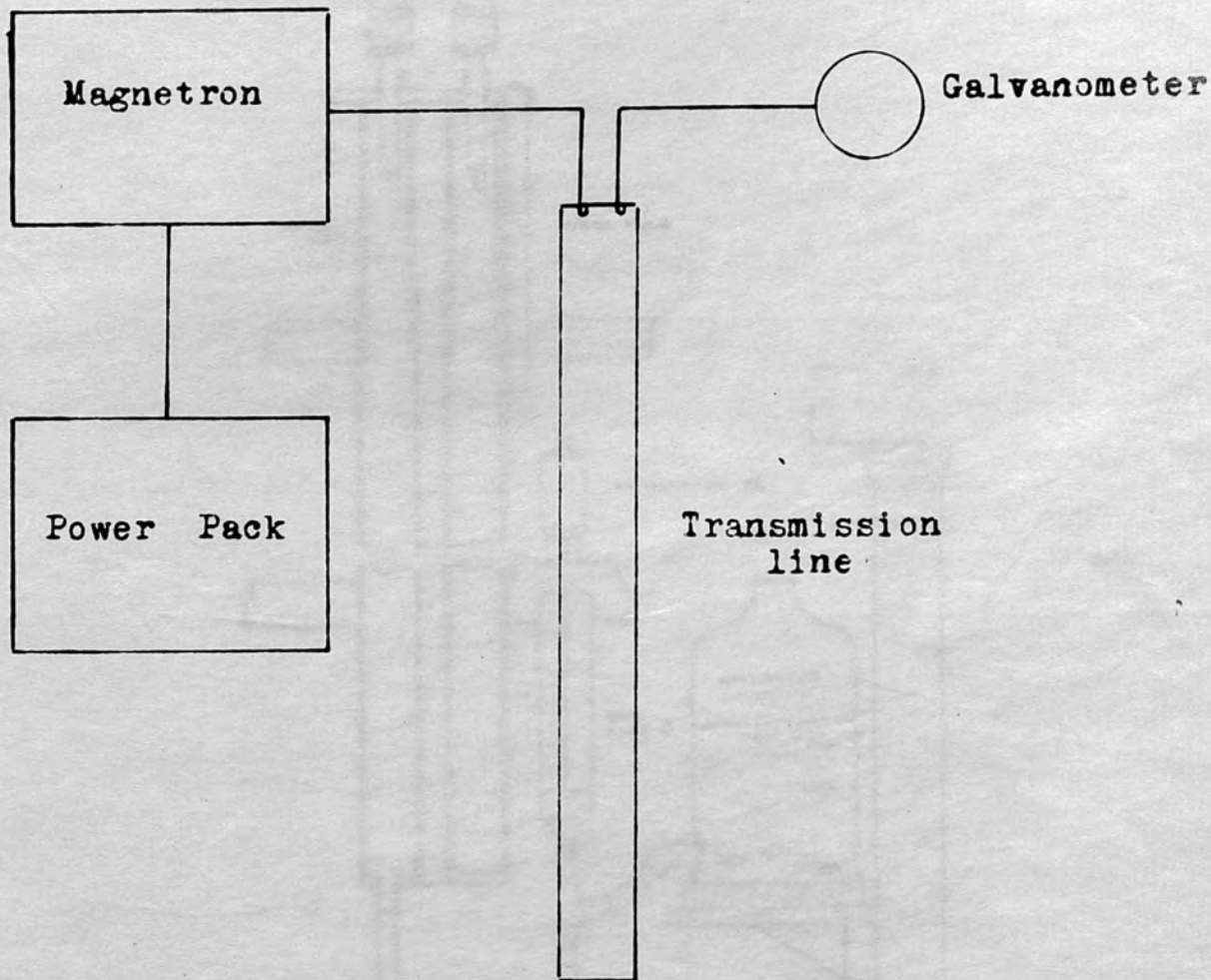
Fig.2.1   Block diagram of the apparatus.
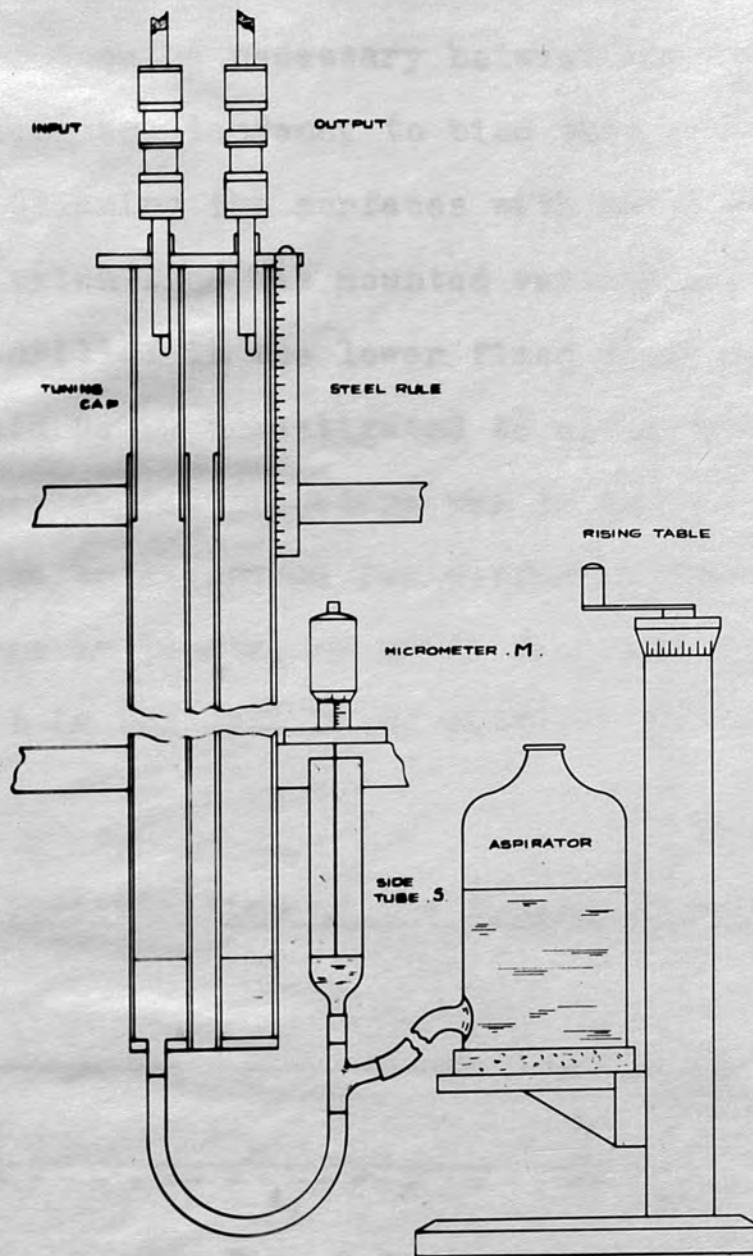
Fig. 2.2   Diagram of apparatus.

slide freely inside the outer conductor of the main line, while its inner conductor could slide freely over the inner of the main transmission line. No lubricant was found to be necessary between the two metal surfaces, but any tendency to bind that occurred was remedied by cleaning the surfaces with metal polish.

The transmission line was mounted vertically and a circular hole drilled in the lower fixed short circuit enabled the liquid being investigated to enter the line. The required experimental procedure was to adjust the length of the line to resonance for different liquid levels, the change in length, or shift Δ , and the depth of liquid 1 in the line being measured as accurately as possible.

2.2 A general investigation of the tuning of such a system.
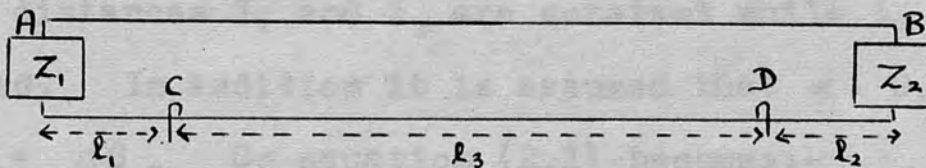
Fig. 2.3 shows a length 1 of transmission line, characteristic impedance Z_o, attenuation coefficient α and phase constant β , terminated at its two ends by impedances Z_1 and Z_2. A voltage injecting loop C is

located at a distance l_1 from Z_1 and a current detecting loop D at a point distant l_2 from Z_2. Willis Jackson[25] derives an expression for the current I in the detector D of this system in equation (5.13) in his monograph on 'High Frequency Transmission Lines', page 108.

$$I = \frac{E}{Z_o} \frac{\cosh(Pl_1 + \psi_1)\cosh(Pl_2 + \psi_2)}{\sinh(Pl + \psi_1 + \psi_2)} \quad \dots\dots\dots(2.1)$$

where E is the E.M.F. injected in series with the line at C,

$$\frac{Z_1}{Z_o} = \tanh\psi_1 = \tanh(u_1 + jv_1)$$

and
$$\frac{Z_2}{Z_o} = \tanh\psi_2 = \tanh(u_2 + jv_2)$$

In the system described in section 2.1, Z_1 of fig. 2.3 represents the movable short circuited tuning cap and Z_2 the fixed short circuit.

Hence $Z_1 = Z_2 = 0$ and $\psi_1 = \psi_2 = 0$

and the distances l_2 and l_3 are constant while l_1 can be varied. In addition it is assumed that $\alpha = 0$, so that $P = j\beta$. So equation (2.1) becomes:-

$$I = \frac{E}{Z_o} \frac{\cos\beta l_1 \cos\beta l_2}{j\sin\beta(l_1 + l_2 + l_3)} \quad \dots\dots\dots\dots(2.2)$$

Movement of the tuning cap, the alteration of l_1, affects terms in both the numerator and denominator of equation (2.2). The term $\cos\beta l_2$ affects only the scale

of the standing wave pattern, while the effect of l_3

in the denominator is to alter the positions of the

maxima.

Multiplying equation (2.2) by its complex conjugate,

$$|I|^2 = \frac{E^2}{Z_o^2} \frac{\cos^2\beta l_1 \cos^2\beta l_2}{\sin^4\beta l} \quad \ldots\ldots\ldots\ldots(2.3)$$

where $l = l_1 + l_2 + l_3$

Differentiating equation (2.3) with respect to the

variable l_1,

$$\frac{d|I|^2}{dl_1} = - \frac{E^2}{Z_o^2} \frac{2\beta\cos^2\beta l_2 (\sin^2\beta l\cos\beta l_1\sin\beta l_1 + \cos^2\beta l_1\sin\beta l\cos\beta l)}{\sin^4\beta l}$$

$$= - \frac{E^2}{Z_o^2} \frac{2\beta\cos\beta l_1\sin\beta l\cos^2\beta l_2 (\sin\beta l_1\sin\beta l + \cos\beta l_1\cos\beta l)}{\sin^4\beta l}$$

$$= - \frac{E^2}{Z_o^2} \frac{2\beta\cos^2\beta l_2\cos\beta l_1\cos\beta(l-l_1)}{\sin^3\beta l}$$

$$= - \frac{E^2}{Z_o^2} \frac{2\beta\cos^2\beta l_2\cos\beta(l_2 + l_3)\cos\beta l_1}{\sin^3\beta l}$$

$$= \text{Constant} \frac{\cos\beta l_1}{\sin^3\beta l} \quad \ldots\ldots\ldots\ldots(2.4)$$

Hence $\frac{d|I|^2}{dl_1} = 0$ when $\cos\beta l_1 = 0$. This condition

only gives the minimum value of $|I|^2$.

From equation (2.3) it is clear that $|I|^2$ will be a

maximum when $\sin^2\beta l$ is zero, and the tuning will be

sharpest if, in addition, $\cos^2\beta l_1$ is 1. That is if,

$$\beta(l_1 + l_2 + l_3) = \beta l_1 + n\pi$$

i.e. $$\beta(l_2 + l_3) = n\pi$$

or $$l_2 + l_3 = n\lambda/2$$

The most unfavourable condition for sharp tuning occurs when,

$$\beta(l_1 + l_2 + l_3) = \beta l_1 + (2n+1)\pi/2$$

i.e. $$\beta(l_2 + l_3) = (2n+1)\pi/2$$

or $$l_2 + l_3 = (2n+1)\lambda/4$$

Then $$\cos^2\beta l_1 = \sin^2\beta l$$

and $|I|^2$, equation (2.3), is independent of the variation of l_1.

To consider a general case, let $\beta(l_2 + l_3) = \theta$

Equation (2.3) becomes,

$$|I|^2 = \frac{E^2}{Z_o^2} \frac{\cos^2\beta l_2 \cos^2(\beta l - \theta)}{\sin^2\beta l} \quad \dots\dots\dots(2.5)$$

$$= \frac{E^2}{Z_o^2} \frac{\cos^2\beta l_2 (\cos\beta l\cos\theta + \sin\beta l\sin\theta)^2}{\sin^2\beta l} \quad \dots(2.5)$$

$$= \frac{E^2}{Z_o^2} \cos^2\beta l_2 (\cot\beta l\cos\theta + \sin\theta)^2 \quad \dots\dots(2.5)$$

The maximum signal should be infinite whatever the value of $\cos\theta$. The tuning however will be sharper the more closely $\cos\theta$ approaches one. The affect of the added term $\sin\theta$ is negligible as $\cot\beta l$ approaches infinity but it will make the maximum asymmetrical.

If, however, the apparatus can be designed so that

$$l_1 = l_3 = 0$$

that is with input and output loops C and D located
in the short circuiting tuning cap, or probes at a
distance of $\lambda/4$ from it, then $\theta = \beta l_2$, and equation
(2.5) becomes:-

$$|I|^2 = \frac{E^2}{Z_o^2} \cot^2 \beta l_2 \quad \ldots\ldots\ldots\ldots(2.6)$$

and the sharpness of the tuning is no longer affected
by the length of the line.   (Fig. 2.4).
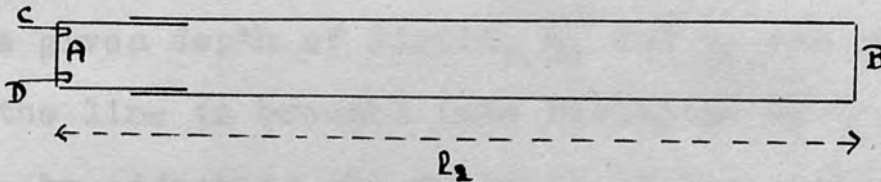
Fig. 2.4

2.2.1   The effect on the tuning of the introduction
of liquid into the line.

Let Z_2 be the impedance at B caused by the
introduction of a depth l of liquid.
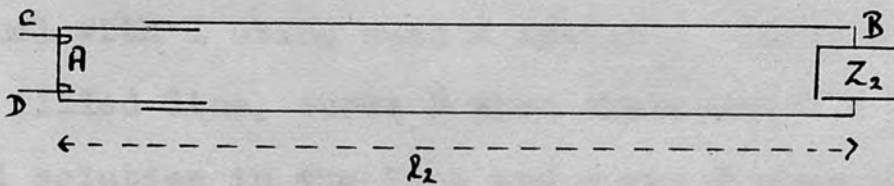
Fig. 2.5

Then $\dfrac{Z_2}{Z_o} = \tanh \psi_2 = \tanh(u_2 + jv_2)$

and equation (2.1) becomes:-

$$I = \frac{E}{Z_0} \frac{\cosh(\beta l_2 + \psi_2)}{\sinh(\beta l_2 + \psi_2)} \quad \dots\dots\dots(2.7)$$

$$= \frac{E}{Z_0} \frac{\cosh(\beta l_2 + u_2 + jv_2)}{\sinh(\beta l_2 + u_2 + jv_2)} \quad \dots\dots\dots(2.7)$$

Multiplying equation (2.7) by its complex conjugate,

$$|I|^2 = \frac{E^2}{Z_0^2} \left\{ \frac{ch^2(\beta l_2 + u_2)\cos^2 v_2 + sh^2(\beta l_2 + u_2)\sin^2 v_2}{sh^2(\beta l_2 + u_2)\cos^2 v_2 + ch^2(\beta l_2 + u_2)\sin^2 v_2} \right\} \quad \dots(2.8)$$

$$= \frac{E^2}{Z_0^2} \left\{ \frac{sh^2(\beta l_2 + u_2) + \cos^2 v_2}{sh^2(\beta l_2 + u_2) + \sin^2 v_2} \right\} \quad \dots\dots\dots(2.8)$$

For a given depth of liquid, u_2 and v_2 are constant,
and the line is brought into resonance by the variation
of l_2 by adjusting the position of the tuning cap until
a maximum signal is obtained on the galvanometer.   The
sharpness of tuning will thus vary as Z_2 is altered,
i.e. as the liquid depth is varied, but by placing the
input and output loops in the tuning cap the most
favourable conditions may be obtained.

Fig. 2.6 shows the variation of the galvanometer
signal with l using such a system.   Curve A is for an
air filled line, curve B when there are 1.2 cms of a 2N
NaCl solution in the line and curve C when there are 0.21
cms of a 0.66N NaCl solution in the line.   Curve B
represents the tuning sharpness usually observed while
curve C represents poor tuning conditions.   The coupling
of the input and output loops to the line was not the

0.5   0   0,5   1 cms.

A   B   C
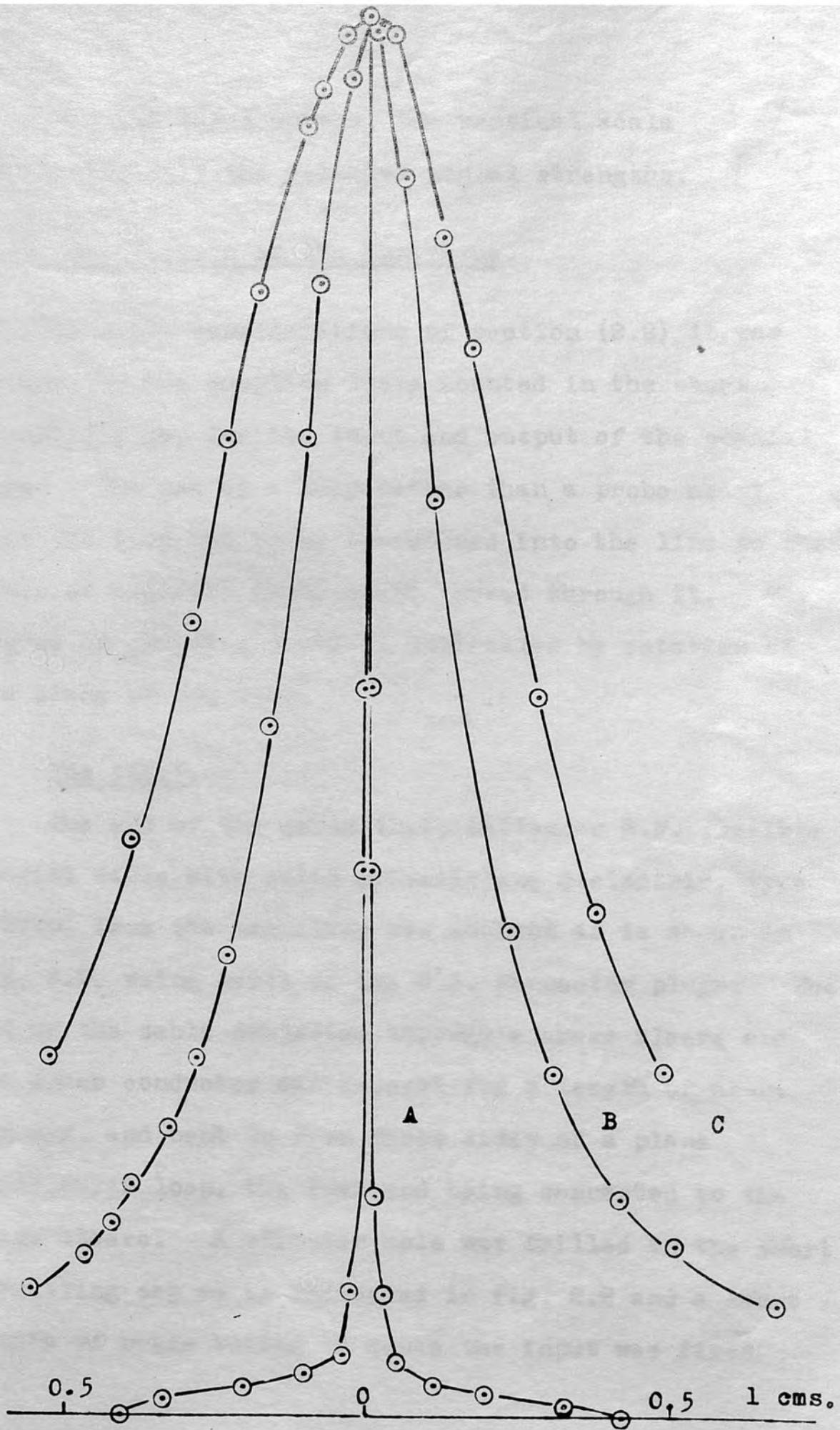
Fig. 2.6   The variation of the Q of the line.

same for the three curves, the vertical scale
indicating only the relative signal strengths.

2.3   Description of the apparatus.

From the considerations of section (2.2) it was
decided to use coupling loops mounted in the short
circuiting cap for the input and output of the coaxial
line.   The use of a loop rather than a probe meant
that the loop had to be introduced into the line so that
lines of magnetic force could thread through it.   The
degree of coupling could be controlled by rotation of
the plane of the loop.

The input.

The end of the cable (B.I. Callender R.F. flexible
coaxial cable with solid polyethylene dielectric, type
T 3020) from the magnetron was mounted as is shown in
fig. 2.2, using parts of the W.S. connector plugs.   The
end of the cable projected through a brass sleeve and
the inner conductor was exposed for a length of about
two cms. and bent to form three sides of a plane
rectangular loop, the free end being connected to the
brass sleeve.   A circular hole was drilled in the short
circuiting cap as is indicated in fig. 2.2 and a short
length of brass tubing to house the input was fixed

Cable to
galvanometer

Spring

Detector

Crystal

Polythene
washer

Polythene

Brass
tubing
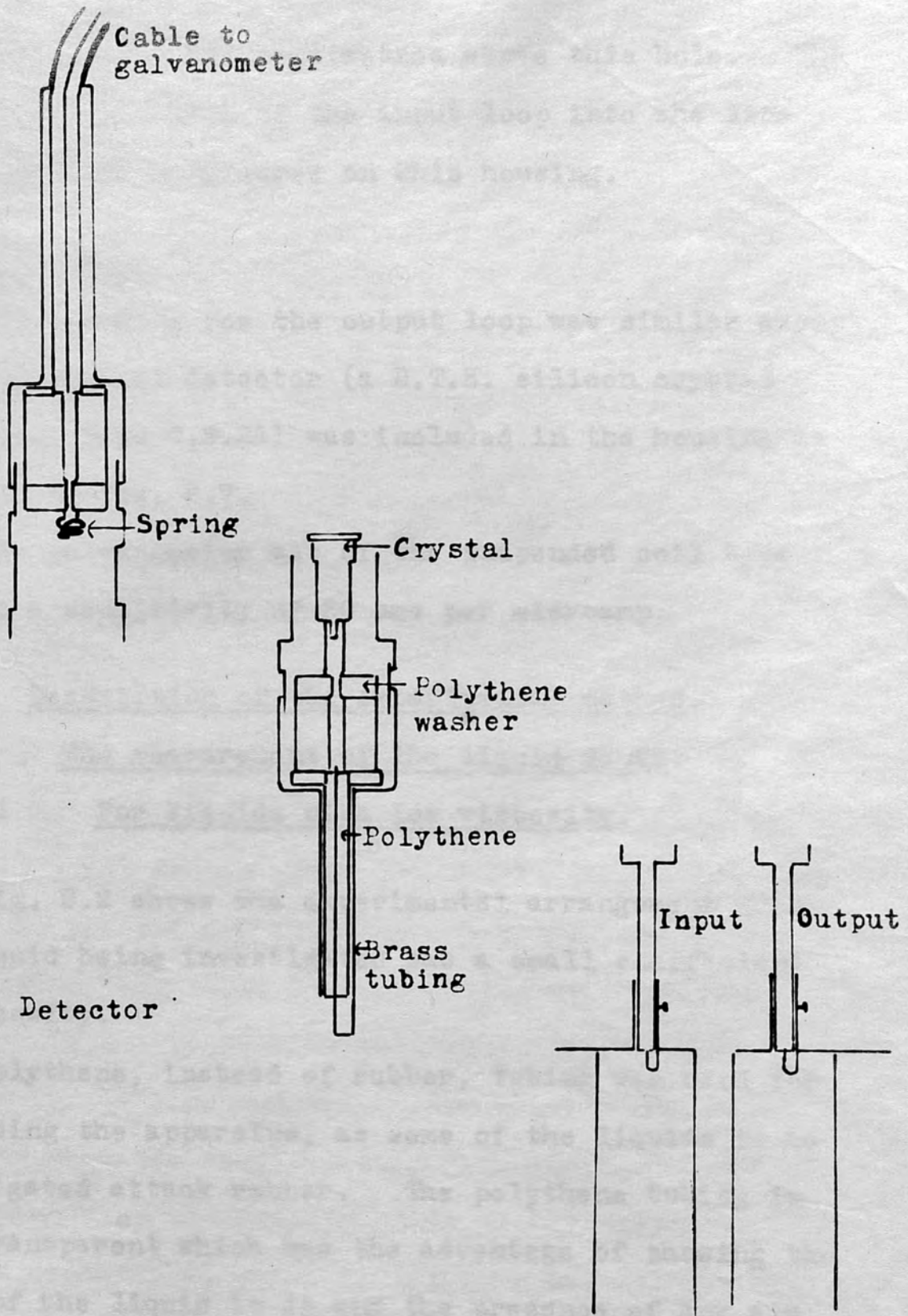
Fig. 2.7

Input   Output

Part of tuning cap,

showing mounting of coupling

loops.

projecting about two centimetres above this hole. The depth of penetration of the input loop into the line was controlled by a screw on this housing.

The output.

The mounting for the output loop was similar except that the crystal detector (a B.T.H. silicon crystal rectifier, type C.S.2A) was included in the housing as is shown in fig. 2.7.

The galvanometer was of the suspended coil type and had a sensitivity of 20 cms per microamp.

2.4   Description of the experimental method.

2.4.1   The measurement of the liquid depth.

2.4.1.1   For liquids with low viscosity.

Fig. 2.2 shows the experimental arrangement when the liquid being investigated has a small coefficient of viscosity.

Polythene, instead of rubber, tubing was used for connecting the apparatus, as some of the liquids to be investigated attack rubber. The polythene tubing is semi-transparent which has the advantage of showing the level of the liquid in it and the presence of any air bubbles. Its lack of flexibility compared with rubber made it awkward to handle, but it was found that

satisfactory polythene-glass and polythene-brass joints
could be made by warming the ends to be joined by blowing
hot air through them.

The liquid being investigated was poured into the
250 cc aspirator on the rising table.   One revolution
of the screw on the rising table caused an alteration
of height of 5 mm., and the head was divided into five
equal divisions, each corresponding to 1 mm.   The
setting of the screw could be judged to the nearest
tenth of a millimeter.

The liquid in the transmission line so far described
was not at atmospheric pressure, so that the level of the
liquid in the side tube S gave no indication of the level
in the line.   This difficulty was overcome by drilling
two small holes in the outer conductor of the transmission
line.   To prevent any radiation from the line it was
essential to keep these holes as small as possible, and
so the minimum size of hole that would give atmospheric
pressure in the line was determined experimentally by
observing when the liquid levels were equal.   Two holes
of diameter 0.15 cms were found to be satisfactory.

The depth of liquid 1 in the line was determined
by the change of liquid level in the glass tube S.   The
long metal pointed tipped rod was attached at its other

end to the micrometer head M.   This rod was adjusted
so that its tip just broke the surface of the liquid
in S when the liquid was on the point of entering the
transmission line.   (This was determined either by
observing when the galvanometer deflection fell from
its maximum reading of air resonance, or by the removal
of the tuning cap and direct observation.   The latter
method was found the more satisfactory.)   A certain
amount of liquid was allowed to enter the line by
raising the reservoir on the rising table.   The stick
was withdrawn to just above the liquid level in S by
means of the micrometer screw M and then slowly turned
till it broke the liquid surface again.   The surface
was well illuminated while this adjustment was being
made.   The difference between the two readings of the
micrometer screw gave l.   More consistent results were
obtained by lowering the stick into the liquid surface
and then reading the vernier, rather than by taking the
reading when the pin broke away from the surface.   The
surface tension of the liquid being investigated
effected these readings, but with care the relative
change of liquid depth in the line could be determined.
The scale on the micrometer head M was divided into
five divisions covering a distance of 1.270 cms.   The
vernier enabled each division to be read to the nearest

one hundredth, and since each division corresponded
to 0.254 cms, the depth could be read to 0.002 of a cm.
A screw at the top of the micrometer enabled the stick
to be reset when l was greater than 1.27 cms.

At first the micrometer head had been fixed to
the movable short circuiting cap and a very long stick
was required. It had been hoped with this arrangement
to determine both l and Δ from the readings of the
micrometer, the adjustment of the tuning cap being
made by a screw on a rack and pinion arrangement. The
rack and pinion was not satisfactory and it was found
that it was better to adjust the tuning cap manually.
As the tuning cap had to be capable of fairly easy
movement, it was found that the slight pressure required
in moving the micrometer might alter the position of the
tuning cap, so that the advantage of having the micro-
meter attached to the tuning cap was lost. In addition
the long stick required with this arrangement was
unsatisfactory due to whip in the stick and the con-
sequent circular motion of its tip in the liquid surface
at S. It was found that the shorter the length of this
stick the greater was the accuracy which could be ob-
tained. The micrometer head was therefore removed from
the tuning cap and attached to a lower horizontal bar

on the framework supporting the apparatus. The
length of stick required for a set of observations
depended on the length of liquid it was desired to
investigate. For liquids with high values of α
this length is not likely to exceed 5 or 6 cms, while
for liquids with a low value of α it may be as much
as 20 or 30 cms. So before beginning a series of
readings with a particular liquid the likely maximum
value of 1 required was estimated and the position of
the apparatus on the rigid supporting framework adjusted
so that the total length of the stick could be kept to
this minimum length.

The reading of the screw of the rising table was
recorded during a set of observations in addition to
that of the micrometer head as a check.

2.4.1.2   For liquids with high viscosity.

Fig. 2.8 shows the experimental arrangement when
the liquid being investigated had a large coefficient
of viscosity.

A vessel, with a lip, such as a beaker, of a
suitable depth (the depth depending upon the depth of
liquid to which it was wished to continue the readings)
and a diameter somewhat greater than that of the trans-
mission line, was filled to the brim with the liquid.
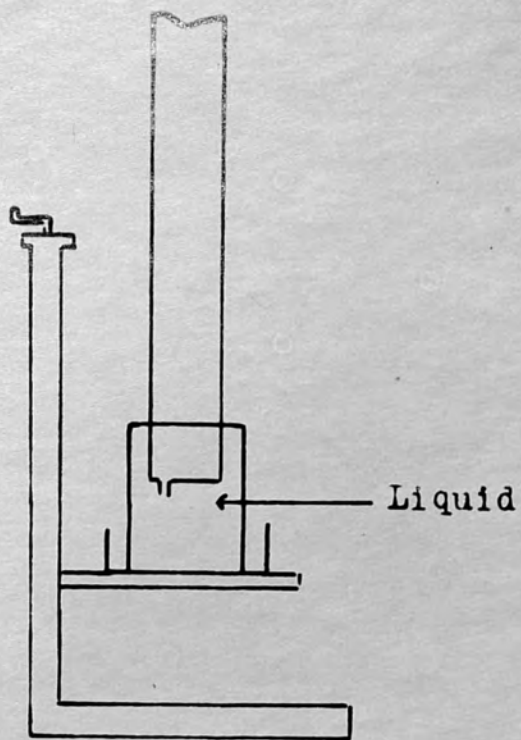
Fig. 2.8   Apparatus used for liquids with a high
viscosity.
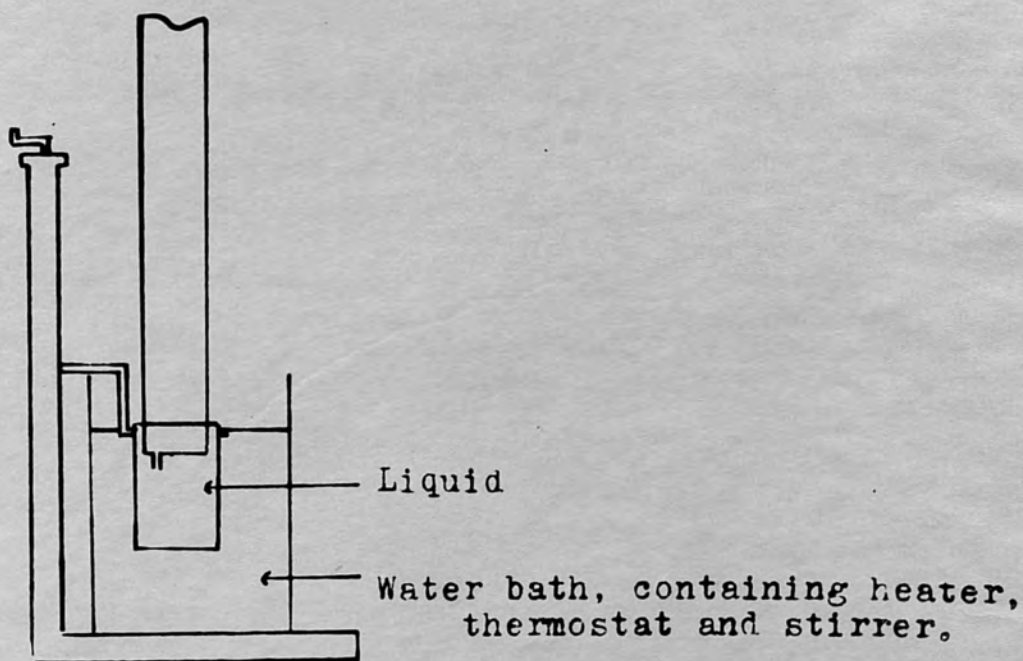
Fig. 2.10   Apparatus used for controlling the

temperature of the liquid.

This vessel was placed on the platform of the rising
table immediately below the short circuited end of the
transmission line.   The lip of the vessel allowed the
liquid displaced by the transmission line as l was
increased to over flow into the trough below and so no
correction for liquid displacement was necessary.   With
this arrangement the depth of liquid filling the line
was determined by the distance travelled by the screw
of the rising table, which could be read to the nearest
tenth of a millimetre.

When the experimental arrangement of section 2.4.1.1
was used for a viscous liquid such as cyclohexanol
(η = 0.2 poise at 39°C) the time taken for the liquid
levels to equalise was so long that the time taken
between each observation made the method impracticable.
With the simpler arrangement of this section however,
the liquid quickly assumed its new level.   This was
indicated by the rapid and definite movement of the
galvanometer mirror after a change in l .

2.4.2   The measurement of the phase shift.

(a)   (b)   (c)   Fig. 2.9
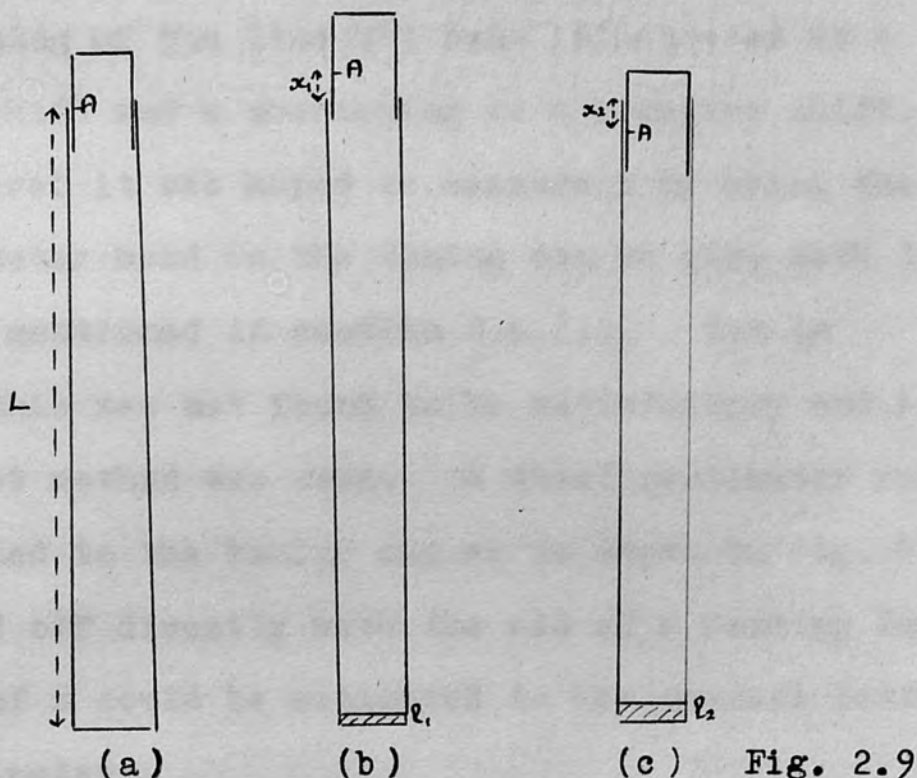
In Fig. 2.9, let L be the length of the line from
the terminating fixed short circuit to its free upper
rim.   When resonance is obtained with an air filled
line, fig. 2.9a, let A be a point on the tuning cap
coinciding with the upper rim of the line.   Suppose
that on the introduction of a depth l_1 of liquid the
line length has to be increased by x_1 to obtain
resonance again.   (Fig.2.9b).   Then

$$L + \Delta_1 = L + x_1 - l_1 \qquad \ldots\ldots\ldots(2.9)$$

If, fig. 2.9c, the introduction of a depth l_2 causes
a reduction x_2 in the length of line necessary for

resonance, then

$$L + \triangle_{2} = L - (x_{2} + l_{2}) \quad \ldots\ldots\ldots\ldots(2.10)$$

A lengthening of the line has been interpreted as a
positive shift and a shortening as a negative shift.

At first it was hoped to measure x by using the
one micrometer head on the tuning cap to give both l
and x, as mentioned in section 2.4.1.1. But in
practice this was not found to be satisfactory and a
more direct method was used. A steel centimeter rule
was attached to the tuning cap as is shown in fig. 2.2,
and x read off directly with the aid of a reading lens.
Readings of x could be estimated to the nearest tenth
of a millimeter.

The position of a point on the rim of the fixed
portion of the transmission line against the scale when
air resonance was obtained was recorded. This was the
arbitrary origin A. The new position of the point on
the rim when a certain depth l of liquid was in the line
under resonant conditions was recorded. The difference
in these two readings gave x. The liquid depth l was
either subtracted or added, depending on whether the
length of the line had been lengthened or shortened and
hence, by equations (2.9) and(2.10), the phase shift for
a given depth of liquid obtained.

2.4.3 The entry of liquid into the line.

The entry of a liquid into the line was observed
by removing the tuning cap after the air resonance
position had been recorded.   The short circuited end of
the transmission line was illuminated and the reservoir
raised through small intervals.   If the liquid was
viscous, a short interval of time was allowed to elapse
between each incremental rise of the reservoir.   The
liquid rose up through the small circular hole in the
short circuiting plate, and as it became level with the
short circuiting plate the light reflected from its
surface changed - in a manner similar to that of the
light reflected from the liquid at the end of a capillary
tube when its surface tension is being measured by
Ferguson's method.   The moment it was plane could be
determined accurately.   If the base plate was dry, the
liquid continued to rise in the tube and bulged up a
little way before spreading over the surface.   It then
started to spread over the plate in an irregular manner.
The entry and exit of the liquid into the line was
observed several times.   The effects of surface tension
were greater with a dry tube than with a wet one.   When
the reservoir level was lowered below the level of the
short circuiting plate a little liquid remained in the

line unless the reservoir was lowered very slowly.
The use of a slightly curved plate to enable the liquid
to drain away completely was not however found to be
satisfactory.

It is shown in section 3.2.2 that the gradient of
the tan ϕ curve at $1 = 0$ depends only on β_o.

$$\frac{d\phi}{dl} = -2\beta_o = -\frac{4\pi}{\lambda_o}$$

Differentiating equation (1.19) with respect to 1
gives

$$\frac{d\Delta}{dl} = \frac{\lambda_o}{4\pi} \frac{d\phi}{dl}$$

so that the gradient of the experimental curve of
phase shift against liquid depth should be - 1 at the
origin.   This gave a rough check on whether the
position of the origin had been correctly chosen.

2.4.4   The recording of the galvanometer deflection.

In addition to measuring 1 and x, the galvanometer
deflection at resonance was also recorded.   As is
shown in section 1.6, the galvanometer deflection is
proportional to $|k'|^2$, where k' is the ratio of the
amplitude of the reflected to the incident voltage.

Before the liquid entered the line, the coupling
of the input and output loops was reduced to a minimum
in order to reduce the signal obtained at air resonance

so that the spot of light reflected from the galvano-
meter mirror remained on the scale. As the liquid
entered the line this deflection fell and very shortly
the coupling would have to be increased to obtain a
reasonable signal at resonance. As l increased however
the deflection at resonance became larger and sometimes
it was necessary to decrease the coupling between the
loops in order to keep the spot of light on the scale.
It would then be necessary to increase the degree of
coupling again as l was increased further, and so on
as the liquid level increased. This alteration in the
degree of coupling between the loops is indicated on the
graphs showing the variation of galvanometer signal with
l by a sudden discontinuity in the curve.

These readings, unlike the measurements of the
phase shift, are not independent of any fluctuations
that may occur in the input voltage. The regularity
of the experimental curves shows that the fluctuations
must have been slight.

2.5   The control of temperature.

The majority of the liquids were investigated at
room temperature and at first no special precautions
were made to keep the temperature constant. A

thermometer was placed in the reservoir containing the liquid being investigated and its readings were recorded at intervals throughout the experiment. A series of readings might take five or six hours to complete and variations of one or two degrees in the temperature might be observed during the day. The apparatus was set up in a semi-basement room against an inside wall which accounted for this small fluctuation of temperature.

When it was decided to apply this method to an investigation of the microwave absorption in blood it was realised that the results at body temperature would be of greater value than those at room temperature and so a thermostatic control system was designed. The experimental arrangement of section 2.4.1.2 had to be used. The beaker containing the liquid to be invest-igated was held in a clamp attached to a rod on the rising table. (Fig. 2.10). The beaker was supported so that its neck was just above the level of the water in a glass tank. The tank also contained a heating element, stirrer and the controlling thermometer so that the water in the tank could be maintained at any desired temperature. As the level of the liquid in the transmission line increased the beaker was raised out of the tank , so that before starting a series of

readings the maximum liquid depth to be investigated
was decided and the level of the water in the tank
adjusted so that it was possible to add water to the
tank during the readings so that all the liquid in the
beaker remained in the water tank.

Chapter 3

THE CALCULATION OF α AND β

3.1 Introduction.

From the graph of the experimentally observed variation of phase shift Δ with liquid depth the attenuation coefficient α and the phase constant β of a liquid may be deduced. The methods available depend upon the type of liquid. Before outlining these methods expressions for the gradient of the $\tan \phi$ curve, the turning points of the $|K|^2$ curve and the condition for no standing wave on the line will be derived.

3.2 The gradient of $\tan \phi$.

From equation (1.13)

$$\tan \phi = \frac{2\beta_.(\alpha \sinh 2\alpha l + \beta \sin 2\beta l)}{(\beta_.^2 - \alpha^2 - \beta^2) \cosh 2\alpha l - (\beta_.^2 + \alpha^2 + \beta^2) \cos 2\beta l}$$

Differentiating with respect to l, and putting

$$a = \beta_.^2 - \alpha^2 - \beta^2 \qquad \text{and} \qquad b = \beta_.^2 + \alpha^2 + \beta^2$$

$$\frac{d(\tan\phi)}{dl} = (a\,ch2\alpha l - b\cos2\beta l)4\beta_0(\alpha^2 ch2\alpha l + \beta^2\cos2\beta l)$$

$$-4\beta_0(\alpha\,sh2\alpha l + \beta\sin2\beta l)(a\alpha\,sh2\alpha l + b\beta\sin2\beta l)/D^2$$

where

$$D^2 = a^2 ch^2 2\alpha l - 2ab\,ch2\alpha l\cos2\beta l + b^2\cos^2 2\beta l$$

Therefore

$$\frac{d(\tan\phi)}{dl} = \frac{4\beta_0}{D^2}\left\{ a\alpha^2 - b\beta^2 + (a\beta^2 - b\alpha^2)ch2\alpha l\cos2\beta l \right.$$

$$\left. - (a+b)\alpha\beta\,sh2\alpha l\sin2\beta l\right\} \qquad \ldots\ldots\ldots(3.1)$$

But

$$\frac{d(\tan\phi)}{dl} = \sec^2\phi\,\frac{d\phi}{dl}$$

Therefore

$$\frac{d\phi}{dl} = \frac{1}{(1 + \tan^2\phi)}\,\frac{d(\tan\phi)}{dl} \qquad \ldots\ldots\ldots(3.2)$$

Hence substituting from equations (1.13) and (3.1)

$$\frac{d\phi}{dl} = \frac{D^2}{D^2 + 4\beta_0^2(\alpha\,sh2\alpha l + \beta\sin2\beta l)^2} \times \frac{4\beta_0}{D^2}\left\{ a\alpha^2 - b\beta^2 \right.$$

$$\left. + (a\beta^2 - b\alpha^2)ch2\alpha l\cos2\beta l - (a+b)\alpha\beta\,sh2\alpha l\sin2\beta l\right.$$

$$\qquad \ldots\ldots\ldots(3.2)$$

$$\frac{d\phi}{dl} = \frac{4\beta_0\left\{a\alpha^2 - b\beta^2 + (a\beta^2 - b\alpha^2)ch2\alpha l\cos2\beta l - (a+b)\alpha\beta\,sh2\alpha l\sin2\beta l\right\}}{D}$$

where

$$D = a^2 ch^2 2\alpha l - 2ab\,ch2\alpha l\cos2\beta l + b^2\cos^2 2\beta l + 4\beta_0^2\alpha^2 sh^2 2\alpha l$$

$$+ 8\beta_0^2\alpha\beta\,sh2\alpha l\sin2\beta l + 4\beta_0^2\beta^2\sin^2 2\beta l \qquad \ldots\ldots\ldots(3.2)$$

It was shown in section 1.4 that

$$\Delta = \frac{\lambda_0}{4\pi}\,\phi \qquad \ldots\ldots\ldots\ldots(1.19)$$

so that the gradient of the experimental curve $d\Delta/dl$ is related to $d\phi/dl$ by the expression,

$$\frac{d\Delta}{dl} = \frac{\lambda_o}{4\pi} \frac{d\phi}{dl} \quad \ldots\ldots\ldots(3.3)$$

3.2.1 The gradient of the curve when $\tan\phi = 0$.

When $\tan\phi = 0$, equation (3.2) becomes,

$$\frac{d\phi}{dl} = \frac{d(\tan\phi)}{dl}$$

and equation (1.13) shows that $\tan\phi = 0$ when

$$\sin2\beta l = \frac{\alpha}{\beta} \sinh2\alpha l \quad \ldots\ldots(3.4)$$

but for small liquid depths and low values of α,

$$\frac{\alpha}{\beta}\sinh2\alpha l \longrightarrow 0$$

i.e. $\sin2\beta l = 0 \quad \ldots\ldots\ldots(3.5)$

or $2\beta l = n\pi$ where n is $0,1,2,3\ldots\ldots$

and $\cos2\beta l = \pm 1$

depending on whether n is even or odd. Substituting these conditions in equation (3.2),

$$\frac{d\phi}{dl} = \frac{4\beta_o\left\{a\alpha^2 - b\beta^2 \pm (a\beta^2 - b\alpha^2)\,ch2\alpha l\right\}}{(a\,ch2\alpha l \mp b)^2} \quad \ldots\ldots(3.6)$$

$$= \frac{4\beta_o\left\{\beta_o^2(\alpha^2 - \beta^2) - (\alpha^2 + \beta^2)^2 \pm \left[\beta_o^2(\beta^2 - \alpha^2) - (\alpha^2 + \beta^2)^2\right]ch2\alpha l\right\}}{\left[(\beta_o^2 - \alpha^2 - \beta^2)\,ch2\alpha l \mp (\beta_o^2 + \alpha^2 + \beta^2)\right]^2}$$

Thus the gradient of the curve when $\tan\phi = 0$ and n is **even** is,

$$\frac{d\phi}{dl} = \frac{4\beta_o\left\{\beta_o^2(\alpha^2 - \beta^2) - (\alpha^2 + \beta^2)^2 + \left[\beta_o^2(\beta^2 - \alpha^2) - (\alpha^2 + \beta^2)^2\right]ch2\alpha l\right\}}{\left[(\beta_o^2 - \alpha^2 - \beta^2)\,ch2\alpha l - (\beta_o^2 + \alpha^2 + \beta^2)\right]^2}$$

$$\ldots\ldots(3.7)$$

and when n is odd,

$$\frac{d\phi}{dl} = \frac{4\beta_o\{\beta_o^2(\alpha^2 - \beta^2) - (\alpha^2 + \beta^2)^2 - [\beta_o^2(\beta^2 - \alpha^2) - (\alpha^2 + \beta^2)^2] \, ch2\alpha l\}}{[(\beta_o^2 - \alpha^2 - \beta^2) \, ch2\alpha l + (\beta_o^2 + \alpha^2 + \beta^2)]^2} \quad \ldots\ldots(3.8)$$

Since for equation (3.5) to hold α must be small, the effect of α^2 in (3.6) is negligible. Hence equations (3.7) and (3.8) become, when n is even,

$$-\frac{d\phi}{dl} = \frac{4\beta_o\{\beta_o^2\beta^2 + \beta^4 + (\beta^4 - \beta_o^2\beta^2) \, ch2\alpha l\}}{[(\beta^2 - \beta_o^2) \, ch2\alpha l + \beta_o^2 + \beta^2]^2} \quad \ldots\ldots(3.7a)$$

and when n is odd,

$$-\frac{d\phi}{dl} = \frac{4\beta_o\{\beta_o^2\beta^2 + \beta^4 - (\beta^4 - \beta_o^2\beta^2) \, ch2\alpha l\}}{[(\beta^2 - \beta_o^2) \, ch2\alpha l - (\beta_o^2 + \beta^2)]^2} \quad \ldots\ldots(3.8a)$$

Thus, for a given liquid, the gradient when $\tan\phi = 0$ and n is even, is always negative; and as l increases the numerical value of the gradient decreases until the steady negative value of $\tan\phi$ is reached. When, however, n is odd, the gradient at $\tan\phi = 0$ has at first (assuming the value of α is sufficiently low for $\tan\phi$ to go to infinity for small liquid depths) a fairly high negative value which increases with l until a critical depth is reached where the slope of the curve suddenly changes sign, (this corresponds to $\tan\phi$ no longer reaching infinity) and from this point

onwards the curve has positive gradients of decreasing magnitude at points where n is odd, until the steady negative value of tan ϕ is reached. This is illustrated in figs. 1.10 and 1.11. For 'water-type' liquids, (figs. 1.12 and 1.13), the gradients at tan ϕ = 0 are alternately negative and positive, as n is even or odd, and of decreasing numerical value as l increases.

Equations (3.7a) and (3.8a) may be simplified further. It has been assumed that α is small and that $\alpha^2 \ll \beta^2$.

When n is even,

$$- \frac{d\phi}{dl} = \frac{4\beta_o\beta^2\{(\beta^2 - \beta_o^2)\,\text{ch}2\alpha l + \beta_o^2 + \beta^2\}}{[(\beta^2 - \beta_o^2)\,\text{ch}2\alpha l + \beta_o^2 + \beta^2]^2} \qquad \dots\dots\dots(3.7a)$$

$$= \frac{4\beta_o\beta^2}{(\beta^2 - \beta_o^2)\,\text{ch}2\alpha l + \beta_o^2 + \beta^2}$$

But cosh2αl may be expanded as:-

$$\text{ch}2\alpha l = 1 + \frac{(2\alpha l)^2}{2!} + \frac{(2\alpha l)^4}{4!} +$$

hence with the condition that α is small,

$$\text{ch}2\alpha l \longrightarrow 1 \qquad \text{and}$$

$$- \frac{d\phi}{dl} \longrightarrow \frac{4\beta_o\beta^2}{2\beta^2} \longrightarrow 2\beta \qquad \dots\dots\dots(3.7b)$$

Similarly, when n is odd,

$$- \frac{d\phi}{dl} = \frac{4\beta_o\beta^2}{(\beta_o^2 + \beta^2) - (\beta^2 - \beta_o^2)\,\text{ch}2\alpha l} \qquad \dots\dots\dots(3.8a)$$

and as $\cosh 2\alpha l \longrightarrow 1$,

$$- \frac{d\phi}{dl} \longrightarrow \frac{2\beta^2}{\beta_o} \qquad \ldots\ldots(3.8b)$$

3.2.2 The gradient of the curve when $l = 0$.

When $n = 0$ and $l = 0$, $\cosh 2\alpha l = 1$, so that equation (3.7) becomes,

$$\frac{d\phi}{dl} = \frac{4\beta_o \{\beta_o^2(\alpha^2 - \beta^2) - (\alpha^2 + \beta^2)^2 + \beta_o^2(\beta^2 - \alpha^2) - (\alpha^2 + \beta^2)^2\}}{[(\beta_o^2 - \alpha^2 - \beta^2) - (\beta_o^2 + \alpha^2 + \beta^2)]^2}$$

$$= -2\beta_o$$

Thus the gradient at $l = 0$ is independent of the liquid used and depends only on β_o , i.e. on the wavelength λ_o of the oscillator. Hence the longer λ_o , the less steep is the gradient at $l = 0$, and the shorter λ_o , the steeper is the gradient. Thus for a given λ_o , the gradient at $l = 0$ on the graph of Δ against l must be the same for all liquids. This fact, as was shown in section 2.4.3, may be used to indicate when the liquid enters the line.

3.2.3 The turning points of the curve.

The turning points of the $\tan\phi$ curve occur when $d\phi/dl = 0$. From equation (3.2) this is when:-

$$4\beta_o\{a\alpha^2 - b\beta^2 + (a\beta^2 - b\alpha)ch2\alpha l cos2\beta l - (a+b)\alpha\beta sh2\alpha l sin2\beta l$$

$$= 0 \qquad \ldots\ldots\ldots(3.9)$$

Substituting for a and b,

$$\alpha^2\beta_0^2 - \alpha^4 - 2\alpha^2\beta^2 - \beta_0^2\beta^2 - \beta^4 + (\beta_0^2\beta^2 - 2\alpha^2\beta^2 - \beta^4 - \alpha^2\beta_0^2 - \alpha^4)\,\text{ch}2\alpha l\cos2\beta l$$
$$= 2\beta_0^2\alpha\beta\,\text{sh}2\alpha l\sin2\beta l \qquad \ldots\ldots\ldots(3.9)$$

If, as is generally the case, $\alpha^2 \ll \beta^2$, then equation (3.9) becomes:-

$$-\beta_0^2\beta^2 - \beta^4 - 2\alpha^2\beta^2 + (\beta_0^2\beta^2 - 2\alpha^2\beta^2 - \beta^4)\,\text{ch}2\alpha l\cos2\beta l$$
$$= 2\beta_0^2\alpha\beta\,\text{sh}2\alpha l\sin2\beta l \qquad \ldots\ldots\ldots(3.9a)$$

or neglecting α^2,

$$-\beta_0^2\beta^2 - \beta^4 + (\beta_0^2\beta^2 - \beta^4)\,\text{ch}2\alpha l\cos2\beta l$$
$$= 2\beta_0^2\alpha\beta\,\text{sh}2\alpha l\sin2\beta l \qquad \ldots\ldots\ldots(3.9b)$$

3.3 <u>The gradient of the curve relating $|k|^2$ with l.</u>

From equation (1.24)

$$|k|^2 = \frac{b\,\text{ch}2\alpha l - a\cos2\beta l - 2\beta_0(\beta\,\text{sh}2\alpha l - \alpha\sin2\beta l)}{b\,\text{ch}2\alpha l - a\cos2\beta l + 2\beta_0(\beta\,\text{sh}2\alpha l - \alpha\sin2\beta l)}$$

Differentiating with respect to l, and writing D for the denominator of equation (1.24),

$$D^2\frac{d|k|^2}{dl} = (b\,\text{ch}2\alpha l - a\cos2\beta l + 2\beta_0\beta\,\text{sh}2\alpha l - 2\beta_0\alpha\sin2\beta l)$$
$$\times (2\alpha b\,\text{sh}2\alpha l + 2\beta a\sin2\beta l - 4\beta_0\alpha\beta\,\text{ch}2\alpha l + 4\beta_0\alpha\beta\cos2\beta l)$$
$$- (b\,\text{ch}2\alpha l - a\cos2\beta l - 2\beta_0\beta\,\text{sh}2\alpha l + 2\beta_0\alpha\sin2\beta l)$$
$$\times (2\alpha b\,\text{sh}2\alpha l + 2\beta a\sin2\beta l + 4\beta_0\alpha\beta\,\text{ch}2\alpha l - 4\beta_0\alpha\beta\cos2\beta l)$$
$$\ldots\ldots\ldots(3.10)$$

which, on simplification, reduces to

$$D^2 \frac{d|k|^2}{dl} = 8\beta_o\left\{(\beta_o^2\beta^2 - \beta_o^2\alpha^2 - \beta^4 - \alpha^4 - 2\alpha^2\beta^2)\text{sh}2\alpha l\sin2\beta l \right.$$
$$\left. + 2\beta_o^2\alpha\beta\,\text{ch}2\alpha l\cos2\beta l - 2\beta_o^2\alpha\beta\right\} \quad\ldots\ldots\ldots(3.10)$$

3.3.1 The turning points of the curve relating $|k|^2$ with l.

The turning points of the curve occur when $d|k|^2/dl = 0$. From equation (3.10) this is when:-

$$(\beta_o^2\beta^2 - \beta_o^2\alpha^2 - \beta^4 - \alpha^4 - 2\alpha^2\beta^2)\text{sh}2\alpha l\sin2\beta l + 2\beta_o^2\alpha\beta\,\text{ch}2\alpha l\cos2\beta l$$
$$= 2\beta_o^2\alpha\beta \quad\ldots\ldots\ldots(3.11)$$

If α is sufficiently small for α^2 to be neglected equation (3.11) may be simplified. Dividing by $2\beta_o^2\alpha\beta$, and omitting terms in α^2 and α^4,

$$\frac{\beta^2(\beta_o^2 - \beta^2)}{2\,\beta_o^2\alpha\beta}\text{sh}2\alpha l\sin2\beta l + \text{ch}2\alpha l\cos2\beta l - 1 = 0 \quad\ldots\ldots(3.12)$$

Since $\text{sh}2\alpha l = 2\alpha l + \dfrac{(2\alpha l)^3}{3!} + \text{higher terms}$

and $\text{ch}2\alpha l = 1 + \dfrac{(2\alpha l)^2}{2!} + \text{higher terms}$

equation (3.12) may be written, neglecting α^2 and higher powers of α,

$$\frac{\beta(\beta_o^2 - \beta^2)}{\beta_o^2}l\sin2\beta l + \cos2\beta l - 1 = 0 \quad\ldots\ldots\ldots(3.12)$$

$$\frac{\beta(\beta_o^2 - \beta^2)}{\beta_o^2}2l\sin\beta l\cos\beta l - 2\sin^2\beta l = 0 \quad\ldots\ldots\ldots(3.12)$$

Therefore, $$\frac{\beta(\beta_0^2 - \beta^2)1}{\beta_0^2} = \tan\beta1$$

provided $\sin\beta1 \neq 0$

Hence the turning points of the curve occur where

$$\beta1 = n\pi + \tan^{-1}\left[\frac{\beta(\beta_0^2 - \beta^2)1}{\beta_0^2}\right]$$

$$= n\pi + \tan^{-1}\left[\beta1(1 - \iota)\right] \qquad \ldots\ldots\ldots(3.13)$$

provided α is small and $\sin\beta1 \neq 0$.

3.3.2 The condition for $|K|^2$ to be greater than one.

From equation (1.24)

$$|K|^2 = \frac{b\,ch2\alpha1 - a\cos2\beta1 - 2\beta_0(\beta\,sh2\alpha1 - \alpha\sin2\beta1)}{b\,ch2\alpha1 - a\cos2\beta1 + 2\beta_0(\beta\,sh2\alpha1 - \alpha\sin2\beta1)}$$

if $\qquad \alpha\sin2\beta1 > \beta\,sh2\alpha1$

then $\qquad\qquad |K|^2 > 1$

This occurs for very small values of 1, and is more noticeable in liquids with large values of α or β, i.e. with the 'water' or 'alcohol-type' liquids. Fig. 4.55 shows the observed increase in the galvanometer signal, which is proportional to $|K|^2$, for a depth of 0.01 cms of physiological saline.

When $|K|^2 = 1$, and $1 \neq 0$, a galvanometer signal equal to that obtained with an air filled line is observed, then, for this depth of liquid,

$$\alpha\sin2\beta1 = \beta\,sh2\alpha1$$

This equation is not however suitable for calculating

α or β if one of these quantities is known. For the value of l at which this effect occurs is so very small (l $\doteq$ 0.016 cms for a 0.15N NaCl solution) that

$$\sin 2\beta l \rightarrow 2\beta l \quad \text{and} \quad \sinh 2\alpha l \rightarrow 2\alpha l$$

so that both sides of the equation approach $2\alpha\beta l$.

3.4 The condition for no standing wave along the line.

If the denominator and numerator of $\tan\phi$, equation (1.13), should both become zero for a certain liquid depth, then $d(\tan\phi)/dl$, equation (3.1), is infinite so that the positions of the turning points are indeterminate. This means that there is no standing wave system along the line, i.e. $Z_T = Z_o$, and the line is correctly terminated. Under these conditions equation (1.10) becomes:-

$$\frac{Z_T}{Z_o} = \frac{j\beta_o}{\alpha + j\beta} \tanh(\alpha + j\beta)l = 1$$

Rationalising:-

$$\frac{Z_T}{Z_o} = \frac{j\beta_o(\alpha - j\beta)}{(\alpha^2 + \beta^2)} \left\{ \frac{\text{sh}2\alpha l + j\sin 2\beta l}{\text{ch}2\alpha l + \cos 2\beta l} \right\} = 1$$

Thus, equating real and imaginary parts, the condition

for Z_T to equal Z_o is that:-

$$\frac{\beta_o \beta sh2\alpha l - \beta_o \alpha sin2\beta l}{(\alpha^2 + \beta^2)(ch2\alpha l + cos2\beta l)} = 1 \quad \ldots\ldots\ldots(3.14a)$$

and

$$\beta_o \alpha sh2\alpha l + \beta_o \beta sin2\beta l = 0 \quad \ldots\ldots\ldots(3.14b)$$

From equation (1.13) the conditions for the denominator and numerator of $\tan \phi$ to be zero are respectively:-

$$(\beta_o^2 - \alpha^2 - \beta^2)ch2\alpha l - (\beta_o^2 + \alpha^2 + \beta^2)cos2\beta l = 0 \quad \ldots\ldots(3.15a)$$

and

$$\alpha sh2\alpha l + \beta sin2\beta l = 0 \quad \ldots\ldots(3.15b)$$

Equations (3.14b) and (3.15b) are identical. If equation (3.15a), with the use of equation (3.15b), can be shown to be identical with equation (3.14a), then there is no standing wave pattern when both the numerator and denominator of $\tan \phi$ are zero. Equation (3.15a) may be written:-

$$(\beta_o^2 - \alpha^2 - \beta^2)ch2\alpha l = (\beta_o^2 + \alpha^2 + \beta^2)cos2\beta l$$

Multiplying both sides by $(\alpha^2 + \beta^2)(ch2\alpha l + cos2\beta l)$,

$$(\alpha^2 + \beta^2)\left\{ (\beta_o^2 - \alpha^2 - \beta^2)ch^2 2\alpha l + (\beta_o^2 - \alpha^2 - \beta^2)ch2\alpha l cos2\beta l \right\}$$

$$= (\alpha^2 + \beta^2)\left\{ (\beta_o^2 + \alpha^2 + \beta^2)ch2\alpha l cos2\beta l + (\beta_o^2 + \alpha^2 + \beta^2)cos^2 2\beta l \right\}$$

or $\quad (\alpha^2 + \beta^2)(\beta_o^2 ch^2 2\alpha l - \beta_o^2 cos^2 2\beta l)$

$$= (\alpha^2 + \beta^2)^2 (ch^2 2\alpha l + 2ch2\alpha l cos2\beta l + cos^2 2\beta l)$$

$$(\alpha^2 + \beta^2)\beta_o^2(1 + sh^22\alpha l - 1 + sin^22\beta l)$$

$$= (\alpha^2 + \beta^2)^2 (ch2\alpha l + cos2\beta l)^2 \quad \ldots\ldots\ldots\ldots(3.15a)$$

$$\beta_o^2(\alpha^2 sh^22\alpha l + \beta^2 sh^22\alpha l + \alpha^2 sin^22\beta l + \beta^2 sin^22\beta l)$$

$$= (\alpha^2 + \beta^2)^2 (ch2\alpha l + cos2\beta l)^2 \quad \ldots\ldots\ldots\ldots(3.15a)$$

Squaring equation (3.15b),

$$\alpha^2 sh^22\alpha l + \beta^2 sin^22\beta l = -2\alpha\beta sh2\alpha l sin2\beta l$$

and substituting in equation (3.15a)

$$\beta_o^2(\beta^2 sh^22\alpha l + \alpha^2 sin^22\beta l - 2\alpha\beta sh2\alpha l sin2\beta l)$$

$$= (\alpha^2 + \beta^2)^2 (ch2\alpha l + cos2\beta l)^2 \quad \ldots\ldots\ldots\ldots(3.15a)$$

or $\quad \beta_o^2(\beta sh2\alpha l - \alpha sin2\beta l)^2 = (\alpha^2 + \beta^2)^2 (ch2\alpha l + cos2\beta l)^2$

$$\ldots\ldots\ldots\ldots(3.15a)$$

which, on taking the square root, is identical with equation (3.14a),

$$\frac{\beta_o(\beta sh2\alpha l - \alpha sin2\beta l)}{(\alpha^2 + \beta^2)(ch2\alpha l + cos2\beta l)} = 1 \quad \ldots\ldots\ldots\ldots(3.15a)$$

Thus the conditions which make $Z_T = Z_o$ are also the conditions which make the numerator and denominator of tan ϕ zero together.

This condition of no standing wave pattern was observed experimentally with a depth of 0.28 cms of distilled water at a temperature of 36.5°C, indicating $\alpha = 0.36$ assuming $\beta = 5.95$.

3.5   The calculation of β , the phase constant.

After the observed values of Δ and l have been
plotted, an approximate value of β is found using
one of the two methods given below.

3.5.1   The calculation of β from the values of l
    when $\Delta = 0$ or $\pm\lambda_0/4$.

From equation (1.13)

$$\tan\phi = \frac{2\beta(\alpha\,sh2\alpha l + \beta\sin2\beta l)}{(\beta_0^2 - \alpha^2 - \beta^2)\,ch2\alpha l - (\beta_0^2 + \alpha^2 + \beta^2)\cos2\beta l}$$

If $\alpha = 0$, $\tan\phi = 0$ when $2\beta l = n\pi$.
This condition holds approximately if α is small or
if n is small.   Thus an approximate value of β can
be obtained from the gradient of the graph of the
values of l at $\Delta = 0$ and $\pm\lambda_0/4$ plotted against n.
The smaller the value of α the better is this
approximation for β .

This method fails for the 'alcohol-type' liquids
for then positive values of Δ are not obtained.

3.5.2   The calculation of β from the values of l
    when $|K|^2$ has maximum and minimum values.

At the turning points of the $|K|^2$ curve, (section
3.3.1) $\sin\beta l \neq 0$, but $\sin2\beta l \rightarrow 0$.   Hence for
liquids that do not give positive values of Δ , the

'alcohol-type' or electrolytic solutions, an approx-
imate value of β may be found from the gradient of
the graph of the values of l when the galvanometer
readings at resonance, (proportional to $|\mathbf{K}|^{2}$), have
maximum and minimum values, against n.

3.6 Methods of determing the limits between which
α must lie.

An approximate value for β having been found,
it is used to determine the limits between which α
must lie, using one or more of the methods described
in this section.

3.6.1 The curve no longer going to infinity method.

For liquids with low values of α , it is seen
from the experimental graphs of Δ against l that for small
liquid depths the curve goes to minus $\lambda_{o}/8$ and plus
$\lambda_{o}/8$, ($\Delta = 1.1375$ cms) alternately, corresponding
to tan $\phi = \mp \infty$. After a certain liquid depth is
reached, which varies with the liquid used, a phase
shift as great as $\lambda_{o}/8$ is no longer obtained, the
curve turning and crossing the axis again. This
change in behaviour of the curve depends upon the
absorption of the liquid and the limits between which
α must lie for a particular liquid may be quickly

determined as follows.

Tan $\phi = \infty$ when the denominator of equation (1.13) is zero, i.e. when

$$(\beta_o^2 - \alpha^2 - \beta^2)\,ch2\alpha l \;=\; (\beta_o^2 + \alpha^2 + \beta^2)\cos2\beta l \qquad \ldots\ldots(3.16)$$

Cosh2αl must always be greater than 1, whereas cos2βl cannot be greater than 1.   Hence tanϕ can go to infinity when

$$ch2\alpha l \quad < \quad \frac{\beta_o^2 + \alpha^2 + \beta^2}{\beta_o^2 - \alpha^2 - \beta^2}$$

but can no longer go to infinity when

$$ch2\alpha l \quad > \quad \frac{\beta_o^2 + \alpha^2 + \beta^2}{\beta_o^2 - \alpha^2 - \beta^2}$$

$$\text{Let } \; k' \quad = \quad \frac{\beta_o^2 + \alpha^2 + \beta^2}{\beta_o^2 - \alpha^2 - \beta^2}$$

Since α must be small for a phase shift of $\lambda_o/8$ to be obtained at all, α^2 may be neglected.   Therefore

$$k' \quad = \quad \frac{\beta_o^2 + \beta^2}{\beta_o^2 - \beta^2} \qquad \ldots\ldots\ldots\ldots(3.17)$$

For the magnetron oscillator used, $\lambda_o = 9.1$ cms giving $\beta_o = 0.69$ so that for a given liquid k' can be calculated using the value of β obtained from the method of section 3.5.1.   The greatest value of 1 for which $\Delta = 1.1375$ cms,

$\tan \phi = \infty$, is read off from the graph of Δ against
1. Let it be l_1. Then α is the only unknown in
the equation

$$ch2\alpha l_1 \; < \; k' \qquad \ldots\ldots\ldots(3.18)$$

and so is readily calculated.

The value of l, l_2 say, when the curve instead
of reaching $\Delta = 1.1375$ cms turns round, is used to
fix a lower limit for α.

$$ch2\alpha l_2 \; > \; k' \qquad \ldots\ldots\ldots(3.18)$$

For liquids with higher values of α, the 'water-type'
liquids, $\tan \phi$ may never reach infinity and then this
method can give only a lower limit for α.

3.6.2 <u>The change of sign of the gradient method.</u>

The change of sign of the gradient at $\tan \phi = 0$
when n is odd for a critical liquid depth was described
in section 3.2.1. It gives a quick method of fixing
limits between which α must lie for liquids with low
values of α.

From equation (3.8a), the gradient when $\tan \phi = 0$
and n is odd, will be negative if:-

$$\beta_o^2\beta^2 + \beta^4 \; > \; (\beta^4 - \beta_o^2\beta^2)ch2\alpha l \qquad \ldots\ldots(3.19)$$

and positive if:-

$$\beta_o^2\beta^2 + \beta^4 \; < \; (\beta^4 - \beta_o^2\beta^2)ch2\alpha l \qquad \ldots\ldots(3.20)$$

In these equations β_o is known and for low absorbing liquids β is obtained with some accuracy by the method of section 3.5.1. The value of 1 at the highest odd value of n giving a negative gradient when $\tan \phi = 0$, or $\Delta = \pm \lambda_o/4$, is read off from the experimental curve of the shift Δ against 1 and substituted in equation (3.19). Similarly, the value 1 at the next higher odd value of n position where the gradient has become positive is substituted in equation (3.20), and thus the limits between which α must lie can be determined.

3.6.3 <u>The curve no longer taking positive values method.</u>

From equation (1.13)

$$\tan \phi = \frac{2\beta_o(\alpha \, sh2\alpha 1 + \beta \sin 2\beta 1)}{(\beta_o^2 - \alpha^2 - \beta^2) \, ch2\alpha 1 - (\beta_o^2 + \alpha^2 + \beta^2) \cos 2\beta 1}$$

Since β is always greater than β_o, the cosh term in the denominator of equation (1.13) is always negative, its value increasing with 1. The cosine term in the denominator is alternately positive and negative, its maximum value being $(\beta_o^2 + \alpha^2 + \beta^2)$. Thus for sufficiently large values of 1 the denominator becomes negative and remains negative as 1 is increased still further. When the liquid depth is such that the denominator is

always negative as l increases, positive values of
tan ϕ or Δ will occur when the numerator is also
negative. The first term of the numerator, $\alpha \sinh 2\alpha l$,
is always positive, while the second term, $\beta \sin 2\beta l$,
is alternately positive and negative. The maximum
value of this second term is $\pm \beta$. Thus when

$$\alpha \sinh 2\alpha l \; > \; \beta \qquad \ldots\ldots\ldots (3.21)$$

Δ can no longer take positive values.

The value of l when Δ no longer assumes positive
values, l_3, is read off from the experimental graph
and substituted in equation (3.21)

$$\alpha \sinh 2\alpha l_3 \; > \; \beta \qquad \ldots\ldots\ldots (3.21)$$

α is then the only unknown in this equation which
can be solved graphically.

This method is applicable to all types of liquids,
but it is likely to be more accurate for liquids with
higher absorptions. This is because with high values
of α the depth of liquid required for the curve to
assume only negative values is small, of the order of
3 or 4 cms, and with the method of measurement used at
present any inaccuracies in the measurements increase
with l.

3.6.4   The constant galvanometer signal method.

$|K|^2$ approaches a constant value as $\sinh 2\alpha l$ and

cosh2αl approach infinity.   This occurs when 2αl $>$ 5.
Thus if the value of l when the galvanometer signal
remains constant for increasing liquid depths is
substituted in this condition, a lower limit is obtained
for α .

3.7   Methods of finding α .

After the limits for α have been found from one
or more of the methods in section 3.6, one or more of
the following methods is used to find α exactly.   Any
of these methods may equally well be used to determine
β exactly, assuming a value for α .

3.7.1   From the maximum and minimum values of Δ .

The value of l at a clearly defined turning point
of the experimental graph is substituted in equation
(3.9a) or (3.9b) of section 3.2.3.   Neglecting α^2,

$$- \beta_o^2\beta^2 - \beta^4 + (\beta_o^2\beta^2 - \beta^4)\,\text{ch}2\alpha l\cos2\beta l$$

$$= 2\beta_o^2\alpha\,\beta\,\text{sh}2\alpha l\sin2\beta l \qquad \ldots\ldots\ldots(3.9b)$$

Assuming β is known this equation can then be solved
graphically for α .

3.7.2   From the gradient of the curve when tanϕ = 0.

Equations (3.7a) and (3.$\$$0) of section 3.2.1 give
expression for the gradients of the curve at tanϕ = 0

when α^2 is neglected.

When n is <u>even</u>,

$$- \frac{d\phi}{dl} = \frac{4\beta_o \left\{ \beta_o^2 \beta^2 + \beta^4 + (\beta^4 - \beta_o^2 \beta^2) \, ch2\alpha l \right\}}{\left[(\beta^2 - \beta_o^2) \, ch2\alpha l + (\beta_o^2 + \beta^2) \right]^2} \quad \ldots\ldots(3.7a)$$

and when n is <u>odd</u>,

$$- \frac{d\phi}{dl} = \frac{4\beta_o \left\{ \beta_o^2 \beta^2 + \beta^4 - (\beta^4 - \beta_o^2 \beta^2) \, ch2\alpha l \right\}}{\left[(\beta^2 - \beta_o^2) \, ch2\alpha l - (\beta_o^2 + \beta^2) \right]^2} \quad \ldots\ldots(3.8b)$$

The gradient at $\tan \phi = 0$, $\Delta = 0$ or $\pm \lambda_o/4$, for some value of n of the phase shift against l graph, $d\Delta/dl$ is measured, and the value of l noted. It was found necessary to draw the portion of the curve near $\tan \phi = 0$ on a large scale to obtain an accurate value of $d\Delta/dl$. From equation (3.3)

$$\frac{d\phi}{dl} = \frac{4\pi}{\lambda_o} \frac{d\Delta}{dl}$$

On substitution of these values in either (3.7a or b)³·⁸ᵃ, according to whether n is even or odd, a quadratic in $cosh2\alpha l$ is obtained, which on solution gives α.

3.7.3 Substitution of experimental values in the tanϕ equation.

α can be found by substitution of β and any pair of experimental values of Δ and l in equation (1.13)

$$\tan \phi = \frac{2\beta_o(\alpha\,sh2\alpha l + \beta sin2\beta l)}{(\beta_o^2 - \alpha^2 - \beta^2)\,ch2\alpha l - (\beta_o^2 + \alpha^2 + \beta^2)\,cos2\beta l}$$

and

$$\phi = \frac{4\pi}{\lambda_o}\Delta \qquad \ldots\ldots\ldots(1.19)$$

The resulting equation can then be solved graphically
for α.

3.7.4   From the steady negative value of Δ.

In section 1.5.2 it was shown that for sufficiently
large values of l

$$\tan \phi \rightarrow - \frac{2\beta_o\alpha}{\beta^2 + \alpha^2 - \beta_o^2} \qquad \ldots\ldots\ldots(1.23)$$

The steady negative value of Δ is read off from the
graph, $\tan \phi$ calculated and substituted in equation
(1.23) with β_o and β, so giving a value for α.

3.7.5   From the value of l when $\Delta = 0$.

The method of section 3.6.3 may sometimes be
used to give a definite value for α.   If the value
of Δ approaches zero very closely but does not become
positive for a particular value of l, (see the results
for alcohol, fig. 4.80) then this value of l may be
substituted with β in the equation

$$\alpha\,sh2\alpha l + \beta sin2\beta l = 0$$

and the resulting equation solved for α.

Chapter 4

EXPERIMENTAL RESULTS

4.1   Introduction.

In the experimental method used so far a very
large number of readings of Δ are taken corresponding
to different values of l.   The accuracy with which Δ
may be measured depends upon the sharpness of the
tuning, which, as was shown in section 2.2, varies for
different values of l.   In general the tuning is sharp
except at the first turning point of the $\tan \phi$ curve.
Uncertainty in the exact value of Δ is indicated on
the experimental graphs by a vertical line extending
over the range of values of Δ giving maximum signal.
Tables 2, 10 and 14 in the Appendix give the readings
recorded for representative liquids of the three main
types.

The methods available for the calculation of α
and β have been outlined in chapter 3.   The methods
used depend upon the type of liquid, the deduction of
α and β amounting to a method of successive approx-
imations.

The experimental disadvantages of the present method are:-

(i)   Measurements of 1 and Δ are not independent, so that any inaccuracy in 1 affects Δ and increases with increasing depth.

(ii) The uncertainty of knowing when the liquid covers the short circuiting base.

(iii) The time necessary to make the required number of observations.

A modified form of the apparatus is described in chapter 5 which would give independent measurements of Δ and 1, and would permit the use of a very much quicker experimental method for determining the critical points of the $\tan\phi$ curve. This method, however, involves the use of a movable short circuiting piston and has not yet been used owing to the difficulties of designing a movable piston which would act as a true short circuit. This problem is discussed more fully in chapter 5.

The accuracy to which values for α and β may be calculated from the experimental curves depends upon the method used, and the estimated error is indicated in the tables of results. The final values of α and β obtained by this phase shift method are compared with those obtained by other workers using different

methods.   To facilitate comparison the real and imaginary parts of the dielectric constant ε' and ε'' have been calculated from these values of α and β. The relationships [36] between ε', ε'' and the propagation constants for materials in coaxial lines or waveguides and carrying travelling or standing waves are:-

$$\varepsilon' = \left[\beta^2 - \alpha^2 + \left(\frac{2\pi}{\lambda_c}\right)^2 \right] \left(\frac{\lambda_o}{2\pi}\right)^2 \quad \dots\dots\dots(4.1)$$

$$\varepsilon'' = 2\alpha\beta\left(\frac{\lambda_o}{2\pi}\right)^2 \qquad\qquad \dots\dots\dots(4.2)$$

where λ_c is the cut-off wavelength.   For coaxial lines $\lambda_c = \infty$ for the principal wave so that equation (4.1) becomes:-

$$\varepsilon' = (\beta^2 - \alpha^2)\left(\frac{\lambda_o}{2\pi}\right)^2 \qquad \dots\dots\dots(4.3)$$

Solving these equations for α and β gives:

$$\alpha = \frac{2\pi}{\lambda_o}\left\{ \frac{\varepsilon'}{2}\left[\pm \sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} - 1 \right]\right\}^{\frac{1}{2}}$$

and $$\beta = \frac{2\pi}{\lambda_o}\left\{ \frac{\varepsilon'}{2}\left[1 \pm \sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2} \right]\right\}^{\frac{1}{2}}$$

4.2   Experimental results for a non-absorbing liquid.

The non-absorbing liquid benzole was used;   the variation of phase shift with liquid depth is shown in

Fig. 4.1   Benzole   18.5 to 17.8°C   27.3.52.

Fig. 4.1 continued.   Benzole.

l cms.

15

10

5

Method 3.5.1

$$\frac{n}{l} = \frac{2\beta}{\pi} = \frac{11.3}{17}$$

$$\beta = 1.043$$

5   10   n

Fig. 4.2   Benzole.

fig. 4.1. Measurements of Δ were taken at frequent intervals as the liquid depth was increased from 0 to 7.5 cms. The liquid depth was then increased to 13.4 cms and the readings of Δ were again taken at frequent intervals till l = 18 cms.

The wavelength λ in benzole was found by plotting the values of l when $\tan \phi$ = 0 against n. (Fig. 4.2). For non-absorbing liquids, such as benzole, $\tan \phi = 0$ when $2\beta l = $ nπ exactly.

The refractive index of benzole was measured using an Abbé refractometer. This gave

$$\mu = 1.4999 \quad \text{at } 19.5°C$$

or $\varepsilon_3 = 2.25$

From fig. 4.2 $\beta = 1.043 \pm 1\%$, $\lambda = 6.02$

giving $\varepsilon' = \left[\dfrac{9.1}{6.02}\right]^2 = 2.28$

Abadie[26] obtains $\varepsilon' = 2.29 \pm 1\%$, $\varepsilon'' = $ practically zero, for benzene in the wavelength range 3 to 17 cms.

4.3 Experimental results for liquids with low absorptions.

The liquids investigated were:-

(i) Chlorobenzene.

(ii) A solution of 200 cc of the polar substance, chlorobenzene, in 100 cc of the non-polar

solvent, benzole.   (63.3 mole % of

chlorobenzene.

(iii)   Cyclohexanol,

at room temperature.

Cyclohexanol is a highly viscous liquid (η = 0.2

poise at 39°C) so that the experimental arrangement

described in section 2.4.1.2 was used for this liquid.

Table 2 in the Appendix gives the experimental

readings recorded when chlorobenzene was used.

Figs. 4.3, 4.4 and 4.5 show the experimentally observed

variations in phase shift with liquid depth for the

three liquids.   The variation in galvanometer signal

with increasing liquid depth is also plotted for

cyclohexanol in fig. 4.6.   Table 3, 4 and 5 summarise

the values of β , the limits of α and values of α

obtained by the application of some of the methods

outlined in chapter 3.

Tracing of Fig 1.10   Theoretical curve $\beta_o = 0.69$, $\alpha = 0.11$, $\beta = 1.6$

Fig. 4.3   Chlorobenzene   19°C   4.3.52.

Fig. 4.4 Mixture of Chlorobenzene and Benzole. 18°C 24.3.52.

Cyclohexanol.   18·7 to 19·8°C
20.1.53.

Fig. 4.5

Cyclohexanol.

Fig. 4.5 continued.

Fig. 4.6   Cyclohexanol.

Cyclohexanol.

Fig. 4.6 continued.

Table 3

β calculated by the method of section 3.5.1

Liquid	β	Fig.
Chlorobenzene	1.60	4.7
63.3 mole % chlorobenzene	1.45	4.8
Cyclohexanol	1.23	4.9

Table 4

The limits of α by the methods of section 3.6

Liquid	Method of 3.6.1	Method of 3.6.2
Chlorobenzene	$0.093 < \alpha < 0.148$	$0.096 < \alpha < 0.161$
63.3 mole % chlorobenzene	$0.063 < \alpha < 0.093$	$0.069 < \alpha < 0.096$
Cyclohexanol	$0.105 < \alpha < 0.156$	$0.10 < \alpha < 0.18$

Table 5

α calculated by the methods of section 3.7

Liquid	Method of 3.7.1	Method of 3.7.2	Method of 3.7.3
Chlorobenzene	0.113 (Fig.4.10)	0.116 (Fig.4.11)	0.116 (Fig.4.12)
63.3 mole % chlorobenzene	0.071 (Fig.4.13)	0.08 (Fig.4.14)	0.088 (Fig.4.15)
Cyclohexanol	0.149 (Fig.4.16)	0.14 (Fig.4.17)	0.147 (Fig.4.18)

l cms.

10

5

Method 3.5.1

$$\frac{n}{l} = 1.02$$

$$\beta = 1.60$$

5   10   n

Fig. 4.7   Chlorobenzene.

Method 3.5.1.

$$\frac{n}{l} = \frac{7.4}{8}$$

$$\beta = 1.45$$

Fig. 4.8   Chlorobenzene - benzole mixture.

Fig. 4.9   Cyclohexanol.

Method 3.5.1.

$$\frac{n}{1} = \frac{7}{8.4}$$

$$\beta = 1.235$$

Method 3.7.1.

$l = 5.1$

$y_1 = 7.768 - 4.39 \, ch \, 10.2\alpha$

$y_2 = 0.83\alpha \, sh \, 10.2\alpha$

$\alpha = 0.113$

Fig. 4.10   Chlorobenzene.

Method 3.7.3.

$l = 2.47, \quad \Delta = -0.578$

$y_1 = 2.084 \, ch \, 4.94\alpha - 2.34$

$y_2 = 1.38\alpha \, sh \, 4.94\alpha$

$\alpha = 0.116$

Fig. 4.12   Chlorobenzene.

Method 3.7.2.

Gradient when $\triangle = 0$, $l = 1.97$,

$n = 2$,

$\dfrac{d\triangle}{dl} = -0.925$

$\alpha = 0.116$

Fig. 4.11   Chlorobenzene.

Method 3.7.1.

l = 7.7 at fourth turning
point.

$y_1 = 5.41 - 3.21 \mathrm{ch} 15.4\alpha$

$y_2 = 0.45\alpha \, \mathrm{sh} 15.4\alpha$

$\alpha = 0.0714$

Fig. 4.13   Mixture of Chlorobenzene and Benzole.

Method 3.7.3.

$l = 2.5, \quad \Delta = -0.35$

$y_1 = 0.85 \mathrm{ch} 5\alpha - 0.88$

$y_2 = 1.38\alpha \, \mathrm{sh} 5\alpha$

$\alpha = 0.088$

Fig. 4.15   Mixture of Chlorobenzene and Benzole.

Method 3.7.2.

Gradient when $\Delta = 0$, $1 = 2.15$,

$$n = 2,$$

$$\frac{d\Delta}{dl} = -0.985$$

$$\alpha = 0.08$$

Fig. 4.14 Mixture of Chlorobenzene and Benzole.

Method 3.7.1.

$l = 6.1$ cms at first turning point.

$y_1 = 1.045\,ch\,12.2\alpha - 2.93$

$y_2 = 0.84\alpha\,sh\,12.2\alpha$

$\alpha = 0.149$

Fig. 4.16   Cyclohexanol.

Method 3.7.3.

$l = 2,\qquad \Delta = 0.54$

$y_1 = 1.24 - 0.95\,cn\,4\alpha$

$y_2 = 1.38\alpha\,sh\,4\alpha$

$\alpha = 0.147$

Fig. 4.18   Cyclohexanol.

Method 3.7.2.

Gradient when $\Delta = 0$, $n = 2$

$l = 2.525$

$\dfrac{d\Delta}{dl} = -0.925$

$\alpha = 0.14$

Fig. 4.17   Cyclohexanol.

Table 6 gives the final values for α and β
for the three liquids together with the calculated
values of ε' and ε'' for comparison with other results.

Table 6

Liquid

Chlorobenzene $\qquad$ $\beta = 1.60 \pm 2\%$ $\qquad$ $\alpha = 0.115 \pm 2\%$

$\qquad\qquad\qquad$ $\varepsilon' = 5.34$ $\qquad\qquad$ $\varepsilon'' = 0.771$

63.3 mole %
chlorobenzene $\qquad$ $\beta = 1.45 \pm 2\%$ $\qquad$ $\alpha = 0.080 \pm 2\%$

$\qquad\qquad\qquad$ $\varepsilon' = 4.40$ $\qquad\qquad$ $\varepsilon'' = 0.486$

Cyclohexanol $\qquad$ $\beta = 1.23 \pm 2\%$ $\qquad$ $\alpha = 0.15 \pm 2\%$

$\qquad\qquad\qquad$ $\varepsilon' = 3.26$ $\qquad\qquad$ $\varepsilon'' = 0.778$

The values $\beta = 1.60$, $\alpha = 0.11$ and $\beta_\bullet = 0.69$
were substituted in equation (1.13) and the calculated
values of Δ for different values of l are plotted in
fig. 1.10. A tracing of this curve has been made to
facilitate comparison with the experimental curve
obtained for chlorobenzene.

Hill, (unpublished results), using a double probe
transmission line method and a wavelength of 9.35 cms
obtained the following results for chlorobenzene at
room temperature:-

$\qquad\qquad$ $\beta = 1.56 \pm 2\%$ $\qquad\qquad$ $\alpha = 0.11 \pm 2\%$

and for the 63.3 mole % chlorobenzene

$$\beta = 1.42 \pm 2\% \qquad \alpha = 0.79 \pm 2\%$$

which, taking into account the slight difference in

wavelength, agrees exactly with the present results.

Branin and Smyth,[22] using an accurate null method

and a wavelength of 10 cms give the following results

for chlorobenzene:-

Temperature	ε'	ε''
1°C	5.73	0.88
25	5.50	0.64
40	5.26	0.50
55	5.06	0.39

4.4 Experimental results for liquids with high values for both α and β.

4.4.1 Results for distilled water.

The variation of phase shift with liquid depth for

distilled water at a room temperature varying between

17° and 19.3°C is plotted in fig. 4.19. The variation

of galvanometer signal is shown in fig. 4.20.

Fig. 4.21 shows the experimental curve for

distilled water at 37°C, obtained by the experimental

arrangement of section 2.5.

Fig. 4.19

Water 17 to 19°C   28.4.52.

Fig. 4.20   Water.

Fig. 4.20 continued.   Water.

Fig. 4.21

Water 37°C 1.7.52

Table 7 gives the values obtained for β and α
using some of the methods described in sections 3.5,
3.6 and 3.7.

Table 7

Method	Water at 17° to 19° C	Fig.	Water at 37° C	Fig.
3.5.1	$\beta = 5.99$	4.22	$\beta = 5.95$	4.23
3.6.1	$\alpha > 0.46$		$\alpha = 0.38$	
3.6.2	-		$\alpha = 0.38$	
3.6.3	$\alpha > 0.53$		-	
3.7.2	$\alpha = 0.61$		-	
3.7.3	$\alpha = 0.57$	4.24	$\alpha = 0.34$	4.26
	$\alpha = 0.56$	4.25	$\alpha = 0.39$	4.27

Figs. 1.12 and 1.13 show curves plotted from
calculated values of Δ for different liquid depths
for $\alpha = 0.46$ and $\alpha = 0.7$ respectively, the values
of β_o and β being 0.69 and 5.9 for both curves.
The most noticeable effect of increasing α is in the
damping of the pattern. The positions of the turning
points of the curves do not vary appreciably with the
two values of α, so that the method of section 3.7.1
is not likely to yield an accurate value of α. The

Fig. 4.22   Distilled water at room temperature.

l cms

2

1.5

1

0.5

Method 3.5.1

$$\frac{n}{1} = \frac{8.5}{2.25}$$

$\beta = 5.95$

5   10   n

Fig. 4.23   Distilled water at 37°C

Method 3.7.3.

$1 = 0.9, \quad \triangle = 0.12$

$y_1 = 9.83 - 5.67 ch 1.8\alpha$

$y_2 = 1.38\alpha\, sh 1.8\alpha$

$\alpha = 0.57$

Fig. 4.24   Water at 17° to 19° C.

Method 3.7.3.

$1 = 1.2, \quad \triangle = -0.1$

$y_1 = 4.72 ch 2.4\alpha - 8.25$

$y_2 = 1.38\,\alpha\, sh 2.4\alpha$

$\alpha = 0.56$

Fig. 4.25   Water at 17° to 19° C.

Method 3.7.3.

$l = 0.87, \quad \Delta = 0.22$

$y_1 = 13.37 - 11.02 \mathrm{ch} 1.74\alpha$

$y_2 = 1.38\alpha \, \mathrm{sh} 1.74\alpha$

$\alpha = 0.342$

Fig. 4.26   Water at 37°C.

Method 3.7.3.

$l = 1.2, \quad \Delta = -0.135$

$y_1 = 6.58 \mathrm{ch} 2.4\alpha - 9.198$

$y_2 = 1.38\alpha \, \mathrm{sh} 2.4\alpha$

$\alpha = 0.398$

Fig. 4.27   Water at 37°C.

maximum value of Δ obtained experimentally is about
0.5 cms for liquid depths of the order of 0.2 cms.
At this point the value of Δ is varying very rapidly
with l, so that great care has to be taken that the
liquid level is not disturbed as the line is adjusted
to resonance.

The theoretical curves, (1.12) and (1.13), show
that the present experimental method is unlikely to
give very accurate values of α and β for the 'water-
type' liquids.   The experimental curves, (4.19) and
(4.21), however do show immediately the marked decrease
in α with increased temperature.

From the results shown in table 7 it is concluded
that at a wavelength of 9.1 cms, distilled water at a
temperature of 17° to 19° C has

$\alpha = 0.57 \pm 5\%$, $\beta = 5.99 \pm 3\%$

and at 37° C

$\alpha = 0.38 \pm 5\%$, $\beta = 5.95 \pm 3\%$

From equations (4.3) and (4.2), these values correspond
to

$\varepsilon' = 74.7$ $\varepsilon'' = 14.4$ at 17° - 19° C

and

$\varepsilon' = 74$ $\varepsilon'' = 9.5$ at 37° C

The dielectric behaviour of water at microwave
frequencies has been widely investigated by many authors,

and, as was mentioned in the Introduction, the values
obtained have not always been in agreement.   The
results obtained by Conner and Smyth[28]   using a wave-
length of 9.72 cms when investigating the variation of
ε' and ε'' with temperature for water are plotted in
fig. 4.28.   They made measurements on the standing wave
pattern set up in a coaxial line filled with a variable
amount of water and calculated the wavelength in the
water and the loss factor ε'' , using a method based on
Drude's[16]   'First method', from a graph showing the
variation of detector signal with liquid depth.

In 1947 Collie, Hasted and Ritson[11]   introduced a
method of measuring the absorption coefficient by
direct transmission through two waveguides, one just
below and the other just above cut-off dimensions, and
carried out an extensive investigation of the dielectric
properties of water using this method in addition to a
resonant cavity method.   The absorption coefficient k
and the real part of the refractive index measured by
Collie et. al. is different from that defined in the
Introduction, being related to the complex refractive
index by the relationship

$$\bar{n} = n - jk$$

so that $\varepsilon' - j\varepsilon'' = (n - jk)^2$

and $\quad\quad\quad\quad\quad \varepsilon' = n^2 - k^2$

and $\quad\quad\quad\quad\quad \varepsilon'' = 2nk$

n and k are related to α and β by

$$\alpha = \frac{2\pi}{\lambda_o} k$$

$$\beta = \frac{2\pi}{\lambda_o} n$$

Fig. 4.29 shows their results for a wavelength of
10 cms expressed in terms of α and β. The
accuracy is estimated as $\pm$ 1%. This curve shows
the very rapid variation of α with temperature.

The rapid variation of α and β with wavelength
is illustrated by the results obtained by Little[29]
using a double probe method of measuring the standing
wave pattern obtained when the depth of water above a
short circuit in a waveguide is varied. These results
for a temperature of 21°C are plotted in fig. 4.30
the estimated accuracy being $\pm$ 1% at the longer and
$\pm$ 2% at the shorter wavelengths. Hill[30], using a
similar method with a coaxial line and a wavelength
of 9.35 cms obtained the following results,

$\quad\quad\quad\quad \alpha = 0.58 \pm 2\% \quad\quad\quad \beta = 5.9 \pm 2\% \quad$ at 17°C

$\quad\quad\quad\quad \alpha = 0.57 \pm 2\% \quad\quad\quad \beta = 5.9 \pm 2\% \quad$ at 19°C

Cook[13] recently carried out a series of measure-
ments on water covering a wavelength range from 10 to
1.262 cms, and a temperature range from 0° to 60°C.

At 10 and 6.48 cms he used a coaxial line version of Drude's 'first method' and at 3.195 and 1.262 cms the Buchanan[21] method. He checked the 10 cms results by using a cylindrical waveguide method similar to Collie et al. Fig. 4.31 shows the values of ε' and ε'' at 18° and 37°C extrapolated from his results and plotted on a logarithmic frequency scale. The values of ε' and ε'' at a wavelength of 9.1 cms are:-

$$\varepsilon' = 77.2 \qquad \varepsilon'' = 15 \qquad \text{at } 18°C$$

and $\qquad \varepsilon' = 73.8 \qquad \varepsilon' = 9.1 \qquad \text{at } 37°C$

The present results are in fair agreement with these values. This is shown more readily by the Cole-Cole plots of fig. 4.32. The Cole-Cole semi-circles are drawn using $\varepsilon_s = 80.82$ at 18°C, $\varepsilon_s = 74.5$ at 37°C and $\varepsilon_\infty = 5.0$, which data Cook found gave the best average fit to his experimental results. The results at the higher temperature are in better agreement with those of Cook than the ones at room temperature, as might be expected with the smaller value of α. The discrepancy at 18°C seems to be due to too small a value for β.

Water.

Wavelength 9.72 cms.

Fig. 4.28   Conner and Smyth's results.

Fig. 4.29   Collie, Hasted and Ritson's results.

Fig. 4.30   Little's results for water at 21°C.

Fig. 4.31   Cook's results for water.

Temperature 18°C

ε''

40

20

20 40 60 80 ε'

Temperature 37°C

ε''

40

20

20 40 60 80 ε'

Fig. 4.32   Results for water.

4.4.2   Results for ox blood and serum.

Fig. 4.33 shows the results obtained using serum from defibrinated ox blood at a room temperature which varied between 17.7 and 19.6 C using the usual experimental method described in section 2.4.1.1.   This graph compared with that for water (Fig. 4.19) shows immediately that the wavelength in the serum is very similar to that in water and that the attenuation coefficient is greater.

At first the same method was used for obtaining results with whole ox blood at room temperature.   The higher viscosity of the blood meant that a longer interval had to be allowed after each alteration in 1 before finding the resonance position.   The opaqueness of the blood made it difficult to observe when the tip of the micrometer pin was in the surface of the blood in the glass side tube, so it was found better to use the readings of the screw on the rising table for determining 1.   In addition the micrometer pin picked up fibres from the blood which also made readings with it inaccurate.   The whole blood results were very similar to those obtained with serum.   It was decided to use the method described in section 2.4.1.2 placing the blood in a beaker on the rising table directly below the transmission line.   Successive readings

could then be taken much more rapidly.   Fig. 4.34
shows the results obtained.

Ox serum was used again using the simplified
experimental arrangement of section 2.4.1.2 to give a
better comparison with whole blood.   The results are
plotted in fig. 4.35.

As the attenuation coefficient for water decreases
so rapidly with temperature, it was thought that the
values of α and β at the inner body temperature of
37°C would be of more value than those at room temperature.
The thermostatic control arrangement described in section
2.5 was used.   The results are plotted in figs. 4.36
and 4.37.   Satisfactory results with whole ox blood at
37°C were not obtained, as a skin was found to form on
the surface of the blood at this higher temperature
which prevented the displaced blood from overflowing as
l increased, which accounts for the larger positive
values obtained owing to the inaccuracy in l.   The
general shape of the curve indicates a similar value of
α to serum at this temperature.

Table 8 summarises the values of α and β calculated
for serum and whole ox blood at the two temperatures.
Figs. 4.38, 4.39 and 4.40 show the galvanometer reading
plotted against liquid depth for ox serum and blood at
room temperature, and serum at 37°C respectively.

Fig. 4.33

Ox Serum 17.7 to 19.6°C

30. 4. 52.

Fig. 4.34

Ox Blood. 15·3 to 16·8 °C

14. 5. 52.

Fig. 4.35

Ox Serum.   17° to 19°C
27. 6. 52.

Fig. 4.36

Ox Serum 37°C   26. 6. 52.

Fig. 4.37

Ox Blood.   37°C

28.5.52.

Fig. 4.38   Serum   17·7 to 19·6°C.

Fig. 4.39   Ox Blood.   15·3 to 16·8°C

Fig. 440

Serum 37°C.

Table 8

Ox Serum   17° to 19° C

Method		Fig.
3.5.1	$\beta = 5.71$	4.41
3.5.2	$\beta = 5.71$	4.41
3.6.3	$\alpha \geqslant 0.7$	
3.7.3	$\alpha = 0.688$	4.42
	$\alpha = 0.663$	4.43

Whole Ox Blood   15.3° to 16.8° C

3.5.2	$\beta = 5.24$	4.44

Ox Serum   37° C

3.5.1	$\beta = 5.44$	4.45
3.5.2	$\beta = 5.50$	4.46
3.6.4	$\alpha > 0.52$	
3.7.3	$\alpha = 0.556$	4.47
	$\alpha = 0.598$	4.48

From these results it is concluded that the dielectric behaviour of whole ox blood and serum is similar, and the attenuation coefficient is greater than that for water at the same temperatures, (see section 4.4.1). The value of α decreases with temperature, but not so rapidly as for water.

Methods 3.5.1 & 3.5.2.

$$\frac{n}{1} = \frac{10}{2.75} = 0.09$$

$$\beta = 5.71$$

Fig. 4.41   Ox serum at room temperature.

Method 3.7.3.

$l = 0.4,$ $\Delta = 0.16$

$y_1 = 8.88 - 7.2\,\mathrm{ch}\,0.8\alpha$

$y_2 = 1.38\alpha\,\mathrm{sh}\,0.8\alpha$

$\alpha = 0.688$

Fig. 4.42   Ox Serum at 17° to 19°C.

Method 3.7.3.

$l = 0.9,$ $\Delta = 0.09$

$y_1 = 8.67 - 4.05\,\mathrm{ch}\,1.8\alpha$

$y_2 = 1.38\alpha\,\mathrm{sh}\,1.8\alpha$

$\alpha = 0.663$

Fig. 4.43   Ox Serum at 17° to 19°C.

l cms.

Method 3.5.2.

$$\frac{n}{l} = \frac{1.5}{0.45}$$

$$\beta = 5.24$$

Fig. 4.44   Ox blood at room temperature.

Fig. 4.45   Ox Serum at 37°C

Fig. 4.46   Ox serum at 37°C

Method 3.7.3.

$l = 0.9$, $\quad \Delta = 0.055$

$y_1 = 4.35 - 2.24\,ch\,1.8\alpha$

$y_2 = 1.38\alpha\,sh\,1.8\alpha$

$\alpha = 0.556$

Fig. 4.47   Ox Serum at 37°C.

$l = 1.6 \quad \Delta = -0.03$

$y_1 = 7.28 - 1.3\,ch\,3.2\alpha$

$y_2 = 1.38\alpha\,sh\,3.2\alpha$

$\alpha = 0.598$

Fig. 4.48   Ox Serum at 37°C.

Table 9 sets out the concluded values of α, β, ε' and ε'', together with those obtained for water for comparison.

Table 9

Room temperature

	β	α	ε'	ε''
Serum	$5.71 \pm 3\%$	$0.68 \pm 5\%$	67.2	16.3
Blood	$5.24 \pm 3\%$	$0.7 \pm 8\%$	56.6	15.3
Water	$5.99 \pm 3\%$	$0.57 \pm 5\%$	74.7	14.4

Inner body temperature 37°C

	β	α	ε'	ε''
Serum	$5.44 \pm 3\%$	$0.58 \pm 5\%$	61	13.2
Blood		$0.6 \pm 8\%$		
Water	$5.95 \pm 3\%$	$0.38 \pm 5\%$	74	9.5

Very little reliable work on the dielectric properties of blood at microwave frequencies had been done until the recent investigations by England and Sharples[23,24] and Cook.[13] England and Sharples used the von Hippel[18] method of measurement in a waveguide at wavelengths of 10, 3.18 and 1.27 cms. Their results for human blood at 10 cms were:-

Specimen at 37°C	α	β
Whole human blood	0.65 ± 5%	4.6 ± 2%
Blood serum	0.84 ± 5%	5.3 ± 2%
Water	0.3	5.4

Cook made a thorough investigation of whole human blood at four different wavelengths, 16.93, 10.03, 3.195 and 1.262 cms and four different temperatures, 15°, 25°, 35° and 45°C, using the same methods as he had used in his investigation of water.   His results at 10.03 cms for whole human blood were:-

15°C $\varepsilon' = 59.9 ± 2\%$ $\varepsilon'' = 19.9 ± 5\%$

35°C $\varepsilon' = 56.0 ± 2\%$ $\varepsilon'' = 15.9 ± 5\%$

and for water at 10 cms

15°C $\varepsilon' = 78.6 ± 1\%$ $\varepsilon'' = 15.0 ± 2\%$

35°C $\varepsilon' = 74.0 ± 1\%$ $\varepsilon'' = 8.7 ± 2\%$

The present results show that the dielectric properties of ox blood are similar to those of human blood.

4.4.3   Results for NaCl solutions of varying concentrations.

Figs. 4.49 to 4.60 show the results obtained using NaCl solutions of concentrations varying from 0.15N up to 2N in distilled water.   Physiological saline, Ringer Tyrode solution, was used for the 0.15N solution.   This consists of:-

NaCl	0.8	gr
KCl	0.02	
CaCl$_2$	0.02	
NaHCO$_3$	0.1	
NaH$_2$PO$_4$	0.005	
MgCl$_2$	0.01	
Glucose	0.1	

in 100 gr of water.

The variation of galvanometer signal with liquid depth is shown for each solution.   Table 10 in the Appendix gives the experimental readings recorded when Physiological saline was used.   They are typical of the results obtained for the 'water-type' liquids.

It is immediately seen from these graphs that α increases very rapidly with increased concentration. This is almost entirely due to the conductivity of the liquid.   The total effective value ε_c'' of the imaginary

Physiological Saline. 19 to 20°C.

18. 2. 53.

Fig. 4. 49

Fig. 4.50   0.33 N NaCl solution.   17.2° to 18.5°C
10.2.53.

Fig. 4.51   0.66N NaCl solution.   18.2° to 19.2°C
11.2.53.

Fig. 4.55 Physiological Saline

Fig. 4.52   N   NaCl solution.   18·1 to 18·6°C.   3.3.53.

Fig. 4.53   1·5 N NaCl solution.   18·9 to 19°C.   17.2.53.

Fig. 4.54   2 N NaCl solution.   17·8 to 17°C.   2.2.53.

Fig. 4.55   Physiological Saline.

Fig. 4.56   0.33 N NaCl solution.

Fig. 4.57   0.66N NaCl solution.

Fig. 4.58   Normal   NaCl solution.

Fig. 4.59   1.5N   NaCl solution.

Fig. 4.60   2N NaCl solution.

part of the dielectric constant is made up of two terms,
one being the contribution from dipole relaxation ε''_d,
and the other from the ionic conductivity of the
solution.

$$\varepsilon''_t = \varepsilon''_d + \frac{2\sigma}{f} \qquad \ldots\ldots\ldots\ldots(4.6)$$

where σ is the specific conductivity in e.s.u. and
f is the frequency.

ε''_t is then the value of the imaginary part of the
dielectric constant calculated from the value of α
obtained from the experimental graphs.   Fig. 4.61
shows the variation of conductivity with concentration
for NaCl solutions at 18 C as given in the International
Critical Tables.   It is assumed that there is no change
in these static conductivity values at microwave
frequencies.   The justification for this is discussed
by both Hasted, Ritson and Collie,[31] and Lane and Saxton[32]
in their investigations of dielectric properties of
electrolytes.

Tables 11 and 12 summarise the values of α and β
calculated for the different solutions using the methods
given in sections 3.5, 3.6 and 3.7.   Values of β were
obtained from the values of l when the galvanometer
had maximum or minimum signals, as the values of α
were too large for the method of section 3.5.1 to be

valid. Approximate values of α could be obtained,
either by inspection of the experimental curves, or by
one of the methods of section 3.6. If $\alpha \gg 1$, it is
not permissable to neglect terms in α^2 when applying
the methods of section 3.7; so that this approximate
value of α was substituted for α^2 and the equation
solved for α. The resulting value of α was then
used for α^2 in subsequent calculations.

Table 11

Values of β calculated by the method of section
3.5.2 for NaCl solutions.

Concentration	β	Fig.
0.15 N	6.03	4.62
0.33N	5.75	4.63
0.66N	5.88	4.64
1 N	6.3	4.65
1.5N	6.28	4.66
2 N	5.72	4.67

Fig. 4.61   Conductivity of NaCl solutions at 18°C

from I.C.T.

Fig. 4.62   Physiological saline.

Fig. 4.63   0.33N

Method 3.5.2.

$$\frac{n}{1} = \frac{5.5}{1.5}$$

$$\beta = 5.75$$

Fig. 4.64   0.66N

Method 3.5.2.

$$\frac{n}{1} = \frac{6}{1.6}$$

$$\beta = 5.88$$

l cms.

1

Method 3.5.2.

$$\frac{n}{l} = \frac{2}{5}$$

$$\beta = 6.3$$

0.5

1   2   3   4   n

Fig. 4.65   1 N

l cms.

1

Method 3.5.2.

$$\frac{n}{l} = \frac{2.2}{5.5}$$

$$\beta = 6.28$$

0.5

1   2   3   4   n

Fig. 4.66   1.5N

l cms.

0.5

Method 3.5.2.

$$\frac{n}{l} = \frac{1.1}{0.3}$$

$$\beta = 5.72$$

1   2   3   n

Fig. 4.67   2 N

Table 12

Values of α for NaCl solutions

Method	α	Fig.
Concentration 0.15N		
3.6.1	$\alpha > 0.5$	
3.6.2	$\alpha > 0.5$	
3.6.3	$\alpha > 0.74$	
3.7.3	$\alpha = 0.95$	4.68
	$\alpha = 0.98$	4.69
	$\alpha = 0.895$	4.70
3.7.5	$\alpha = 0.91$	
Concentration 0.33N		
3.6.3	$\alpha > 0.91$	
3.6.4	$\alpha > 1.05$	
3.7.3	$\alpha = 1.09$	4.71
	$\alpha = 1.08$	4.72
Concentration 0.66N		
3.6.3	$\alpha > 1.38$	
3.6.4	$\alpha > 1.25$	
3.7.3	$\alpha = 1.43$	4.73
	$\alpha = 1.56$	4.74
3.7.4	$\alpha = 1.81$	

Fig. 4.68   0.15N

Method 3.7.3

$1 = 0.2$, $\triangle = -0.36$

$y_1 = 19.9\,\mathrm{ch}0.4\alpha - 20.82$

$y_2 = 1.38\alpha\,\mathrm{sh}0.4\alpha$

$\alpha = 0.95$

Fig. 4.69   0.15N

$1 = 0.3$ $\triangle = 0.21$

$y_1 = 13.77 - 10.97\,\mathrm{ch}0.6\alpha$

$y_2 = 1.38\alpha\,\mathrm{sh}0.6\alpha$

$\alpha = 0.98$

Fig. 4.70   0.15N

$1 = 0.85$ $\triangle = 0.04$

$y_1 = 7.59 - 2.04\,\mathrm{ch}1.7\alpha$

$y_2 = 1.38\alpha\,\mathrm{sh}1.7\alpha$

$\alpha = 0.895$

Method 3.7.3.

$1 = 0.3$ $\triangle = 0.12$

$y_1 = 7.9 - 5.6\,ch\,0.6\alpha$

$y_2 = 1.38\alpha\,sh\,0.6\alpha$

$\alpha = 1.09$

Fig. 4.71   0.33N

Method 3.7.3.

$1 = 0.85,$ $\triangle = 0.01$

$y_1 = 6.17 - 0.49\,ch\,1.7\alpha$

$y_2 = 1.38\alpha\,sh\,1.7\alpha$

$\alpha = 1.08$

Fig. 4.72   0.33N

Method 3.7.3.

$l = 0.85,$ $\quad \triangle = -0.02$

$y = 1.04\,ch1.7\alpha + 5.06$

$y = 1.38\,\alpha\,sh1.7\alpha$

$\alpha = 1.43$

Fig. 4.73   0.66N

Method 3.7.3.

$1 = 0.3, \quad \triangle = 0.028$

$y_1 = 4.36 - 1.39 ch 0.6\alpha$

$y_2 = 1.38 \alpha sh 0.6\alpha$

$\alpha = 1.56$

Fig. 4.74   0.66N   NaCl solution.

$1 = 0.3, \quad \triangle = 0.03$

$y_1 = 6.56 - 1.72 ch 0.6\alpha$

$y_2 = 1.38 \alpha sh 0.6\alpha$

$\alpha = 1.88$

Fig. 4.75   Normal NaCl solution.

Fig. 4.76   1.5N NaCl solution.

Method 3.7.3.

$1 = 0.3,\quad \triangle = -0.025$

$y_1 = 1.53\, \text{ch}\, 0.6\alpha - 3.88$

$y_2 = 1.38\alpha\, \text{sh}\, 0.6\alpha$

$\alpha = 2.525$

Method 3.7.3.

$l = 0.3,$ $\quad \triangle = -0.08$

$y_1 = 4.95 \mathrm{ch} 0.6\alpha - 2.63$

$y_2 = 1.38\alpha \, \mathrm{sh} 0.6\alpha$

$\alpha = 3.17$

Fig. 4.77   2N NaCl solution.

Method 3.7.3.

$l = 0.2,$ $\quad \triangle = -0.17$

$y_1 = 10.35 \mathrm{ch} 0.4\alpha - 12.85$

$y_2 = 1.38\alpha \, \mathrm{sh} 0.4\alpha$

$\alpha = 3.29$

Fig. 4.78   2N NaCl solution.

Method	α	Fig.
Concentration 1N		
3.6.3	$\alpha > 1.4$	
3.6.4	$\alpha > 1.5$	
3.7.3	$\alpha = 1.88$	4.75
Concentration 1.5N		
3.6.3	$\alpha > 2.4$	
3.6.4	$\alpha > 2.3$	
3.7.3	$\alpha = 2.52$	4.76
3.7.4	$\alpha = 2.54$	
Concentration 2N		
3.7.3	$\alpha = 3.17$	4.77
	$\alpha = 3.29$	4.78
3.7.4	$\alpha = 3.25$	

This method does not give accurate results for liquids with high values of α , but the results obtained for the lower concentrations of NaCl are estimated to be accurate to within $\pm$ 5% and those for the higher concentrations to within $\pm$ 10 or 15%. Fig. 4.79 shows the increase in α (uncorrected for conductivity) with concentration. Table 13 gives these concluded values of α and β , the calculated

Fig. 4.79   Variation of α for NaCl solutions.

values of ε' , ε''_t and ε''_d and the conductivity σ of
the different solutions in e.s.u. taken from fig. 4.61
for a temperature of 18°C. The experiments were
performed at room temperature which varied between 17°
and 20°C. Although the values of α and β vary rapidly
with temperature, it was thought better to use the
experimental arrangement of section 2.4.1.1, which gives
a more accurate method of measuring 1, than that of
section 2.4.1.2 and a temperature controlled water bath.

Table 13

N	α	β	ε'	ε''_t	$10^6 K/c$	$\sigma 10^8$	ε''_d
0.15	0.93 ± 5%	6.03 ± 3%	74.5	23.58	89.6	121	16.27
0.33	1.08 ± 5%	5.75 ± 5%	67	26.1	84	250	10.95
0.66	1.5 ± 5%	5.88 ± 5%	67.8	37.0	78.25	475	8.2
1	1.88 ±10%	6.3 ±10%	73.5	49.5	74.19	666	9.5
1.5	2.5 ±10%	6.28 ±10%	69.6	66.0	69.25	938	9.2
2	3.3 ±15%	5.72 ±15%	46	79.0	64.66	1160	8.6

Hasted, Ritson and Collie[31] investigated the
dielectric properties of NaCl solutions at 10 and 3 cms,
and at different temperatures, using the same method of
measuring the absorption coefficient as they had used
for water. Their results at 10 cms and 21°C are given

below.

Normalacy	Equivalent conductivity	ε' ± 3%	ε_t'' ± 3%	ε_d''
0.5	85.6	72.8	38.28	12.6
1	78.7	67.4	60.3	13.1
1.5	73.0	61.5	78.4	12.7
2	68.6	56.2	95	11.8

More recently Haggis, Hasted and Buchanan[33] have
published results for a large number of different salts
in solutions using the Buchanan[21] null method at wave-
lengths of 1.264, 3.175 and 9.22 cms. Only one
measurement on a NaCl solution at 9.22 cms was reported.

Molarity	Conductivity	ε'	ε_t''	ε_d''	Temperature
0.33	97	72.4	33.5	14.3	25°C

Saxton and Lane[32] have also recently published
results at wavelengths of 0.62, 1.24 and 3.2 cms of the
dielectric properties of electrolytes including NaCl
solutions using a wave guide technique similar to that
of Collie et al. The method involved the observation
of the attenuation in transmission through waveguides
of differing cross-sectional dimensions containing
the liquid. In highly absorbing liquids the

measurements of the absorption coefficient k are more
accurate than those of n the real part of the refractive
index, and only the variation of k with concentration
is given.   At the very much shorter wavelengths their
results are not comparable with those obtained by the
present method.

4.5   Experimental results for liquids with a high
      value of α and a low value of β .

4.5.1   Results for ethyl alcohol.

Table 14 in the Appendix gives the experimental
readings recorded when ethyl alcohol at a temperature
of 18°C was used.   These results are plotted in
fig. 4.80.   The tuning was very flat for liquid depths
between 0.7 and 1.2 cms.   Fig. 4.81 shows the variation
of galvanometer signal with liquid depth.   The value
of β can be deduced by the method of section 3.5.2
and fig. 4.82 shows the values of l when the galvanometer
readings have maxima or minima values plotted against n.
Figs. 1.14 and 1.17 show the variation of Δ with l and the
square of the modulus of the reflection coefficient $|K|^2$
with l calculated from equations (1.13) and (1.24)
respectively for $\beta_o = 0.69$, $\alpha = 0.7$ and $\beta = 1.86$.

Fig. 4.80   Ethyl Alcohol   18°C   12.2.52.

Fig. 4.81   Ethyl Alcohol.

Comparison between the theoretical and experimental
curves shows general agreement and indicates a value
of α very slightly greater than 0.7 for ethyl alcohol.
Table 15 summarises the values of α and β obtained by
the methods of sections 3.5, 3.6 and 3.7.

Table 15

Values of α and β calculated for ethyl alcohol

Method		Fig.
3.5.2	$\beta = 1.84 \pm 2\%$	4.82
3.7.1	$\alpha = 0.78$	4.83
3.7.3	$\alpha = 0.68$	4.84
3.7.4	$\alpha = 0.84$	
3.7.5	$\alpha = 0.70$	

It is concluded that the values of β, α, $\varepsilon' \& \varepsilon''$
for ethyl alcohol at 18°C are:-

$$\beta = 1.84 \pm 2\% \qquad \alpha = 0.74 \pm 8\%$$
$$\varepsilon' = 5.95 \qquad \varepsilon'' = 5.75$$

Hill[30], using a wavelength of 9.35 cms and a double
probe coaxial line method, obtained the following results
for alcohol at 19°C

$$\beta = 1.83 \pm 2\% \qquad \alpha = 0.73 \pm 2\%$$

l cms.

-1.5

-1

Method 3.5.2.

$$\frac{n}{l} = \frac{1.75}{1.5}$$

$$\beta = 1.84$$

-0.5

1   2   n

Fig. 4.82   Ethyl alcohol.

Method 3.7.1.

$l = 1.12$ cms.

$y = 13.03 - 5.48\,\mathrm{ch}\,2.24\alpha$

$y = 1.44\alpha\,\mathrm{sh}\,2.24\alpha$

$\alpha = 0.788$

Fig. 4.83   Alcohol.

Method 3.7.3.

$l = 0.4, \quad \triangle = -0.45$

$y = 2.41\,\mathrm{ch}\,0.8\alpha - 2.23$

$y = 1.38\alpha\,\mathrm{sh}\,0.8\alpha$

$\alpha = 0.68$

Fig. 4.84   Alcohol.

Little[29] using the same method as Hill but applied to a waveguide found it was impossible to reproduce results for absorption measurements more closely than $\pm$ 5% for ethyl alcohol.

These results do not lie on the Debye semi-circle (ε_s = 25, ε_∞ = 1.9) given by Bolton[37] from his measurements at 12.6 cms using a standing wave method in a waveguide, and earlier work by other authors. Although Bolton estimates an accuracy of 1% for his values, the remaining data is all from work done before 1940. The temperature at which the measurements were made is not stated. Bolton, however, used freshly dried and distilled ethyl alcohol, while no such precautions were taken with the present method. This might account for the relatively greater absorption (ε' = 5.5, ε'' = 7.8 at 12.6 cms), although, if the alcohol used in the phase shift method had absorbed water vapour from the atmosphere it would probably have increased both ε' and ε''.

4.6 Conclusions.

In deducing values for α and β from the methods of sections 3.5, 3.6 and 3.7, it was found that the results obtained by some methods were more reliable

than those obtained by other methods. The method of
section 3.7.2 for example, the value of the gradient
to the experimental curve at $\tan \phi = 0$, does not give
very reliable results owing to the difficulty of
drawing the tangent to the curve. A slight variation
in the value of $d\Delta/dl$ may easily give an insoluble
quadratic equation or a value of $\cosh 2\alpha l < 1$. Again,
the accuracy of the substitution method, 3.7.3, depends
upon the position of the selected point, since $\tan \phi$
varies very rapidly when $\phi > 1.3$ or $\Delta > 0.9$. Substituted
values of l and Δ if only very slightly out will give
an insoluble equation for α. These points must be
assessed in arriving at final values for α and β.

A method of dielectric measurement in the micro-
wave region must yield values of α and β correct to
at least $\pm 2\%$. This phase shift method, with the
present apparatus only fulfills this requirement for
liquids such as Chlorobenzene with low values of α.
The higher the value of β the greater the accuracy
obtainable. With Cyclohexanol, for example, the small
value of β, 1.23 cms, means that the depth of liquid
required before the pattern repeats is large, 5 cms,
and owing to the method of measuring used at present,
where the shift of the tuning cap is added to, or

subtracted from l, any inaccuracy in l, due to uncert-
ainty as to when the liquid covers the short circuiting
plate, increases with l.   Thus the larger value of β
(5.9 cms^{-1}) for water compared with alcohol ($\beta = 1.86$)
makes measurements with the 'water-type' liquids more
reliable than those with the 'alcohol-type' liquids.

Chapter 5

CRITICISM OF THE METHOD

5.1 Introduction.

The apparatus and method were devised originally
to test if the equation

$$\tan \phi = \frac{2\beta_0(\alpha \sinh 2\alpha l + \beta \sin 2\beta l)}{(\beta_0^2 - \alpha^2 - \beta^2)\cosh 2\alpha l - (\beta_0^2 + \alpha^2 + \beta^2)\cos 2\beta l}$$

which had been derived from theoretical considerations,
did actually represent the behaviour of liquid dielectrics.
The results (chapter 4) obtained for a wide variety of
liquids confirmed this relationship. It was found in
addition that the values of α and β derivable from
the graph of the observed phase shift Δ against the
liquid depth l compared well with those values obtained
by other methods. This chapter discusses the use of
this variation of phase shift with liquid depth for
microwave absorption measurements in liquids.

5.2   A suggested modification to the apparatus and the experimental method.

For this phase shift method to be suitable for measurements of α and β a quicker experimental procedure is necessary.   The present method is tedious and inaccuracies may be introduced by temperature variation and evaporation during the readings.   A limitation on the accuracy obtainable with the present experimental arrangement is that the measurements of Δ and l are not independent.   These difficulties could be overcome by the use of a movable short circuiting plunger in place of the fixed short circuit.   Fig. 5.1 shows the suggested design.

In this new design the maximum length of the air filled portion of the line would be $3/4\lambda_o$ (6.825 cms for $\lambda_o = 9.1$cms) while the length of the liquid filled portion would depend upon the type of liquid it was required to use.   Even for liquids with a low absorption a total length of transmission line of about 20 cms should be sufficient.   The movement of the short circuiting plunger would be controlled by a micrometer on a rack and pinion.   This micrometer would give the value of l.   There would now be no need of a tuning cap, so that the input and output coupling loops would be

housed in the form of a ... heated to a
fixed short circuit ... would, however, be necessary
to be able to ... the ... elevation ... for
conservation of the ... surface and for the cleaning
of the line. ... To make to compensate for the liquid
displaced at the top of the element, the inner of the
coaxial line, where a ... ... the slot Fig. 5 would
be attached to the movable ... governing the motion
of the slider, so ... and the slider would move ...
together. The ... ... function of the glass tube
S must be the same as ... of the inner. This would
compensate the effect of ... displacement. Any ...
surface tension effects would tend to ... themselves throughout
the readings.

The experimental procedure would be as follows.
With a liquid in the slider, the slider is moved to the
first resonance point, ... from that end of λ/2 from the
upper short circuit. ... this is required to bring
the ... by raising or ... such ... making sure that ...
to ... ... so that ... the ... ... the cleaning.
The ... ... and the ... between ...
... ... thickness of
the liquid ... ... the slider is lowered
into the liquid until resonance is obtained and l is

Fig. 5.2.

Fig. 5.1   Diagram illustrating the suggested modification
to the apparatus.

housed in the same way as before but attached to a
fixed short circuit. It would however, be necessary
to be able to remove this short circuiting cap for
observation of the liquid surface and for the cleaning
of the line. In order to compensate for the liquid
displaced as the rod of the plunger, the inner of the
coaxial line, enters the liquid, the side tube S would
be attached to the rack and pinion governing the motion
of the plunger so that S and the plunger would move
together. In addition the diameter of the glass tube
S must be the same as that of the inner. This would
compensate the effect of liquid displacement. Any
surface tension effects would remain constant throughout
the readings.

The experimental procedure would be as follows.
With no liquid in the line, the plunger is moved to the
first resonance position at a distance of $\lambda_o/2$ from the
upper short circuit. Liquid is now allowed to enter
the line by raising the reservoir until it comes just up
to the upper surface of the short circuiting piston.
This gives the zero position and it is marked by means
of the micrometer pin being adjusted to the surface of
the liquid in the side tube S. The piston is lowered
into the liquid until resonance is obtained and l is

read off from the plunger micrometer.   The plunger is
lowered further into the liquid, the value of l at each
resonant position being recorded.   These readings give
the points A, B, C, ... in fig. 5.2, so that if the
absorption is not too high β can be deduced.   The
plunger is then returned to its zero position and the
depth of liquid in the line is altered, by raising or
lowering the reservoir, by an amount x as measured by
the micrometer pin in S.   The new resonant positions
are found, A', B', C', ... and marked on the graph.   This
process is repeated until sufficient points have been
obtained to enable one of the methods of determining α
to be used.

This method would enable limits between which
α must lie to be quickly determined.   For, after
finding the positions where the phase shift is zero,
the positions corresponding to $\Delta = \pm \lambda_o/8$, i.e. $\tan \phi = \pm \infty$,
could then be plotted.   The greatest value of l, for
a given liquid, at which a phase angle of $\pi/2$ occurs
and the least value of l when the phase angle is less
than $\pi/2$ is found, and the limits between which α
must lie can be calculated by the method of section
3.6.1.

This modification has not so far been adopted
owing to the difficulty of producing a movable short

circuiting piston at these wavelengths.   Cook, who
used a simple short circuiting plunger at 10 and 6.48 cms
in his measurements with water and blood, was consulted.
He said that he had had no difficulties with this plunger
for high loss liquids such as water, but that in his
method, which was a coaxial version of Drude's 'first
method', it had not been necessary to raise the plunger
nearer to the liquid surface than about six centimeters.
Although he had not tried making measurements with
smaller liquid depths it seems likely that the plunger
would not act as a complete short circuit as it approaches
the surface.   As the variation of $\tan \phi$ with l has
reached a constant negative value for liquid depths
greater than four or five centimeters for the 'water-
type' liquids it does not seem that a simple plunger,
making good contact with the walls of the outer conductor
by means of a fringe of small phosphor bronze spring
contacts, would be satisfactory for this phase shift
method of measurement.   It is felt that the difficulties
in the design of a suitable movable piston which would
act as a true short circuit in low loss liquids or very
small depths of high loss liquids, would detract from
the very simple apparatus required for this phase shift
method of measurement.

Although this suggested modification enables the phase shift and liquid depth to be measured independently it does not overcome the difficulty of knowing the exact moment that the liquid enters the line, or covers the short circuiting piston. This means that the exact location of the origin of the Δ/l graph is in doubt. The silver or gold plating of the inner surface of the line, in addition to increasing the conductivity, should improve the at present irregular flow of liquid across the short circuit, and so minimise this source of error.

The maximum phase shift for liquids with a low absorption is $\pm \lambda_o/4$, which at the frequency investigated is $\pm$ 2.275 cms, while for the 'water' and 'alcohol-type' liquids it is $\pm \lambda_o/8$, i.e. $\pm$ 1.138 cms. Table 1 of section 1.5.4 shows that the measurement of Δ should be capable of greater accuracy at a longer wavelength.

The replacement of the galvanometer by a super-heterodyne receiver might enable measurements of the very slight variations of Δ to be made at sufficiently large liquid depths for the suggested modifications of the apparatus using a simple short circuiting plunger to be used.

It seems then that for equation (1.13) to give a method of dielectric absorption measurement yielding

values of α and β accurate to at least $\pm$ 2%, the liquids should have values of α in the range 0 to 0.5, and of β greater than 1.5, and that the modified apparatus must be used with a much more sensitive detector.

5.3   Comparison with other methods.

In chapter 10 of 'Technique of Microwave Measurements' Redheffer[20] summarises the many different methods used for microwave dielectric measurements.   These methods may be divided into three general classes:

(a) Transmission methods, (transmission line or free space).

(b) Reflection methods,   (transmission line or free space), including the short circuited line method.

(c) Resonant cavity methods.

In many of these methods it is in general necessary to make use of two equations and to measure two distinct quantities, such as amplitude and phase angle, in order to calculate the two parameters, α and β , n and k, or tan δ and σ , which give the real and imaginary parts of the dielectric constant.

In the method developed here only one quantity phase shift, is measured, and α and β are deduced from

the variation of phase shift with liquid depth.   The
advantages of the method are:-

(1)   The system is closed, so that there is no loss
       of radiation from slots, as with the travelling
       probe methods, and the position of the observer
       with respect to the line does not affect the
       readings.

(2)   The method is independent of any variation in
       the strength of the input signal;   for the
       detector is only used to indicate when the line
       is resonant.

(3)   The method is independent of the crystal law
       for the same reason.

(4)   Only small quantities of the liquid under
       investigation are required.   (In the present
       apparatus about 300 cc of the 'water-type'
       liquids were used, but, if necessary, this
       volume could be considerably reduced.

(5)   The same apparatus can be used to find α and
       β for all types of liquid dielectrics, although
       it is better for liquids with a low value of α
       and a large value of β .

(6)   The apparatus required is very simple.

(7)   There is no limitation ^in principle on the wavelength that
       may be used.

The disadvantages of the method are, that with
the apparatus and experimental arrangement used at
present:-

(1)   The experimental procedure is lengthy and the
       deduction of α and β tedious.

(2)   The accuracy is not greater than $\pm$ 2% and often
       considerably less.

A number of methods, (Drude[16], Cooper[34], Knerr[17], Conner
and Smyth[28], Abadie[26], Cook[13], Little[29], Ichikawa[38]) are based
on the variation in signal strength obtained at a fixed
detector in a resonant line as the depth of liquid above
a short circuit is varied.   Values of α and β are
obtained from the graph showing the variation in
detector deflection against liquid depth.   These methods
are not independent of the detector law or, except where
special precautions have been taken, of any input
fluctuations.   The difference between these methods
and the phase shift method is that in the latter the
line is adjusted to resonance between each incremental

variation of liquid depth.   Although the variation
of detector signal is not used for the determination
of the parameters, (except for finding β for highly
absorbing liquids) since no precautions have been
taken to ensure a constant input or to investigate
the crystal law, it is of interest to compare the
curves obtained.

Abadie's method [26] gives the most easily compar-
able curve.   Fig. 5.3 shows his apparatus for liquids
with normal absorption.   His equation for the
variation δ in signal strength at the detector is

$$\delta = C\left\{1 + \frac{\tanh^2 \frac{2\pi kx}{\lambda} \cdot \tan^2 \frac{2\pi x}{\lambda}}{\tanh^2 \frac{2\pi kx}{\lambda} + \tan^2 \frac{2\pi x}{\lambda}}\right\} \quad \dots\dots(5.1)$$

where $\delta \propto |I|^2$, $\lambda = \lambda_\bullet/n$, $\sqrt{(\varepsilon' - j\varepsilon'')} = n(1 - jk)$
$x = $ liquid depth and $C = $ a constant.

Since $\quad \frac{2\pi}{\lambda} = \beta \quad$ and $\quad k = \frac{\alpha}{\beta}$

this expression may be written

$$\delta = |I|^2 \cdot \frac{C(1 + \tanh^2\alpha x.\tan^2\beta x)}{\tanh^2\alpha x + \tan^2\beta x} \quad \dots\dots(5.2)$$

$$= \frac{C(ch^2\alpha x\cos^2\beta x + sh^2\alpha x \sin^2\beta x)}{sh^2\alpha x\cos^2\beta x + ch^2\alpha x \sin^2\beta x}$$

$$= \frac{C(\cosh 2\alpha x + \cos 2\beta x)}{\cosh 2\alpha x - \cos 2\beta x} \quad \dots\dots(5.2)$$

Micrometer screw

Short circuiting
piston

Liquid $\rightarrow$

x

Fibre ring

$\dfrac{m\lambda_o}{2}$

Crystal detector

$(2n-1)\dfrac{\lambda_o}{4}$

Fig. 5.3   Coaxial line used by Abadie for liquids
with normal absorption.

Fig. 5.4   Abadie's equation

$$|I|^2 \propto \frac{\cosh 2\alpha x + \cos 2\beta x}{\cosh 2\alpha x - \cos 2\beta x}$$

plotted for

$$\alpha = 0.7, \qquad \beta = 1.86$$
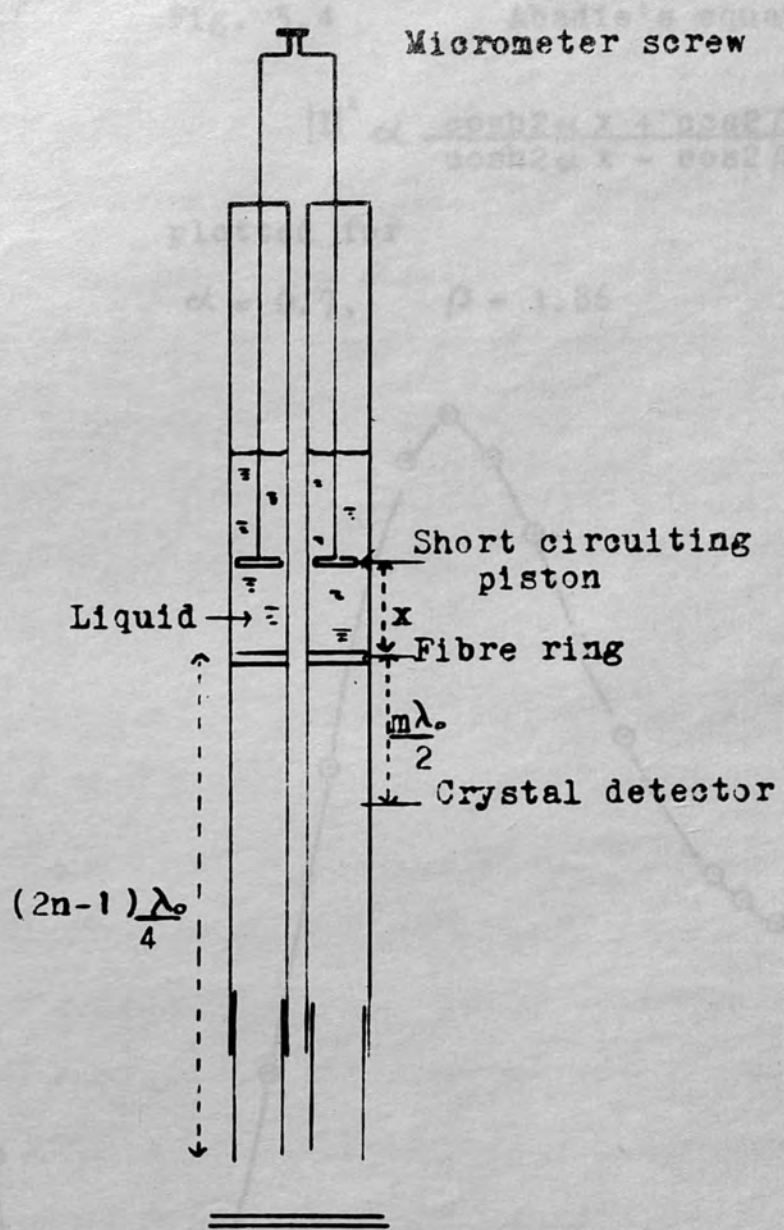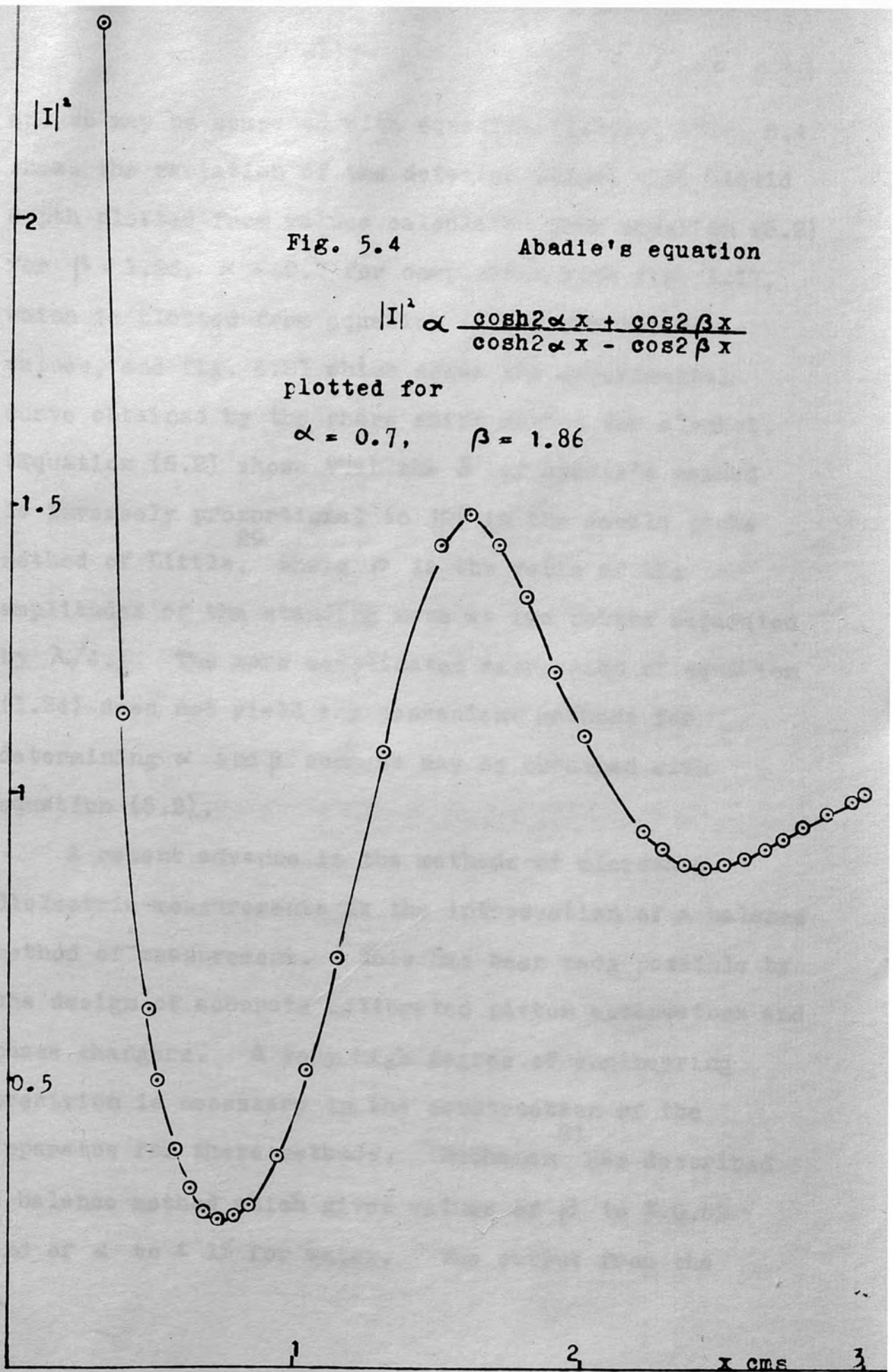
and so may be compared with equation (1.24). Fig. 5.4
shows the variation of the detector signal with liquid
depth plotted from values calculated from equation (5.2)
for $\beta = 1.86$, $\alpha = 0.7$ for comparison with fig. 1.17,
which is plotted from equation (1.24) for the same
values, and fig. 4.81 which shows the experimental
curve obtained by the phase shift method for alcohol.
(Equation (5.2) shows that the δ of Abadie's method
is inversely proportional to $|\rho|^2$ in the double probe
method of Little,[29] where ρ is the ratio of the
amplitudes of the standing wave at two points separated
by $\lambda_o/4$.) The more complicated expression of equation
(1.24) does not yield any convenient methods for
determining α and β such as may be obtained with
equation (5.2).

A recent advance in the methods of microwave
dielectric measurements is the introduction of a balance
method of measurement. This has been made possible by
the design of accurate calibrated piston attenuators and
phase changers. A very high degree of engineering
precision is necessary in the construction of the
apparatus for these methods. Buchanan[21] has described
a balance method which gives values of β to $\pm .0.5\%$
and of α to $\pm 1\%$ for water. The output from the

generator is divided, (fig. 5.5), part being trans-
mitted through a water cell of variable length (this
cell is similar to that used by Collie et al.) and a
phase changer to the H arm of a hybrid tee, and the
remaining part being fed through a cut-off attenuator
to the E arm of the T-junction.   The two outputs are
combined vectorially in the hybrid T-junction and the
resultant fed to a superheterodyne receiver.   By
manipulation of the cut-off attenuator and the phase
changer the receiver output can be made zero for any
liquid depth.   The method can be extended so that α
and β may be measured for low loss liquids and solids.

Branin and Smyth[22] have described a somewhat
similar null method applied to measurements either of
the travelling waves set up in a dielectric filled
slotted line terminated in a matched load, or the
standing waves set up in a short circuited coaxial
line.   A block diagram of their apparatus is shown
in fig. 5.6.   They estimate an accuracy of ±0.1% for
both the travelling and standing wave methods in
medium and low loss dielectrics, the accuracy being
somewhat less (no figure is stated) in high loss
dielectrics.   They give a minimum instrumental error
of ± 0.2% in ε' and ± 1.4% in ε'', whereas Buchanan
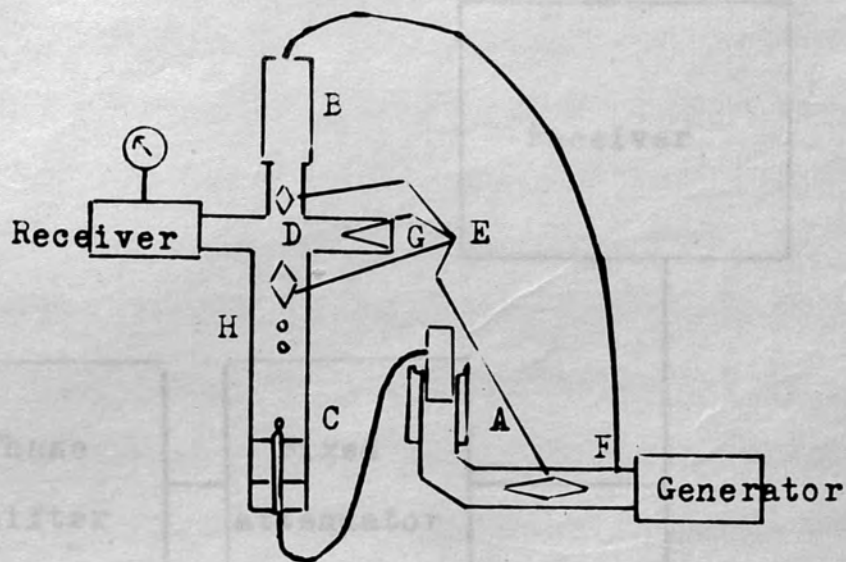
A   Water-cell

B   Cut-off attenuator

C   Phase-changer

D   Hybrid-tee

E   Fixed attenuators

F   Probe

G   Resistive load

H   Tuning stubs

Fig. 5.5   Diagram of Buchanan's apparatus.

Receiver

Phase shifter

Fixed attenuator

Uncalibrated attenuator

Travelling probe

Generator (Klystron)

Calibrated Attenuator

Dielectric filled, slotted line.

Fig. 5.6   Block diagram of Branin and Smyth's apparatus.

estimates an accuracy of better than $\pm$ 1% in both

ε' and ε'' at the two wavelengths, 3.2 and 1.26 cms,

so far investigated using high loss dielectrics.   It

seems likely that this balance technique will supercede

other methods.

5.4   Conclusion.

In view of the successful introduction of these

null techniques for dielectric measurement, it seems

doubtful whether a method based on the variations of

phase angle with liquid depth (equation 1.13) is worth

developing.   Apart from the method (3.6.1) of deter-

mining limits for α , the equation does not offer any

rapid method of deducing α and β accurately.

APPENDIX

Typical Experimental Results

The three tables given in this Appendix show the results obtained with

(a)   a liquid with low absorption, (Table 2)

(b)   a 'water-type' liquid, (Table 10)

(c)   an 'alcohol-type' liquid, (Table 14).

Each is representative of the results obtained for other liquids of the same type.

In these tables:

Column A gives the screw setting on the rising table,

Column B the micrometer reading in inches when the stick

is in the surface of the liquid in S,

Column C the reading of the positions of the tuning cap

on resonance,

Column D the galvanometer deflection,

Column x the change in length of the line to obtain

resonance again after the addition of 1 cm

of liquid.

f indicates that the tuning was flat and the

figure given C is then an average one.

O.S. indicates that the galvanometer deflection

went off the scale.

C.I. indicates coupling between the input and

   output loops was increased.

C.D. indicates coupling between the input and

   output loops was decreased.

Table 2

Results for Chlorobenzene

Temperature 19° C ± 1° C   Date: 4th and 5th March 1952

A	B	1 cms	C cms	x cms	Δ cms
	4.02	0	5.97	0	0
3	2.91	.282	5.98	.01	-.292
4	2.66	.345	6.00	.03	-.375
0	2.29	.44	6.03	.06	-.5
1 C.I.	2.01	.51	6.08	.11	-.62
2	1.80	.564	6.11	.14	-.704
3	1.49	.644	6.22	.25	-.894
3.5		.688	6.255	.285	-.973
4 C.I	1.14	.733	6.34	.37	-1.103
4.5		.774	6.46	.49	-1.264
0	.80	.816	6.575	.605	-1.421
0.5		.853	6.69	.72	-1.573
1	.50	.89	6.80	.83	-1.72
	4.65°		2.25*		
1.5		.915	2.34	.92	-1.835
2	4.44	.945	2.475	1.055	-2

° The micrometer had come to the end of its thread so the stick was shortened and the micrometer reset.

* This represents a shift of $\lambda/2$, 4.55 cms, to an adjacent maximum.

Results for chlorobenzene continued

A	B	l cms	C cms	x cms	$\triangle$ cms		
2.5		.979	2.60	1.18	-2.159		
3	4.145	1.018	2.80	1.38	-2.398	or	+2.152[†]
3.25		1.038	2.90	1.48	-2.518	or	2.032
3.5		1.059	2.975	1.555	-2.614	or	1.936
3.75		1.079	3.10	1.68	-2.759	or	1.791
4	3.83	1.1	3.19	1.77	-2.87	or	1.68
4.25		1.12	3.30	1.88	-3	or	1.55
4.5		1.14	3.35	1.93	-3.07	or	1.48
4.75		1.16	3.44	2.02	-3.18	or	1.37
0	3.51	1.18	3.50	2.08	-3.26	or	1.29
0.5		1.215	3.59	2.17	-3.385	or	1.165
1	3.23	1.25	3.675	2.255	-3.505	or	1.045
2	2.985	1.31	3.77	2.35	-3.66	or	.89
3	2.69	1.385	3.875	2.455	-3.84	or	.71
4	2.35	1.475	3.94	2.52	-3.995	or	.555
0	2.015	1.56	3.99	2.57	-4.13	or	.42
1	1.735	1.65	3.99	2.57	-4.22	or	.33
2	1.50	1.69	4.01	2.59	-4.28	or	.27
3	1.20	1.77	4.01	2.59	-4.36	or	.19
4	.86	1.85	4.01	2.59	-4.44	or	.11
0	.53	1.91	4.00	2.58	-4.49	or	.06
0.5		1.97	4.00	2.58	-4.55	or	0
1	.25°	2	3.99	2.57	-4.57	or	-.02
	4.47						
3	3.94	2.134	3.99	2.57	-4.704	or	-.145
0	3.28	2.302	4.00	2.58	-4.882	or	-.332
2	2.77	2.435	4.055	2.635	-5.07	or	-.52
3	2.45	2.514	4.1	2.68	-5.194	or	-.644
3.5		2.558	4.11	2.69	-5.248	or	-.698
4	2.10	2.602	4.15	2.73	-5.332	or	-.782
4.5		2.641	4.20	2.78	-5.421	or	-.871
0	1.79	2.68	4.22f	2.8	-5.48	or	-.93
0.5		2.712	4.28to 4.32	2.86to 2.90	-5.572 or -5.612	or	-1.022to -1.062

[†] $\triangle$ = -2.398 is a shift of more than $\lambda/4$, (2.275 cms)
and it was decided in section (1.4) to confine the
variations of $\triangle$ to between plus and minus $\lambda/4$ which
corresponds to the phase angle ϕ varying between plus
and minus π.

Results for chlorobenzene continued

A	B	l cms	C cms	x cms	Δ cms		
1	1.54	2.745	4.25to	2.83to	-5.575	or	-1.025to
			4.35	2.93	-5.675	or	-1.125
1.5		2.775	4.35f	2.93	-5.705	or	-1.155
2	1.30	2.805	4.35to	2.93to	-5.735	or	-1.185to
			4.4	2.98	-5.785	or	-1.235
2.25		2.825	4.38to	2.96to	-5.785	or	-1.235to
			4.55	3.13	-5.955	or	-1.405
2.5		2.845	4.45to	3.03to	-5.875	or	-1.335to
			4.6	3.18	-6.025	or	-1.475
2.75		2.866	4.55to	3.12to	-5.986	or	-1.436to
			4.7	3.28	-6.146	or	-1.596
3	.98	2.886	4.55to	3.12to	-6.0^06	or	-1.456to
			4.8	3.38	-6.266	or	-1.716
3.25		2.908	4.65to	3.23to	-6.138	or	-1.588to
			4.85	3.43	-6.338	or	-1.788
3.5		2.93	4.85to	3.43to	-6.36	or	-1.81to
			5.1	3.68	-6.61	or	-2.06
3.75		2.953	5.1to	3.68to	-6.633	or	-2.083to
			5.4	3.98	-6.933	or	2.167
4	.64	2.975	5.3to	3.88to	-6.858	or	2.252to
			5.5	4.08	-7.055	or	2.045
4.5		3.01	5.7to	4.28to	-7.29	or	1.81to
			5.8	4.38	-7.39	or	1.71
0	.33°	3.05	6 to	.03to	-3.08	or	1.47to
	4.62		6.2	.23	-3.28	or	1.27
0.5		3.084	1.6f	.18	-3.264	or	1.286
1	4.35	3.118	1.75	.33	-3.448	or	1.102
2	4.10	3.182	1.85f	.43	-3.612	or	.938
3	3.78	3.254	1.95	.53	-3.784	or	.766
4	3.43	3.36	2.02to	.6 to	-3.96	or	.59 to
			2.05	.63	-3.99	or	.66
0	3.11	3.43	2.10	.68	-4.11	or	.44
1	2.86	3.496	2.09	.67	-4.166	or	.384
2	2.60	3.562	2.10	.68	-4.242	or	.308
3	2.275	3.644	2.10	.68	-4.324	or	.226
4	1.91	3.737	2.10	.68	-4.417	or	.133
0	1.62	3.81	2.10	.68	-4.49	or	.06
1	1.38	3.87	2.09	.67	-4.54	or	.01
1.5		3.903					
2	1.12	3.937	2.08	.66	-4.597	or	-.047
3	.80	4.02	2.06	.64	-4.66	or	-.11
4	.455	4.11	2.04	.62	-4.73	or	-.18

Results for chlorobenzene continued

A	B	l cms	C cms	x cms	Δ cms
0	.15° 4.385	4.21	2.05	.63	-4.84 or -.29
1	4.12	4.277	2.02	.60	-4.877 or -.327
2	3.88	4.338	2.04	.62	-4.958 or -.408
3	3.56	4.42	2.01	.59	-5.01 or -.46
3.5		4.461	2.04f	.62	-5.081 or -.531
4	3.23	4.502	2.03	.61	-5.112 or -.562
4.5		4.534	2.05	.63	-5.164 or -.614
0	2.97	4.566	2.0 to 2.05	.58 to .63	-5.146 or -.596 to -5.196 or -.646
0.5		4.599	1.96 to 2.04	.54 to .62	-5.139 or -.589 to -5.219 or -.669
1	2.72	4.633	2.0f	.58	-5.213 or -.663
1.5		4.667	2.0 to 1.95	.58 to .53	-5.247 or -.697 to -5.197 or -.647
2	2.45	4.701	1.95f	.53	-5.231 or -.681
2.5		4.74	1.93f	.51	-5.25 or -.70
3	2.13	4.78	2.0 to 1.75	.58 to .33	-5.36 or -.81 to -5.11 or -.56
3.5		4.826	1.95 to 1.7	.53 to .28	-5.356 or -.806 to -5.106 or -.556
4	1.77	4.873	1.8 to 1.3	.38 to -.12	-5.253 or -.703 to -4.753 or -.203
4.5		4.909	5.55 to 5.8	-.42 to -.17	-4.489 or .061 to -4.732 or -.189
0	1.485	4.946	5.3 to 5.8	-.67 to -.17	-4.276 or .274 to -4.776 or -.226
0.5		4.976	5.0 to 5.25	-.97 to -.72	-4.006 or .544 to -4.256 or .294
1	1.235	5.006	5.0f	-.97	-4.036 or .514f
2	.96	5.08	5.0 to 4.85	-.97 to -1.12	-4.11 or .44 to -3.96 or .59
3	.635	5.157	4.85f	-1.12	-4.037 or .513f

5.3.52 A little liquid evaporated overnight, so the
rising table was raised until the pin was again in the
surface of the liquid.

4.5	.635		4.9		
0.5	.37° 4.46	5.23	4.85	-1.12	-4.11 or .44

Results for chlorobenzene continued

A	B	l cms	C cms	x cms	Δ cms
1.5	4.225	5.289	4.85	-1.12	-4.169 or .381
2.5	3.925	5.366	4.85	-1.12	-4.246 or .306
3.5	3.565	5.458	4.85	-1.12	-4.338 or .212
0.5	2.995	5.602	4.79	-1.18	-4.422 or .128
1.5	2.75	5.664	4.75 to 4.8	-1.22 to -1.17	-4.444 or .106to / -4.494 or .056
2.5	2.45	5.743	4.75	-1.22	-4.523 or .027
3.5	2.10	5.829	4.725	-1.245	-4.584 or -.03
4.5	1.78	5.91	4.70	-1.27	-4.63 or -.08
0.5	1.51	5.978	4.67	-1.3	-4.678 or -.128
2.5	.96	6.12	4.63	-1.34	-4.78 or -.23
4.5	.335° 4.77	6.276	4.55	-1.42	-4.856 or -.306
1.5	4.26	6.405	4.50	-1.47	-4.935 or -.385
2.5	3.95	6.484	4.50	-1.47	-5.014 or -.464
3.5	3.60	6.573	4.3 to 4.4	-1.67 to -1.57	-4.903 or -.353to / -5.003 or -.453
4.5	3.30	6.649	4.25 to 4.3	-1.72 to -1.67	-4.929 or -.379to / -4.979 or -.329
0.5	3.055	6.711	4.05 to 4.25	-1.92 to -1.72	-4.971 or -.241to / -4.991 or -.441
1.5	2.805	6.774	3.95 to 4.1	-2.02 to -1.87	-4.754 or -.204to / -4.904 or -.354
2.5	2.465	6.862	3.6 to 3.8	-2.37 to -2.17	-4.492 or .058to / -4.692 or -.142
3.5	2.14	6.943	3.45 to 3.55	-2.52 to -2.42	-4.423 or .127to / -4.523 or .027
4		6.976	3.45 to 3.35	-2.52 to -2.62	-4.456 or .094to / -4.356 or .194
4.5	1.88	7.01	3.35 to 3.3	-2.62 to -2.67	-4.39 or .16 to / -4.34 .21
0		7.039	3.25	-2.72	-4.319 or .231
0.5	1.65	7.068	3.25	-2.72	-4.348 or .202
1.5	1.395	7.131	3.19	-2.78	-4.351 or .199
3.5	.71	7.306	3.04	-2.93	-4.376 or .174
4.5	.42	7.386	2.97	-3.0	-4.386 or .164
0.5	.18° 4.80	7.436	2.95	-3.02	-4.416 or .134
2.5	4.21	7.586	2.85	-3.12	-4.466 or .084
3.5	3.87	7.672	2.85	-3.12	-4.552 or -.002
4.5	3.59	7.743	2.78	-3.19	-4.553 or -.003
0.5	3.35	7.804	2.76	-3.21	-4.594 or -.044

Results for chlorobenzene continued

A	B	l cms	C cms		x cms		Δ cms		
1		7.838	2.76		-3.21		-4.628	or	-.078
1.5	3.08	7.873	2.75		-3.22		-4.653	or	-.103
3.5	2.41	8.043	2.65		-3.32		-4.723	or	-.173
4.5	2.12	8.116	2.61		-3.36		-4.756	or	-.206
0.5	1.89	8.172	2.60	to	-3.37	to	-4.802	or	-.252 to
			2.55		-3.42		-4.752	or	-.202
1.5	1.62	8.242	2.55		-3.42		-4.822	or	-.272
2.5	1.30	8.323	2.50		-3.47		-4.853	or	-.303
3.5	.97	8.407	2.40		-3.57		-4.837	or	-.287
4.5	.69	8.476	2.37		-3.6		-4.876	or	-.326
0.5	.42	8.546	2.30		-3.67		-4.876	or	-.326
1.5	.14°	8.616	2.15	to	-3.82	to	-4.796	or	-.246 to
	4.98		2.25		-3.72		-4.896	or	-.346
2.5	4.64	8.702	1.95	to	-4.02	to	-4.682	or	-.132 to
			2.05		-3.92		-4.782	or	-.232
3.5	4.35	8.776	1.80	to	-4.17	to	-4.606	or	-.056 to
			1.75		-4.22		-4.556	or	-.006
4.5	4.08	8.844	1.70	to	-4.27	to	-4.574	or	-.024 to
			1.75		-4.22		-4.524	or	.026
0		8.879							
0.5	3.81	8.914	1.60	to	-4.37	to	-4.544	or	.006 to
			1.55		-4.42		-4.494	or	.056
1.5	3.525	8.985	1.45	to	-4.52	to	-4.465	or	.085 to
			1.55		-4.42		-4.565	or	-.015
3.5	2.92	9.139	1.20	to	-4.77	to	-4.369	or	.181 to
			1.3		-4.67		-4.469	or	.081
4.5	2.645	9.209	5.8		-.17		9.039	or	.061
0.5	2.31	9.292	5.7		-.27		9.022	or	.078
2.5	1.76	9.434	5.6		-.37		9.064	or	.036
3.5	1.49	9.501	5.5		-.47		9.031	or	.059
4.5	1.18	9.582	5.5		-.47		9.112	or	-.012
0.5	.93	9.646	5.45		-.52		9.126	or	-.026
1.5	.61	9.726	5.40		-.57		9.156	or	-.056
2.5	.33	9.796	5.35		-.62		9.176	or	-.076
3.5	.02°	9.876	5.30		-.67		9.206	or	-.106
	4.84								
0.5	4.27	10.021	5.25		-.72		9.301	or	-.201
2.5	3.68	10.17	5.15		-.82		9.35	or	-.25
0	2.985	10.346	4.95	to	-1.02	to	9.326	or	-.226 to
			5.0		-.97		9.376	or	-.276
1	2.73	10.411	4.9		-1.07		9.341	or	-.241
3	2.125	10.562	4.7		-1.27		9.292	or	-.192

Results for chlorobenzene continued

A	B	1 cms	C cms		x cms		Δ cms		
0	1.59	10.701	4.4		-1.57		9.131	or	-.031
1	1.325	10.766	4.4		-1.57		9.196	or	-.096
2		10.8	4.2		-1.77		9.03	or	.07
2.5	.89	10.876	4.2	to	-1.77	to	9.106	or	-.006to
			4.15		-1.82		9.056	or	.044
3.5	.49°	10.982	4.05	to	-1.92	to	9.062	or	.038to
	4.65		4.15		-1.82		9.162	or	-.062
4.5	4.42	11.04	3.95		-2.02		9.02	or	.08
0.5	4.125	11.115	3.9		-2.07		9.045	or	.055
1.5	3.73	11.215	3.85		-2.12		9.095	or	.005
2.5	3.47	11.282	3.78		-2.21		9.072	or	.028
3.5	3.21	11.346	3.70		-2.27		9.076	or	.024
4.5	2.87	11.435	3.67		-2.3		9.135	or	-.035
0.5	2.65	11.489	3.625		-2.345		9.144	or	-.044
1.5	2.24	11.594	3.55		-2.42		9.174	or	-.074
2.5	1.99	11.657	3.45		-2.52		9.137	or	-.037
3.5	1.71	11.727	3.42		-2.55		9.177	or	-.077
4		11.767							
4.5	1.40	11.807	3.40		-2.57		9.237	or	-.137
0.5	1.125	11.878	3.35		-2.62		9.258	or	-.158
1.5	.76	11.970	3.27		-2.70		9.27	or	-.17
2.5	.47	12.042	3.23		-2.74		9.302	or	-.202

Table 10

Results for Ringer Tyrode physiological saline, (0.15N)

Temperature 21.2° - 20°C   Date: 18th February 1953

A	B	l cms	C cms	Δ cms	D cms
3.5	4.66	0	6.01	0	39
3.6		0.008	6.01	-0.018	46
3.7		0.016	6.015	-0.021	46
3.8		0.024	6.02	-0.044	42
3.9		0.032	6.025	-0.057	41
4	4.50	0.041	6.03	-0.071	41
4.1		0.048	6.03	-0.077	37
4.2		0.055	6.03	-0.084	34
4.3		0.062	6.03	-0.091	32
4.4		0.069	6.03	-0.098	27
4.5	4.36	0.076	6.035	-0.111	23.5
4.75		0.092	6.045	-0.137	14.5
0	4.20	0.117	6.055	-0.172	8.5
.25		0.136	6.075	-0.211	4 CI 80
.5		0.154	6.10	-0.254	38
.6		0.162	6.11	-0.272	27.5
.7		0.169	6.12	-0.289	20
.8		0.176	6.12	-0.296	13
.9		0.184	6.15	-0.334	9.4
1	3.91	0.191	6.17	-0.361	6 CI 24
1.1		0.198	6.15	-0.348	21
1.2		0.205	6.15	-0.355	14.5
1.3		0.212	6.15	-0.362	10
1.4		0.219	6.10 to 6.06	-0.309 to -0.279	7
1.5		0.227	6.07f	-0.297	5.25
1.6		0.234	6.01 to 5.96	-0.244 to -0.194	4
1.7		0.241	5.88f	-0.121	3.5
1.8		0.248	5.825 to 5.80	-0.073 to -0.048	3.3
1.9		0.255	5.70	0.045	3.5
2	3.63	0.262	5.70 to 5.65	0.038 to 0.088	3.6
2.25		0.280	5.53	0.190	4.9

Results for Ringer Tyrode physiological saline continued

A	B	l cms	C		Δ cms		D cms
2.5		0.298	5.49		0.213		8.5
2.75		0.315	5.495		0.190		13.4
3	3.35	0.333	5.5		0.167		19.5
3.25		0.343	5.5		0.157		27
3.5		0.374	5.5		0.129		36.5
3.75		0.391	5.5		0.109		47.5
4	3.045	0.41	5.5		0.09		59.5
4.25		0.430	5.5		0.071		72
4.5		0.449	5.5		0.051		86.5
4.75		0.468	5.495		0.037		OS CD 46
0	2.74	0.488	5.5		0.012		49.5
0.25		0.506	5.5		-0.006		53
0.5		0.525	5.5		-0.025		54
0.75		0.543	5.5		-0.043		54
1	2.45	0.561	5.494		-0.056		51.5
1.25		0.58	5.5		-0.08		47
1.5		0.599	5.49		-0.089		42
1.75		0.618	5.49		-0.108		36.5
2	2.155	0.637	5.47		-0.107		29.5
2.25		0.654	5.47		-0.124		23.8
2.5		0.672	5.47		-0.142		18.4
2.75		0.689	5.465		-0.154		13.4
3	1.875	0.706	5.44		-0.146		10.1
3.25		0.726	5.40		-0.126		7.7
3.5		0.747	5.35		-0.097		5.9
3.75		0.767	5.34 to		-0.157 to		5.2
			5.30		-0.057		
4	1.56	0.787	5.28 to		-0.067 to		4.7
			5.24		-0.027		
4.25		0.807	5.21		-0.017		5
4.5		0.828	5.13		0.043		5.6
4.75		0.848	5.11		0.042		6.8
0	1.24	0.868	5.10		0.032		8
.25		0.884	5.10		0.016		9.7
.5		0.900	5.07		0.03		12
.75		0.916	5.065		0.019		14.2
1	0.985	0.932	5.055		0.013		16.1
1.25		0.953	5.04		0.007		18.5
1.5		0.974	5.02		0.006		20.5
1.75		0.995	5.00		0.005		22
2	0.66	1.016	4.99		-0.008		23.3
2.25		1.037	4.98		-0.017		24
2.5		1.059	4.98		-0.039		24.25

Results for Ringer Tyrode physiological saline continued

A	B	l cms	C	Δ cms	D cms
2.75		1.080	4.975	-0.055	23.8
3	0.32	1.101	4.97	-0.071	23
3.25		1.118	4.945	-0.063	20.4
3.5		1.135	4.95	-0.085	19
3.75		1.152	4.94 to 4.92	-0.092 to -0.072	17.6
4	0.6- 4.60	1.169	4.90	-0.069	16.2
4.25		1.185	4.90	-0.085	14.6
4.5		1.201	4.895	-0.096	13
4.75		1.217	4.88	-0.097	11.5
0	4.35	1.233	4.85	-0.083	10.5
.25		1.251	4.86	-0.111	9.2
.5		1.270	4.82	-0.09	8.7
.75		1.289	4.80	-0.089	8.2
1	4.055	1.308	4.74	-0.048	8.2
1.25		1.326	4.72	-0.046	8.3
1.5		1.345	4.70	-0.045	8.6
1.75		1.363	4.67	-0.033	9.2
2	3.76	1.382	4.64	-0.022	9.7
2.25		1.401	4.60	-0.001	10.4
2.5		1.420	4.59	-0.010	11.4
2.75		1.439	4.58	-0.019	12.1
3	3.46	1.459	4.56	-0.019	13.1
3.25		1.478	4.545	-0.023	14
3.5		1.498	4.53	-0.028	15
3.75		1.518	4.51	-0.028	15.6
4	3.145	1.538	4.50	-0.038	16.2
4.25		1.556	4.49	-0.046	16.5
4.5		1.573	4.475	-0.049	16.7
4.75		1.591	4.45	-0.041	16.8
0	2.8675	1.609	4.44	-0.049	16.3
.25		1.627	4.43	-0.057	16.1
.5		1.646	4.42	-0.066	15.5
.75		1.664	4.40	-0.064	14.8
1	2.58	1.682	4.39	-0.072	13.8
1.5		1.721	4.34	-0.061	12.2
2	2.265	1.760	4.285	-0.045	11.2
2.25		1.780	4.27	-0.050	10.9
2.5		1.800	4.255	-0.055	10.65
2.75		1.819	4.21	-0.029	10.15
3	1.96	1.839	4.20	-0.039	10.4
3.5		1.879	4.15	-0.029	10.5
4	1.65	1.918	4.10	-0.018	11

Results for Ringer Tyrode physiological saline continued

A	B	l cms	C	△ cms	D cms
4.5		1.955	4.06	-0.015	12
0	1.36	1.991	4.01	-0.002	12.4
.5		2.029	3.99	-0.019	12.8
1		2.067	3.95	-0.017	13.4
1.5		2.105	3.94	-0.045	14
1.75		2.124	3.93	-0.054	14.2
2	0.765	2.143	3.90	-0.043	13.7
3	0.47	2.22	3.85	-0.07	13
4	0.17-4.23	2.295	3.78	-0.075	12
4.5		2.332	3.75	-0.082	11.6
4.75		2.351	3.70	-0.051	11.5
0		2.369	3.67	-0.039	11.4
.25		2.388	3.68	-0.068	11.5

Table 14

Results for Ethyl Alcohol

Temperature 18°C   Date: 12th February 1952

$A^{\times}$	l cms	C	x cms	Δ cms
0	0	5.87	0	0
2	.156	5.875	0.005	-0.161
4	.312	5.90	0.03	-0.342
0	.390	5.91	0.04	-0.43
0.5	.429	5.93	0.06	-0.489
1	.468	5.94	0.07	-0.538
1.5	.507	5.96	0.09	-0.597
2	.546	5.99 to 6.0	0.12 to 0.13	-0.666 to -0.676
2.5	.585	5.98	0.11	-0.695
3	.624	5.975	0.095	-0.719
3.5	.663	5.95 to 5.94	0.08 to 0.07	-0.743 to -0.733
4	.702	5.92	0.05	-0.752
4.5	.741	5.85 to 5.83	0.02 to 0.04	-0.721 to -0.701
0	.780	5.80 to 5.75	0.07 to 0.12	-0.71 to -0.66
0.5	.819	5.80 to 5.65	0.07 to 0.22	-0.749 to -0.599
1	.858	5.7 to 5.5	0.17 to 0.37	-0.688 to -0.488
1.5	.897	5.6 to 5.4	0.27 to 0.47	-0.627 to -0.427
2	.936	5.5 to 5.3	0.37 to 0.57	-0.566 to -0.366
2.5	.975	5.3 to 5.1	0.57 to 0.77	-0.405 to -0.205
3	1.014	5.1 to 5.0	0.77 to 0.87	-0.244 to -0.144
3.5	1.053	5.05 to 4.90	0.82 to 0.97	-0.233 to -0.083
4	1.092	4.92 to 4.89	0.95 to 0.98	-0.142 to -0.112

+ One division of the screw on the rising table corresponds to a change of 0.078 cm in the liquid level.

Results for Ethyl Alcohol continued

A	l cms	C	x cms	$\triangle$ cms
0	1.17	4.76 to 4.73	1.11 to 1.14	-0.06 to -0.03
1	1.248	4.75 to 4.65	1.12 to 1.22	-0.128 to -0.028
2	1.326	4.66	1.21	-0.116
3	1.404	4.625	1.245	-0.159
4	1.482	4.55	1.32	-0.162
0	1.56	4.50	1.37	-0.19
1	1.638	4.46	1.41	-0.228
2	1.716	4.40	1.47	-0.246
3	1.794	4.34	1.53	-0.264
4	1.872	4.32	1.55	-0.322
0	1.95	4.22	1.65	-0.3
1	2.028	4.17	1.7	-0.328
2	2.106	4.10	1.77	-0.336
3	2.184	4.04	1.83	-0.354
4	2.262	3.98	1.89	-0.372
0	2.34	3.87	2.0	-0.34
1	2.418	3.79	2.08	-0.338
2	2.496	3.72	2.15	-0.346
3	2.574	3.66	2.21	-0.364
4	2.652	3.55 to 3.50	2.32 to 2.37	-0.332 to -0.282
0	2.73	3.48	2.39	-0.34
1	2.808	3.40	2.47	-0.338
2	2.886	3.34	2.53	-0.356
3	2.964	3.21	2.66	-0.304
4	3.042	3.16	2.71	-0.332
0	3.12	3.05	2.82	-0.3
1	3.198	3.02	2.85	-0.348
2	3.276	2.93	2.94	-0.336
3	3.354	2.84	3.03	-0.324
4	3.432	2.75	3.12	-0.312
0	3.51	2.70	3.17	-0.34
1	3.588	2.67	3.20	-0.388
2	3.66	2.56	3.31	-0.356
3	3.744	2.51	3.36	-0.384
4	3.822	2.37	3.50	-0.322
0	3.9	2.31	3.56	-0.34
1	3.978	2.24	3.63	-0.348
2	4.056	2.18	3.69	-0.366
4	4.212	2.04	3.83	-0.382

Results for Ethyl Alcohol continued

A	l cms	C	x cms	Δ cms
1	4.368	1.90	3.97	-0.398
3	4.524	1.70	4.17	-0.354
0	4.68	1.55	4.32	-0.36
2	4.836	1.40	4.47	-0.366
4	4.992	1.25	4.62	-0.372
1	5.148	1.10	4.77	-0.378

References

1   Cole, K.S. and Cole, R.H., J. Chem. Phys.
        9, 341, (1941)

2   Debye, P., 'Polar Molecules' (New York.,
        The Chemical Catalog. Co. Inc. 1929)

3   Onsager, L., J. Amer. Chem. Soc., 58, 1486, (1936)

4   Perrin, F., J. Phys. Radium, (7), 5, 497, (1934)

5   Oncley, J.L., Chem. Revs., 30, 433, (1942)

6   Kirkwood, J.G., J. Chem. Phys. 4, 592, (1936)

7   Trans. Faraday Soc., 'Discussion on Dielectrics'
        42A, (1946)

8   Frohlich, H., 'Theory of Dielectrics', (Oxford 1949)

9   Böttcher, C.J.F., 'Theory of Electric Polarisation'
        (Elsevier Publishing Co. 1952)

10 Lane, J.A. and Saxton, J.A., Proc. Royal Soc. A,
        213, 400, (1952)
    Saxton, J.A., Proc. Royal Soc. A, 213, 473, (1952)

11 Collie, C.H., Ritson, D.M. and Hasted, J.B.,
        Proc. Phys. Soc., 60, 145, (1948)

12 'The Principles of Microwave Circuits' edited by
        C.G. Montgomery, p. 380, Fig. 11.7, (McGraw-
        Hill, 1948)

13 Cook, H.F., Brit. J. App. Phys. 3, 249, (1952)

14   Baz, G. Physik. Z., 40, 394, (1939)

15   Kebbel, W., Hoch: tech. u Elek akus 53, 81 (1939)

16   Drude, P., Wied. Ann 55, 633 (1895); 61, 466 (1897)

17   Knerr, H.W., Phys. Rev., 52, 1054, (1937)

18   Roberts, S., and von Hippel, A., J. App. Phys.,
         17, 610, (1946)

19   Horner, F., Taylor,   Dunsmuir, R., Lamb, J.,
         Willis Jackson, J.I.E.E., 93, Pt.III, 33 (1946)

20   Redheffer, R.M., 'Technique of Microwave Measurements',
         Chapter 10 (McGraw-Hill, 1947)

21   Buchanan, T.J., J.I.E.E., 99, Pt.III, 61, (1952)

22   Branin, F.H., and Smyth, C.P., J. Chem. Phys.,
         20, 1121, (1952)

23   England, T.S., and Sharples, N.A., Nature, 163,
         487, (1949)

24   England, T.S., Nature, 166, 480, (1950)

25   Jackson, W., 'High Frequency Transmission Lines'
         (Methuen Monograph 1945)

26   Abadie, P., Trans. Faraday Soc., 42A, 143, (1946)

28   Conner, W.P. and Smyth, C.P., J. Amer. Chem. Soc.,
         65, 382, (1943)

29   Little, V.I., Proc. Phys. Soc., 66, 175, (1953)

30   Hill, N., Thesis, University of London, (1953)

31   Hasted, J.B., Ritson, D.M. and Collie, C.H.,
         J. Chem. Phys. 16, 1, (1948)

32   Lane, J.A., and Saxton, J.A., Proc. Roy. Soc. A,
         214, 531 (1952)

33   Haggis, G.H., Hasted, J.B., and Buchanan, T.J.
         J. Chem. Phys., 20. 1452 (1952)

34   Cooper, R., J.I.E.E., 93, Pt.III, 69, (1946)

36   Jackson, W. Trans. Faraday Soc., 42A, 91, (1946)

37   Bolton, H.C., Proc. Phys. Soc., 61, 294, (1948)

38   Ichikawa, T., Yamamura, H. and Negita, H.,
         J. Sci. Hiroshima Univ., A 15, 73, (1951)