NUMBER RECOGNITION IN DIFFERENT FORMATS[1]

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Number recognition in different formats

An interesting aspect about numbers is that they can be presented in different formats. Although numbers are associated spontaneously with arabic digits, they can also be represented as Roman numerals, (e.g., MMIV), sequences of words (both spoken and written), or in an analog form (e.g., dots on a die, tallies on a sheet of paper, or bar graphs). This raises the question of how numbers in the different formats are processed. What are the commonalities and what are the differences? I will first deal with the analog displays, which have a meaning both for humans and animals; and then continue with the verbal and the arabic numerals, which are uniquely human achievements. In line with McCloskey and Macaruso (1995), I will use the term *number* for format-independent aspects of numerical cognition, and the term *numeral* to refer to modality-specific representations (i.e., analog, verbal, and arabic numerals).

Perceiving analog displays of numbers

The basic function of numbers is to represent quantities (also called numerosities when the elements are clearly separated). By counting how many similar elements there are in a scene, we can assess their number. Because 5-year olds regularly make errors in their counting (e.g., Gelman & Gallistel, 1978), for a long time it was thought that knowledge of numerosities required formal education to be mastered. However, research in the 1980-1990s has indicated that this is not true for the apprehension of small numerosities. It is now well established that young babies, just like all kinds of animals (rats, pigeons, pigs, …) can easily discriminate numerosities smaller than four (see the chapters of Brannon and Spelke, Barth, & Lipton). In addition, they can compare two quantities when the differences between the quantities are big. For instance, Antell and Keating (1983) reported that newborns who were habituated to successive displays with two elements each (and, therefore, barely looked at them any more), showed increased interest when a display with three elements was presented. Using a similar habituation technique, Xu and Spelke (2000) reported that 6-month olds can discriminate between 8 and 16 items, but not between 8 and 12.

Human adults also show a distinction between the perception of a small number of items and the perception of a large number. Whereas it only takes some 50 ms longer to decide that a display contains three dots than to decide that it contains one dot, the time needed to detect nine dots is more than 600 ms longer than the time to detect seven dots. The difference is even consciously felt by the participants. Whereas they “see” the numerosity directly when the display contains less than four items, they have to “count” in order to correctly assess larger numbers. In addition, the assessment of larger numerosities is easier when the items are presented in a canonical form (e.g., a six represented by 2 rows of 3 dots, as on a die) than when they are presented in a random configuration (Mandler & Shebo, 1982; Wender & Rothkegel, 2000). The immediate apprehension of small numerosities (up to 3 to 4 elements) has been called *subitizing* (Kaufman, Lord, Reese, Volkman, 1949; Jensen, Reese, & Reese, 1950). Figure 1 shows the typical results of a study on subitzing.

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Further interesting observations are made when numerosities larger than four are presented and mathematically literate participants are prevented from counting them (for instance by a brief display of the stimulus pattern). Under these circumstances, participants have to come up with an educated guess, and they again show behavior that very much resembles that of animals.

A first finding is that participants spontaneously underestimate the number of elements in the display. The underestimate increases as the numerosity grows. For instance, Krueger (1982) showed each participant one sheet of paper with some Xs on. Participants were asked to give an estimate of the number of Xs on the page. When 50 Xs were present on the sheet, participants estimated them to be around 40; when 100 Xs were shown, estimates hinged around 75; when 200 were shown, the average estimate was some 135; and when 300 Xs were shown, participants estimated them to be around 200. So, there was a compressive function between the estimates given by the participants and the actual number presented (the former increased less rapidly than the latter). The compressive function was best captured by a power function with an exponent of .8 (i.e., in-between a linear function – exponent 1 – and a square root function – exponent .5).

A second finding when adults estimate numerosities on the basis of analog displays, is that the estimates show variability. For instance, van Oeffelen and Vos (1982) showed participants tachistoscopic displays with random configurations of dots and asked them to estimate whether or not there were exactly 12 dots in the display. The interesting variable was how often the participants would think there were 12 elements, given that another number had been presented. The results of this study are shown in the lower left part of Figure 2. When the number of elements presented was 11 or 13, participants made some 35% false alarms. When 10 or 14 elements were on the display, participants made some 24% errors. For a distance of 3, they made 17% errors, and for a distance of 4 they made 8% errors. This pattern of mistakes is very similar to the patterns of errors shown by animals in similar designs (see the two upper parts of Figure 2).

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Finally, a third finding with numerosity estimates is that the variability in estimates increases with growing target numbers. This was particularly clear in an experiment reported by Whalen, Gallistel, and Gelman (1999). Participants were presented with a dot that repeatedly flashed on and off in one location, and were asked to say approximately how many times the dots flashed, without verbal counting (special precautions were taken to prevent the counting). Both the response means and the standard deviations increased in direct proportion to the target number, which ranged from 7 to 25. Again, these findings were very similar to previous results obtained with rats and shown in Figure 3.

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On the basis of these findings, a considerably number of researchers nowadays assume that animals and humans are born with a *preverbal numerical system* (based on analogue magnitudes) that is capable of apprehending small numerosities precisely and larger numerosities approximately (e.g., Butterworth, 1999; Dehaene, Dehaene-Lambertz, & Cohen, 1998; Gallistel & Gelman, 1992; Wynn, 1998).

Gallistel and Gelman (1992), for instance, hypothesized that subitizing is nothing else than fast counting, based on the preverbal representations. In their model, each time an element of a display is encountered, a quantity is added to an accumulator (the authors compare this process to pouring cups of water in a bucket). At the end of the count, the accumulator is emptied into memory, and the total quantity is read. However, because there is some noise in the unit quantities added and/or in the reading from memory, there will be variability in the outcomes. This variability grows the more units (cups) have been added. Therefore, only for small numbers of units is it possible to rapidly assess the exact quantity. For larger numbers either mistakes are made, or a more laborious process must be used, which consists of verbal counting.

The idea of an innate, preverbal numerical system has also been defended by Dehaene and colleagues (e.g., Dehaene, 1992; Dehaene, Dehaene-Lambertz, & Cohen, 1998). They use the metaphor of a number line for this system. Numerical representations are thought to be ordered from small to large, and numbers are recognized by looking at which part of the number line is activated. The number line is thought to be compressed (e.g., according to a logarithmic function or a power function), so that the part of the line devoted to the number 1 is larger than the part devoted to the number 2, which in turn is larger than the part devoted to the number 3, and so on. Because of this characteristic, the representations of small numbers are more easily discernable than those of large numbers, and from a certain magnitude on the numerical representation can no longer be determined with certainty. It can only be estimated, unless an explicit verbal counting process is initiated.

This interpretation, however, is not shared by everyone (e.g., Mix, Huttenlocher, & Levine, 2002; Simon, 1997). There are two main points of contention. First, there is the question to what extent the empirical evidence of numerical knowledge in children and animals is due to the numerosity of the items (i.e., to the abstract notion of number: “two-ness, three-ness”), or to some confounded perceptual factor, such as the area covered by the items, the duration of the stimulus display (when the items are presented in time), or the density of the elements in the display. For instance, Feigenson, Carey, and Spelke (2002) in a first experiment replicated Antell & Keating’s (1983) experiment with the use of animallike objects made of Lego bricks: Infants of 7 months old who were habituated to successive displays of one object, showed increased interest when a display with two objects was shown, and vice versa. However, in four subsequent experiments, the authors failed to find the dishabituation effect when the front surface area of the objects was controlled, so that the task could not be explained on the basis of the total size of the stimulus configuration. For instance, the infants did not show renewed interest when in the habituation phase two small objects were presented, and in the test phase one double-sized object. Apparently, the infants’ behavior was more influenced by the size of the total stimulus configuration than by the number of elements in the display.

The second point of contention is whether one really needs numerical knowledge to perceive numerosities up to 4. It is generally assumed that humans (and animals) can keep 3-4 chunks of information simultaneously in short-term memory. Maybe this is the reason why infants and animals can perceive the difference between 2 and 3 elements, and why human adults show the subitizing effect? All they have to do is to match the second perceptual stimulus to the information of the first stimulus stored in short-term memory (Simon, 1997). This could be done by a simple one-to-one matching process, without any requirement of numerical knowledge (see also Logan & Zbrodoff, in press, for a recent perceptual interpretation of both the subitizing effect and the "counting" effect shown in Figure 1). One specific prediction of the short-term memory account is that infants and animals must not be able to compare numerosities larger than 4 (when perceptual factors are controlled), because these numerosities lay outside the short-term memory span. Needless to say, this is currently a matter of strong debate in the literature (see, e.g., Feigenson, Carey, & Hauser, 2002 vs. Xu & Spelke, 2000; Xu, 2003).

In summary, when mathematically literate humans are confronted with numbers shown in an analog format, they have no problems perceiving numerosities smaller than 4 (subitizing). For larger numerosities, they either start to count or they make a rough estimate. Because the subitizing effect and the rough estimates resemble characteristics of animal cognition (accurate perception of small numerosities, a tendency to underestimate large numerosities, and an increased variability in the estimates of larger numerosities), some authors have suggested that they are based on an innate, preverbal numerical system, which humans share with animals. Other researchers question such a nativistic view of numerical cognition, and point to the fact that much empirical evidence can be explained by perceptual factors unrelated to numerical cognition.

The finding that people spontaneously start to count numerosities larger than 4, shows how important symbolic representations are for human numerical cognition. In the following sections, I review the main findings on the processing of these symbolic representations.

Recognizing verbal numerals

The words for the small numbers are among the first acquired, and research has shown that nearly half of the 3-year olds are capable of using the words up to seven in a sensible way (e.g., to count a row of objects; Gelman & Gallistel, 1978). Needless to say, knowledge of number words dramatically facilitates the mathematical competence of humans, and a look at the number words themselves reveals some of the hurdles that had to be overcome in inventing them (Ifrah, 1998). For instance, the fact that the words "one, two, and three" have the same stem in German and Roman languages indicates that they have a common, more ancient origin. Similarly, nearly all Western languages have a number word related to "new" (nine, neuf), presumably because this number marked a discovery at some moment in our history; and the words for 11 and 12 betray that the base-ten structure of our number system was not yet well established by the time they were coined (although "eleven" and "twelve" originate from the sayings "one-left" and "two-left" – after you've counted all 10 fingers/digits; the fact that our number system has a base 10 also originates from the widespread use of fingers to count).

There are no reasons to assume that the perception and the production of verbal numerals would be any different from that of other words. So, we can take inspiration from the more general models of visual and auditory word recognition and production.

Because not everything can be covered in the space of a chapter, I will limit myself to the recognition of printed words. Readers interested in an introduction to spoken word recognition, may want to have a look at McQueen (in press). Those interested in spoken word production, are referred to Levelt, Roelofs, and Meyers (1999) and the following commentaries. Finally, those interested in written word production, may want to read Bonin, Peereman, and Fayol (2001).

There are three discussions within the literature of visual word recognition that are particularly interesting for number recognition. The first deals with the question of whether or not a mental lexicon is needed for the recognition of word forms; the second concerns the question of how the meaning of words is accessed; and the third addresses the question of how morphologically complex words are recognized.

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Insert Figure 4 about here (Coltheart et al., 2001)
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In models of word recognition it has been customary to make a distinction between a so-called word-form level and a word-meaning level (e.g., Balota, 1994; see Figure 4 for an example of such a model). The flowcharts of these models usually capture the former under the term “lexicon” and the latter under the term “semantic system”. At the lexical level, a match is made between the incoming perceptual information and word-form knowledge stored in memory, to determine whether a given stimulus (either visual or auditory) refers to a known word or not. At the semantic level, the meaning of a known word is derived. Several reasons have been given for the distinction between the lexical and the semantic level. A first reason is that many researchers believe that the lexical level is more differentiated than the semantic level. For instance, many authors are convinced that a distinction should be made between a visual and an auditory lexicon. Some arguments for this distinction are related to the nature of the input (e.g., the letters of short written words are probably processed in parallel, whereas there is a clear serial component in the phonemes of spoken words, which typically take hundreds of milliseconds to be pronounced). Other arguments are derived from priming studies. It has been shown that within-modality repetition of a word (e.g. visual-visual) results in larger facilitation effects than cross-modality repetition (e.g., auditory-visual; Morton, 1979).

A second reason for separating lexical from semantic representations has to do with the lack of one-to-one mappings between words and meanings. For example, the meaning of words to some extent depends on the context: The word *big* has a different meaning in the phrase *the big ant* than in *the big rocket* (Harley, 2001). Also, many words have different meanings (polysemy) or share their meaning with other words (synonyms). It is difficult to explain the resolution of these ambiguities in the mappings from form to meaning within a single layer of representations.

Finally, a third reason for separating the lexical from the semantic system, is that humans can do quite some processing of “words” without understanding them. For instance, a long series of neuropsychological patients have been described who had severe difficulties matching visually presented words to pictures, but who nonetheless knew very well which letter sequences formed existing words and which formed non-words. In addition, they could read the words aloud, even when the words contained irregular letter sound correspondences, as in *blood*, *climb*, and *come* (Coltheart, in press; Gerhand, 2002).

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Insert Figure 5 about here (Seidenberg & McClelland, 1989)
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Although the distinction between word form and word meaning is still dominant in models of visual word recognition, it has been criticized by Seidenberg and McClelland (1989). In their distributed model of visual word recognition (see Figure 5), word knowledge no longer begins when the activation of an entry in the orthographic lexicon exceeds a certain threshold, but consists of the co-activation of processing units that encode the orthographic, phonological, and semantic properties of a word (see also Van Orden, Pennington, & Stone, 1990). A visual word activates a number of orthographic units representing the sequence of input letters. This activation spreads to the semantic and the phonological units that are connected to the activated orthographic units, and feeds back until a stable state is reached. In addition, the various units are no longer devoted to single words (i.e., there are no localist representations any more). Each unit is activated by many words, and the identity of a word is determined by a pattern of activation across multiple units. Seidenberg and McClelland (1989) showed that many features of human visual word recognition can be simulated with such a model that no longer contains a visual lexicon.

With respect to the recognition of number words, the large majority of existing models have taken inspiration from the Coltheart et al. model (Figure 4) and, therefore, contain an orthographic lexicon with localist representations (for a review, see Campbell's chapter in this book). A major exception has been Campbell (1994) who defended a view very similar to Seidenberg and McClelland's. According to his multiple encoding view, numbers are simultaneously encoded in multiple ways (analog, verbal, arabic) through a process of activation that automatically dissipates. In this model, number recognition depends on the pattern of co-activation of the different codes rather than on the activation of one particular, localist code.

The second discussion within the literature of visual word recognition that is pertinent to number recognition, has to do with the question how central the meaning system is within the language architecture. To an outsider, this may seem a strange discussion, because what else is (visual) word recognition for than to access the meaning of a written message? However, researchers discovered that for the two tasks they usually ask participants to perform, meaning can be more or less discarded to explain the results. These tasks are the naming of visually presented, isolated words (word naming) and deciding whether or not a presented string of letters forms a correct English word or not (lexical decision). For word naming, traditionally three routes have been postulated (see Figure 4; but also see Seidenberg & McClelland (Figure 5) who distinguished between two routes only). First, there is a direct conversion from letters to sounds, making it possible to name unknown sequences of letters, such as non-words. The second route goes from the orthographic input lexicon to a phonological output lexicon, enabling the reader to correctly pronounce irregular words such as *come*. Finally, the third route goes from the orthographic input lexicon, through the semantic system, to the phonological output lexicon. However, it is usually assumed that this route is too slow to affect performance. Hence, this route has not been implemented in any of the existing computational models of word naming. Similarly, lexical decision times have been explained by focussing on the activity within the word-form lexicon, with little or no contribution from the words' meanings.

In general, findings with verbal numerals are well in line with the assumption of asemantic routes in visual word processing. Fias, Reynvoet, and Brysbaert (2001), for instance, presented a verbal numeral and an arabic numeral on the same display. Participants were asked to name the verbal numeral and to ignore the arabic numeral. They were perfectly capable of doing so, as evidenced by the fact that the naming latencies were the same when the arabic numerals referred to different magnitudes than the verbal numerals (e.g., *six – 5*), as when they referred to the same magnitudes (e.g., *six – 6*). In contrast, when the participants had to make a response that involved the meaning of the verbal numerals (i.e., indicate whether the verbal numeral was odd or even), they showed faster responses when both numbers referred to the same magnitude than when they referred to different magnitudes. Other evidence for the existence of non-semantic processing routes for verbal numerals comes from the finding that participants do not need more time to indicate that *eight* is written in small letters and *two* in large letters than to indicate that *eight* is written in large letters and *two* in small letters, whereas they do show such a magnitude-size congruity effect with Arabic numerals and other types of non-alphabetic stimuli (e.g., Ito & Hatta, 2003; see the section on arabic numerals for more information about this task).

On the other hand, research on the processing of verbal numerals has also shown that although the semantically mediated route is slightly slower in the naming of words, its importance must not be underestimated within the traditional three-route model. Reynvoet, Brysbaert, and Fias (2002), for instance, showed that the naming of verbal numerals was primed by arabic numerals with a close value. That is, participants named the target word *five* faster when 115 ms before the arabic primes *4* or *6* had been presented tachistoscopically, than when the arabic primes *2* or *8* had been presented (see the section on arabic numerals for more information about this distance-related priming effect). Subsequent research showed that the same effect was obtained with masked primes presented a mere 43 ms before the targets (Reynvoet & Brysbaert, in press). This cross-notation priming effect suggests that it does not take much to pre-activate the number magnitude route enough to find semantically mediated effects in the naming of verbal numerals.

Other evidence for the importance of the semantically mediated route in the naming of verbal numerals comes from Cappelletti, Kopelman, and Butterworth (2001). They reported the case of a semantic dementia patient who could hardly read words any more (21% of the words with regular letter-sound mappings, such as *must*; and12% of the words with irregular mappings, such as *pint*), but who was flawless at reading verbal numerals, due to spared numerical knowledge. Spared numerical knowledge is also often reported in Alzheimer's disease, and is in line with the finding that numerical knowledge is represented separately from many other types of semantic knowledge in the brain (e.g., Pesenti et al., 2000; see also Dehaene et al.'s chapter in this book).

Finally, the third discussion in the visual word recognition literature, that has a particular bearing on number processing, is the question of how morphologically complex words are recognized. In number reading, only the verbal numerals from *zero* to *twelve* are without question monomorphemic (i.e., consisting of one meaning unit only). In contrast, words like *twenty-one* and *one-hundred twenty-six* are clearly polymorphemic (i.e., contain at least two morphemes). In-between, there are some number names of which it is not clear whether they can be considered as polymorphemic because their constituents are different from the original words (e.g., *thirteen, twenty, fifty, …[instead of threeten, twoty, and fivety]*). There are two types of clear polymorphemic number words. The first are derivations obtained by adding a suffix to a simple number word (e.g., *sixty*, *seventy*). The second are compound words that are obtained by combining two or more words (e.g., *twenty-one*). Theoretically, morphologically complex words can be processed in two ways (see, e.g., Bertram & Hyona, 2003). Either they can be decomposed into their constituents which are then used to compute the meaning; or they can be stored as a whole in the mental lexicon. Researchers have offered quite divergent ideas about the relative importance of the two processing pathways and the factors that determine the balance. Variables that have been proposed are semantic transparency, word frequency, and the length of the constituting words. Morphologically-complex words are more likely to be stored and retrieved as a whole when the semantic relation between the word and the constituents is unclear (i.e., more likely for *honeymoon* than for *honeybee*), when the complex word is frequently encountered (i.e, more likely for *honeybee* than for *honeyfungus*), and when the complex word is short (i.e., more likely for *eyelid* than for *watercourse*). These factors allow us to predict that verbal numerals like *fifteen* and *twenty* (high-frequency, short, no clear relationship between the constituents and the complex word) are more likely to be recognized as a whole than numerals like *seventy* and *ninety* (lower frequency, semantically transparent), and that words like *ninety-eight* (long, low-frequency) are bound to be processed through decomposition. However, thus far, virtually no research has been done on this topic.

All in all, research on the processing of written verbal numerals, even though limited, has returned findings that are well in line with what can be expected on the basis of what is known about the processing of visually presented words in general. Most importantly, there is evidence that for many tasks (e.g., number naming and decisions about the size of number words) the meaning of verbal numerals is not activated fast enough to influence the response. This is in line with the assumption of non-semantically mediated routes in models of word processing, an assumption made by both localist (Figure 4) and distributed (Figure 5) models. As the majority of verbal numerals consist of more than one morpheme, any comprehensive theory of verbal numeral recognition will have to address the question of how morphologically complex words are recognized, an issue that has been overlooked so far.

Recognizing arabic numerals

The invention and application of Arabic (actually Hindi) numerals has further advanced the human numerical competence (Ifrah, 1998). It is widely assumed that the use of Roman numerals has prevented the Romans from attaining a mathematical sophistication that matches the sophistication they reached in other knowledge areas (just try to solve the problem CMIX times LI). Interesting features of arabic numerals are the use of a base 10 throughout (remember that the base-ten structure is not completely present in many verbal number systems; see Miller’s chapter for the implications of this), and the use of place coding. Units are always written rightmost, tens are second, hundreds third, and so on. This way of coding required the invention of the digit 0, for instance to represent *909* (nine hundreds and nine units, no tens). The power of the arabic notation can be seen in the fact that even for simple arithmetic problems involving the addition or multiplication of single digits, participants are much faster and more accurate when the numerals are presented as digits than as words (Campbell, 1994; Noel et al., 1997), even when the words are spoken (LeFevre et al., 2001).

The existence of arabic numerals begs the question of how they are recognized. As for the verbal numerals, a distinction must be made between small numbers and large numbers. Nearly all numerals with three digits or more require a decomposition (parsing) process. There is nobody defending the idea that a numeral like 4253 with its associated magnitude is stored as a whole in the human brain. The only known exception to this parsing requirement is when a complex numeral is frequently used as a nominal label to refer to a particular entity (e.g., when the participant's car is a Peugeot 206, when the participant is heavily interested in Boeings 747, or when the participant is a postman working near the Belgian village Darion, which has the postcode 4253). For these familiar complex numbers, there is some evidence that they may be stored holistically, as it is possible to prime them with their associated words (e.g., the number 206 is recognized faster after the tachistoscopically presented prime Peugeot than after the tachistoscopically presented prime Boeing; Alameda, Cuetos, & Brysbaert, 2003; Delazer & Girelli, 1997), In general, however, complex numbers must be decomposed into their constituents, and this is a process that is prone to brain damage (due to a stroke or to dementia). Many patients with numerical problems have difficulties reading and writing complex arabic numerals correctly (e.g., writing three-hundred and four as 3004).

Researchers largely agree that small numbers are recognized as a whole, but disagree about (1) whether these small numbers are limited to single digits, or whether they also include two-digit numbers (12, 20, 88), and (2) whether semantic activation is pivotal for the processing of arabic numerals? Before homing in on these two discussions, I will first review the major empirical findings about the processing of small arabic numerals.

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A first robust finding is that the processing is more demanding for larger numbers than for smaller numbers. This is already true for digits. It is easier to indicate which is the smaller of the pair 2-3 than to indicate which is the smaller of the pair 8-9. It is also easier to calculate 2+3 and 2x3 than 8+9 and 8x9. Brysbaert (1995) even found a robust number magnitude effect in a short-term memory experiment. In this experiment, participants first had to read three arabic numerals going from 0 to 99, and then to look at a fourth arabic numeral and to decide whether this fourth numeral was part of the initial set: yes or no. Eye movements of the participants were tracked, and the time was measured participants needed to store the numeral in short-term memory before they proceeded to the next numeral. Figure 6 shows the average reading time for the first numeral seen by the participants as a function of number magnitude. The most important variable to predict the reading times turned out to be the logarithm of the number magnitude, in line with the predictions of the compressed number line model (Dehaene, 1992).

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A second robust finding in arabic numeral processing is that when two numbers are processed together, processing times are influenced by the distance between the numbers. This is particularly clear when both numbers have to be compared, as it is much easier to say which digit is the smaller for the pair 2-8 than for the pair 2-3. More precisely, decision times are a function of the logarithm of the distance between the two numbers (see Figure 7). Another distance-related effect that has been described is the number priming effect. A target digit is recognized faster when it follows a (tachistoscopically presented) prime with a close value than when it follows a prime with a more distant value. Figure 8 shows data obtained by Reynvoet and Brysbaert (1999) with a number naming task and masked primes. Response latencies were fastest when prime and target were the same (e.g., 5 and 5; the font size was manipulated in order to diminish the physical overlap of the stimuli). They were significantly slower when prime and target differed by one unit (e.g., 4 or 6 and 5), and again significantly slower when the distance was 2 or 3. With non-tachistoscopic presentation of the prime, the priming is obtained over a range of more than 10 units (Brysbaert, 1995); with tachistoscopic presentation of the prime it usually ends at a distance of 3. A further intriguing aspect of the distance-related priming effect is that it is symmetric. That is, the priming is equally strong from 6 on 5 as from 4 on 5, despite the fact that the associative strength between 4 and 5 is stronger than between 6 and 5 (when asked to say the first word that comes to mind, participants are more likely to say *five* after hearing *four* than after hearing *six*). A last interesting aspect about the priming effect is that it is equally strong across notations as within notations (Reynvoet et al., 2002). The effect of the prime 6 on the arabic target 5 is the same whether the prime is presented in arabic notation or in verbal notation. This finding has been interpreted as evidence that the interaction between prime and target occurs at an abstract, notation-independent level. The most often cited candidate is the number line of analog magnitudes.

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Insert Figure 8 about here
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A third major finding about the processing of arabic numerals is that the semantic magnitude information of the numeral is activated more rapidly than is the case for verbal numerals. Because of this feature, it is nearly impossible to design a task with arabic input that is not affected by the meaning of the numeral. Henik and Tzelgov (1982) designed one of the first studies that demonstrated this aspect of arabic numeral processing. They asked participants to indicate which numeral of a presented pair of digits had the larger physical size (see also Ito & Hatta, 2003, discussed above). Participants found it more difficult to indicate that 2 was the larger in the pair 2 – 8 than to indicate that 8 was the larger in the pair 2 – 8, thereby effectively showing a Stroop-like interference effect between the numerical size (which was to be ignored) and the physical size. Similar findings have been reported in a counting task: It is easier to say that four digits are present in the stimulus *4 4 4 4* than in the stimulus *3 3 3 3* (Pavese & Umilta, 1998)

A last robust finding is that people in Western cultures have a strong tendency to associate small numbers with left, and large numbers with right (Dehaene et al., 1993; see also the chapter by Fias & Fischer). When participants have to indicate whether a number is odd or even, they can do so faster with the left hand to small numbers (e.g., 1, 3) and with the right hand to large numbers (e.g., 6, 8; see Figure 9). This effect has been linked to the reading direction of the participants and/or to the way in which ordered continua (such as the number line) are taught in school. In the parity judgment task, there is an additional tendency to associate odd numbers with left hand responses and even numbers with right hand responses (Nuerk et al., in press).

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Insert Figure 9 about here

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Some of the above effects have been used to try to find out whether two-digit arabic numerals are processed as a whole or as a syntactic combination of tens and units (see the first issue of discussion mentioned at the beginning of this section). If these numbers are processed as a whole, one would expect them to form some kind of continuous number line as a function of their magnitude. On the other hand, if they are stored as combinations of tens and units, one would expect discontinuities at the transition from one ten to the next. As it turned out, researchers observed evidence for both views.

Brysbaert (1995) argued that the reading times shown in Figure 6 strongly suggested that all numerals between 1 and 99 are part of a single compressed number line. Similarly, Dehaene et al. (1990) obtained a logarithmic distance effect in a magnitude comparison of two-digit numbers (in which participants had to indicate whether numerals like *60* and *59* were smaller than *65*; see Figure 7), and argued on the basis of this that two-digit numbers were compared by looking at the analog magnitude they represented and not by looking at the individual digits (in which case it would be much easier to decide that 59 is smaller than 65 than that 60 is smaller than 65, because the former pair of numbers start with a different digit). Reynvoet and Brysbaert (1998) wondered whether they would find the same priming effect from 10 on 9 as from 8 on 9, and having found so, also concluded that units and teens were part of the same continuum. Finally, Dehaene et al. (1993) noted that the small-left and large-right association extended over the boundary of units and teens, and also concluded that they were part of the same number line.

On the other hand, there are findings that cannot easily be explained by the assumption of a single number line going from 1 to 99, and that seem to indicate that two-digit arabic numerals are rapidly decomposed into a syntactic structure of tens and units [a view most strongly defended by McCloskey (1992)]. Nuerk, Weger, and Willmes (2001) showed that in the comparison of two-digit number pairs not only the distance between the numbers counts (Dehaene et al., 1990), but also whether or not both numbers are unit-ten compatible. A number pair was defined as compatible if the magnitude comparison of the tens and the magnitude comparison of the units led to the same response (e.g. 52 and 67 are compatible, because $5 < 6$ and $2 < 7$), and as incompatible if this was not the case (e.g. 47 and 62 are incompatible, because $4 < 6$ but $7 > 2$). Nuerk et al. (2001) observed a significant compatibility effect. Participants were faster to indicate that $52<67$ than that $47<62$, even though the distances between the numbers are the same. This compatibility effect suggests that the tens and the units were compared in parallel, a finding which is more in line with the view that number magnitudes are represented as composites of powers of 10 (i.e., the meaning of the numeral 28 is represented as $\{2\} \times \{10^1\} + \{8\} \times \{10^0\}$). Other evidence for a rapid decomposition of two-digit arabic numerals into powers of 10 was recently reported by Ratinkcx, Brysbaert, and Fias (submitted). These authors asked participants to name two-digit arabic numerals, which were preceded by tachistoscopically presented primes. They not only observed the expected distance-related priming effect (e.g., prime 37 and target 38), but also priming when the prime and the target shared a single digit in the tens or the units position (e.g. primes 28 and 34 for target 38). In addition, there was an *interference* effect when prime and target shared a digit on different positions (e.g., primes 82, 43, and 83 for target 38).

One way of interpreting the divergent findings on the processing of two-digit arabic numerals (recognized as a whole or as a combination of powers of 10) is to assume that both types of processing occur in parallel. Such a model has been proposed by Dehaene and colleagues (e.g., Cohen, Dehaene, & Verstichel, 1994; Dehaene, 1992; Dehaene & Cohen, 1995). In this model, arabic numerals simultaneously activate an analogue magnitude representation on the number line, and a visual arabic number form, in which numbers are represented as strings of digits on an internal visuo-spatial scratchpad. Another idea could be that simultaneously with the analogue magnitude, a more precise semantic representation consisting of powers of ten is built. This representation is needed anyway for the processing and storing of more complex numbers (i.e., integers with more than two digits, and real numbers with multidigit precision; see the parsing process mentioned above).

Dehaene's model brings us to the second point of discussion in the literature: Whether there exists a lexicon for arabic numerals similar to the orthographic lexicon for visual word recognition, so that quite some processing of arabic numerals can be done before the meaning gets fully activated. Dehaene and colleagues claim there is. For instance, Cohen et al. (1994) described a patient who had difficulties reading complex numbers, except when they were highly familiar (e.g., 1945). They attributed this spared capacity to the existence of an input lexicon for familiar arabic numerals, which has direct, non-semantic connections to the speech output. Similarly, Dehaene and Cohen (1997) described a patient who could name digits, despite the fact that her number understanding was impaired (she made 20% errors when asked to indicate whether digits were larger or smaller than a standard). Also in the literature of visual word recognition, it has been claimed that digits, just like all other logographic symbols in texts (abbreviations, punctuation marks, special characters), are part of the orthographic input lexicon used for text reading (e.g., Coltheart, 1978).

On the other hand, there is very little empirical support for non-semantically mediated processing in arabic numerals. As reviewed above, the meaning of a number can easily be ignored in a font size judgement task when the number is presented as an alphabetic word (i.e., deciding which is the physically smaller stimulus is not more difficult for the pair eight-two than for the pair eight-two). However, this is much less easy (and maybe impossible) when the numbers are presented in arabic format or in another logographic script (Henik & Tzelgov, 1982; Ito & Hatta, 2003 (arabic numerals and Kanji words); Pansky & Algom, 1999). Similarly, Fias et al. (2001) reported that verbal numerals could be read without any interference from an arabic distractor on the same display. However, the very same study showed that this was not true for the naming of digits: Naming latencies to the numeral *5* were longer when the distractor was *four* than when it was *five*.

Because of the rapid and omnipresent activation of the semantic information, it has been claimed that the processing of arabic numerals resembles more the processing of pictures than the processing of words (e.g., Brysbaert, Fias, & Reynvoet, 2000; Fias, 2001; Fias et al., 2001; McCloskey, 1992). In theories of picture processing, it is widely assumed that some perceptual form processing is needed before the meaning can be activated, but the idea of an independent picture lexicon directly connected to the speech output has not found empirical support (e.g., Hodges & Greene, 1998). In this respect, it is important to keep in mind that the meaning of arabic numerals need not be confined to magnitude information (although this obviously is the most important semantic attribute of numbers). It can also be encyclopedic or episodic information related to the arabic numeral, certainly when the numeral is often used as a non-quantitative label (as in Boeing 747; or in the number year 1992). This could explain some of the remaining abilities of neuropsychological patients to name arabic numerals of which they no longer know the exact magnitude (Cohen et al., 1994; Dehaene & Cohen, 1997).

Brysbaert, Fias, and Reynvoet (2000) listed some reasons why they thought the creation of a full-fledged lexicon was less compelling for the recognition of arabic numerals than for the recognition of visual words (see also Seidenberg & McClelland (1989) for the reasons why they claim a lexicon is not needed, even not for the recognition of words). For a start, printed words are quite long combinations of letters, which nevertheless have to be read within roughly a third of a second. Indeed, one of the most striking characteristics of the visual word recognition system is that it does not take notably longer to read a nine-letter word than a three-letter word (e.g., compare *lucrative* and *rat*). The same is not true for arabic numerals: As soon as the number length exceeds two digits, response latencies increase dramatically, indicative a cumbersome parsing process (e.g., compare *582617493* and *617*). Second, all combinations of arabic digits have a meaning, as opposed to only a very few of all possible letters combinations. Third, the meaning of arabic numerals is always the same, independent of the context (as opposed to words; see the previous section). Fourth, arabic numerals only exist in one visual form, whereas words can both be written and spoken, and are language-dependent (for those who master more than one language). Finally, more information is attached to words than simply their meaning. In many languages, words have a gender, can differ in number, and can only take certain syntactic roles within a sentence. Many authors believe this word-form related information is stored in the lexicon. For these reasons, the creation of a lexical system next to a word-meaning system seems more compelling for verbal numerals than for arabic numerals. Arabic numerals can in principle be recognized like objects (or pictures of them): The stimulus is decomposed into a structural description of perceptual features, which activates the corresponding semantic information.

The difference between word and digit processing has also been documented in the neuropsychological literature, where patients have been described who could no longer read printed words (alexia), but who could still recognize arabic numerals and do some rather sophisticated processing on them (e.g., Cohen & Dehaene, 2000). Intriguingly, Pesenti et al. (2000) also described a patient who had major difficulties identifying visually presented objects (visual agnosia), but who nevertheless read arabic numerals fluently. Apparently, the similarities in the processing of pictures and arabic numerals do not imply that they are functionally identical (maybe because the meaning of numbers and the meaning of visual objects are different sources of knowledge?).

All in all, recent research on the recognition of small arabic numerals has revealed a rather intriguing picture. First, digits activate their meaning faster than words, and also seem to require semantic mediation for further processing. In this respect, their processing is closer to that of picture recognition than to that of word processing. The meaning primarily refers to the magnitude of the numeral, but can also involve encyclopedic and episodic knowledge associated with the numeral, certainly if the numeral is frequently used in a non-quantitative way. Arabic numerals of three digits and more virtually always need to be parsed (unless they are familiar labels), a process that is rather demanding and highly susceptible to brain damage. Two-digit numbers form a kind of in-between category with quite some evidence for holistic processing, but also some signs of decomposition into tens and units.

Conclusion

In this chapter, I have reviewed the basic findings in number recognition and their implications for our views of what is happening. Rather than giving a personalized and simplified account, I have tried to keep an eye for the major discussions that are going on, although inevitably I biased the text towards my own convictions. A summary of these convictions is shown in Figure 10. For each of the three types of input, I have tried to sketch a general outline of the steps that are likely to be involved. Attentive readers can, on the basis of the various uncertainties that have been discussed, build their own version of the model (and test this).

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A first choice is to divide the semantic number system in a part dedicated to the processing of the magnitude of the numerical input, and a part dedicated to the encyclopedic and episodic knowledge associated with numbers. In the number magnitude system, as before (Brysbaert, 1995) I make a distinction between the recognition of the core numbers, and the precise representation of each and every possible number (simple and complex), probably in a base 10 format. The core numbers consist of the integers 1-99 (the number line), and some basic multipliers (hundred, thousand, …).

For the perception of numerosities in analog displays, I postulate a visual feature detection stage (needed for the separation of the stimulus from the background), directly connected to the compressed, analog number line. This is a simplification, as it does not deal with the processes needed for the sequential counting of the elements in a display that is shown long enough. Another extension of the model would be the addition of a connection between the visual feature units and stored mental images of triangles, faces of dice and so on, which are probably involved in the apprehension of numerosities presented in a familiar, canonical form.

For the recognition of verbal numerals, I have copied the Coltheart et al. (2001) model, and connected it to the semantic system. As individual number words always represent core numbers, only connections between the orthographic input lexicon and the core numbers are postulated. For the same reason, no direct connections between the orthographic input lexicon and the encyclopedic/ episodic information are accepted (e.g., the stimulus "two hundred and six" is not directly associated with a Peugeot car; this requires mediation of the number magnitude system). Another choice that has been made, is to postulate the feedback mechanisms not from the number line, but from the extended number system (which has more precise representations, certainly for numbers beyond the subitzing range). Because the verbal output for many numbers requires a sequence of multiple words, I have included Levelt et al.'s (1999) stage of lemma retrieval and syntactic parsing between the number magnitude system and the phonological output system.

Arabic numerals are encoded in two different ways: as a sequence of position-specific digits, and as a percept of the complete numeral (probably limited to numerals of 4 digits, the maximum capacity of visual short-term memory). The position-specific digits activate the number line and the extended number magnitude system in parallel (in line with the finding that all types of numerical tasks are easier with arabic input than with verbal input). In addition, the mental images of familiar numbers activate associated information in semantic and episodic memory.

A justified criticism against the box-and-arrow type of model proposed in Figure 10 is that it offers little explanation of the specific processes involved. There is a big gap between a general, verbal description of the processes in the different boxes and arrows, and the actual implementation of them, which would make the model detailed enough to quantitatively simulate the various empirical benchmarks that have been listed in the present chapter. This will be the major challenge for the coming years.

References

Alameda, J.R., Cuetos, F., & Brysbaert, M. (2003). The number 747 is named faster after seeing Boeing than after seeing Levis: Associative priming in the processing of multi-digit Arabic numerals. *Quarterly Journal of Experimental Psychology, 56A*, 1009-1019

Antell, S.E., & Keating, D.P. (1983). Perception of numerical invariance in neonates. *Child Development, 54*, 695-701.

Balota, D.A. (1994). Visual word recognition: The journey from features to meaning. In M.A. Gernsbacher (Ed.), *Handbook of psycholinguistics* (pp. 303-358). San Diego: Academic Press.

Bertram, R., & Hyona, J. (2003). The length of a complex word modifies the role of morphological structure: Evidence from eye movements when reading short and long Finnish compounds. *Journal of Memory and Language, 48*, 615-634.

Bonin, P., Peereman, R., & Fayol, M. (2001). Do phonological codes constrain the selection of orthographic codes in written picture naming? *Journal of Memory and Language, 45*, 688-720.

Brysbaert, M. (1995). Arabic number reading: On the nature of the numerical scale and the origin of phonological recoding. *Journal of Experimental Psychology: General*, *124*, 434-452.

Brysbaert, M., Fias, W., & Reynvoet, B. (2000). The issue of semantic mediation in word and number naming. In F. Columbus (Ed.), *Advances in psychological research, Volume I* (pp. 181-200). Huntington, NY: Nova Science Publishers.

Butterworth, B. (1999). *The mathematical brain*. London: Macmillan.

Campbell, J.I.D. (1994). Architectures for numerical cognition. *Cognition, 53*, 1-44.

Cappelletti, M., Butterworth, B., & Kopelman, M. (2001). Spared numerical abilities in a case of semantic dementia. *Neuropsychologia, 39*, 1224-1239.

Cohen, L., & Dehaene, S. (2000). Calculating without reading: Unsuspected residual abilities in pure alexia. *Cognitive Neuropsychology, 17*, 563-583.

Cohen, L., Dehaene, S., & Verstichel, P. (1994). Number words and number non-words – A case of deep dyslexia extending to Arabic numerals. *Brain, 117*, 267-279.

Coltheart, M. (1978). Lexical access in simple reading tasks. In G. Underwood (Ed.), *Strategies of information processing.* London: Academic Press.

Coltheart, M. (in press). Are there lexicons? *Quarterly Journal of Experimental Psychology (A)*.

Coltheart, M., Rastle, K., Perry, C., Langdon, R., & Ziegler, J.C. (2001). DRC: A dual route cascaded model of visual word recognition and reading aloud. *Psychological Review, 108*, 204-256.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44*, 1-42.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*, 371-396.
Dehaene, S., & Changeux, J.P. (1993). Development of elementary numerical abilities: A neuronal model. *Journal of Cognitive Neuroscience, 5*, 390-407.
Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition, 1*, 83-120.
Dehaene, S., & Cohen, L. (1997). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex, 33*, 219-250.
Dehaene, S., Dehaene-Lambertz, G., & Cohen, L. (1998). Abstract representations of numbers in the animal and human brain. *Trends in Neurosciences, 21*, 355-361.
Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance, 16*, 626-641.
Delazer, M., & Girelli, L. (1997). When “Alfa Romeo” facilitates 164: Semantic effects in verbal number production. *Neurocase, 3*, 461-475.
Feigenson, L., Carey, S., & Hauser, M. (2002). The representations underlying infants' choice of more: Object files versus analog magnitudes. *Psychological Science, 13*, 150-156.
Feigenson, L., Carey, S., & Spelke, E. (2002). Infants’ discrimination of number vs. continuous extent. *Cognitive Psychology, 44*, 33-66.
Fias, W. (2001). Two routes for the processing of verbal numerals: Evidence from the SNARC effect. *Psychological Research, 65,* 250-259.
Fias, W., Brysbaert, M., Geypens, F., & d'Ydewalle, G. (1996). The importance of magnitude information in numerical processing: Evidence from the SNARC effect. *Mathematical Cognition*, *2*, 95-110.
Fias, W., Reynvoet, B., & Brysbaert, M. (2001). Are Arabic numerals processed as pictures in a Stroop interference task? *Psychological Research, 65*, 242-249.
Gallistel C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44 (1-2)*, 43-74.
Gelman, R. & Gallistel, C.R. (1978). The child’s understanding of number. Cambridge, MA: Harvard University Press.
Gerhand, S. (2001). Routes to reading: a report of a non-semantic reader with equivalent performance on regular and exception words. Neuropsychologia, 39, 1473-1484.
Harley, T.A. (2001). *The psychology of language: From data to theory* (2nd edition). Hove: Psychology Press.
Henik, A. & Tzelgov, J. (1982). Is 3 greater than 5: The relation between physical and semantic size in comparison tasks. *Memory & Cognition, 10*, 389-395.
Hodges, J.R., & Greene, J.D.W. (1998). Knowing about people and naming them: Can Alzheimer’s disease patients do one without the other? *Quarterly Journal of Experimental Psychology (A), 51*, 121-134.
Ifrah, G. (1998). *The universal history of numbers: from pre-history to the invention of the computer*. London: Collins and Harvill Press.
Ito, Y. & Hatta, T. (2003). Semantic processing of Arabic, Kanji, and Kana numbers: Evidence from interference in physical and numerical size judgments. *Memory & Cognition, 31*, 360-368.
Jensen, E.M., Reese, E.P., & Reese, T.W. (1950). The subitizing and counting of visually presented fields of dots. *Journal of Psychology, 30*, 363-392.
Kaufman, E.L.; Lord, M.W., Reese, T.W., & Volkmann, J. (1949). The discrimination of visual number. *American Journal of Psychology, 62*, 498-525.

Krueger, L.E. (1982). Single judgments of numerosity. *Perception & Psychophysics, 31*, 175-182.

Lefevre, J.A., Lei, Q.W., Smith-Chant, B.L., & Mullins, D.B. (2001). Multiplication by eye and by ear for Chinese-speaking and English-speaking adults. *Canadian Journal of Experimental Psychology, 55*, 277-284.

Levelt, W.J.M, Roelofs, A., & Meyer, A.S. (1999). A theory of lexical access in speech production. *Behavioral and Brain Sciences, 22*, 1-75.

Logan, G.D., & Zbrodoff, N.J. (in press). Subitizing and similarity: Toward a pattern-matching theory of enumeration. *Psychonomic Bulletin & Review*.

Mandler, G., Shebo, B.J. (1982). Subitizing: An analysis of its component processes. *Journal of Experimental Psychology: General, 111*, 1-22.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: evidence from acquired dyscalculia. *Cognition, 44*,107-157.

McCloskey, M. & Macaruso, P. (1995). Representing and using numerical information. *American Psychologist, 50*, 351-363.

McQueen, J.M. (in press). Speech perception. In K. Lamberts & R. Goldstone (Eds.), *The handbook of cognition*. London: Sage Publications.

Mix, K.S., Huttenlocher, J., & Levince, S.C. (2002). Multiple cues for quantification in infancy: Is number one of them? *Psychological Bulletin, 128*, 278-294.

Morton, J. (1979). Facilitation in word recognition: Experiments causing change in the logogen model. In P.A. Kolers, M.E. Wrolstad, & H. Bouma (Eds.), *Processing of visible language* (pp. 259-268). New York: Plenum.

Noël, M.P., Fias, W., & Brysbaert, M. (1997). About the influence of the presentation format on arithmetical-fact retrieval processes. *Cognition*, *63*, 335-374.

Nuerk, H.C., Iversen, W., & Willmes, K. (in press). Notation modulation of the SNARC and the MARC (linguistic markedness of response codes) effects. *Quarterly Journal of Experimental Psychology*.

Nuerk, H-C., Weger, U., & Willmes, K. (2001). Decade breaks in the mental number line? Putting the tens and the units back in different bins. *Cognition, 82*, 25-33.

Pansky, A. & Algom, D. (1999). Stroop and Garner effects in comparative judgment of numerals: The role of attention. *Journal of Experimental Psychology: Human Perception and Performance, 25*, 39-58.

Pavese, A. & Umilta, C. (1998). Symbolic distance between numerosity and identity modulates Stroop interference. *Journal of Experimental Psychology: Human Perception and Performance, 24*, 1535-1545.

Pesenti, M., Thioux, M., Samson, D., Bruyer, R., & Seron, X. (2000). Number processing and calculation in a case of visual agnosia. *Cortex, 36*, 377-400.

Pesenti, M., Thioux, M., Seron, X., & De Volder, A. (200). Neuroanatomical substrates of Arabic number processing, numerical comparison, and simple addition: A PET study. *Journal of Cognitive Neuroscience, 12*, 461-479.

Plaut, D. C., McClelland, J. L., Seidenberg, M. S., & Patterson, K. (1996). Understanding normal and impaired word reading: Computational principles in quasi-regular domains. *Psychological Review, 103*, 56-115.

Ratinck, E., Brysbaert, M., & Fias, W. (submitted). The mental representation of two-digit Arabic numerals examined with masked priming: 28 rapidly activates {{2} x {10}} + {8} in number naming.

Reynvoet, B. & Brysbaert, M. (1999). Single-digit and two-digit Arabic numerals address the same semantic number line. *Cognition, 72*, 191-201.

Reynvoet, B., & Brysbaert, M. (in press). Cross-notation number priming at different stimulus onset asynchronies in parity and naming tasks. *Experimental Psychology*.

Reynvoet, B., Brysbaert, M., & Fias, W. (2002). Semantic priming in number naming. *Quarterly Journal of Experimental Psychology, 55A*, 1127-1139.

Seidenberg, M.S., & McClelland, J.L. (1989). A distributed, developmental model of word recognition and naming. *Psychological Review, 96*, 523-568.

Simon, T.J. (1997). Reconceptualizing the origins of number knowledge: A "non-numerical" account. *Cognitive Development, 12*, 349-372.

Van Oeffelen, M.P., & Vols, P.G. (1982). A probabilistic model for the discrimination of visual number. *Perception & Psychophysics, 32*, 163-170.

Van Orden, G.C., Pennington, B.F., & Stone, G.O. (1990). Word identification in reading and the promise of subsymbolic psycholinguistics. *Psychological Review, 97*, 488-522.

Wender, K.F., & Rothkegel, R. (2000). Subitzing and its subprocesses. *Psychological Research, 64*, 81-92.

Whalen, J., Gallistel, C.R., & Gelman, R. (1999). Nonverbal counting in humans: The psychophysics of number representation. *Psychological Science, 10*, 130-137.

Wynn, K. (1998). Psychological foundations of number: Numerical competence in human infants. *Trends in Cognitive Sciences, 2*, 296-303.

Xu, F. (2003). Numerosity discrimination in infants: Evidence for two systems of representation. *Cognition, 89*, B15-B25.

Xu, F., & Spelke, E.S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*, B1-B11.

Figure 1 : Mean reaction time needed by participants to say how many white asterisks are presented on a computer screen, as a function of numerosity. The points represent the observed data. The lines represent linear regression functions relating reaction time to numerosity within the subitizing range (1-3) and the counting range (4-10). Notice that the lack of a difference between the enumeration of 3 vs. 1 dots, shown in Figure 1, is not always present. Usually, there is a small positive slope of some 50 ms in the subitizing range (partly dependent on whether or not the numerosity 4 is included in the range). Also notice the slope of over 300 ms in the counting range. (from Logan & Zbrodoff, in press)

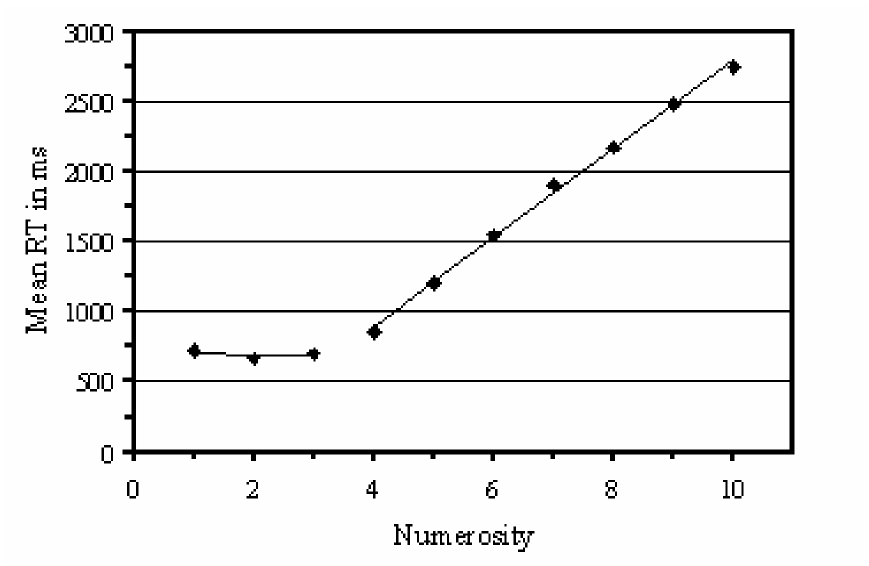


Figure 2 : The distance effect when animals and humans compare numerosities. The data discussed in the text are those of the lower left part (panel C). This panel shows how many times participants wrongly indicated that 12 elements were presented on the screen as a function of the actual number presented. As can be seen, the percentage of errors dropped systematically from a distance of 1 (i.e., when 11 or 13 elements were presented) to a distance of 4. The upper panels show data of animals in similar situations: (A) The deviation between the actual number of pecks made by pigeons and the fixed standard of 50, (B) Chimpanzees selecting the larger of two small numbers of chocolate bits. Finally, panel D shows the number of errors students make when they compare arabic numerals to a fixed standard of 65 (see also Figure 8). Figure copied from Dehaene et al. (1998).

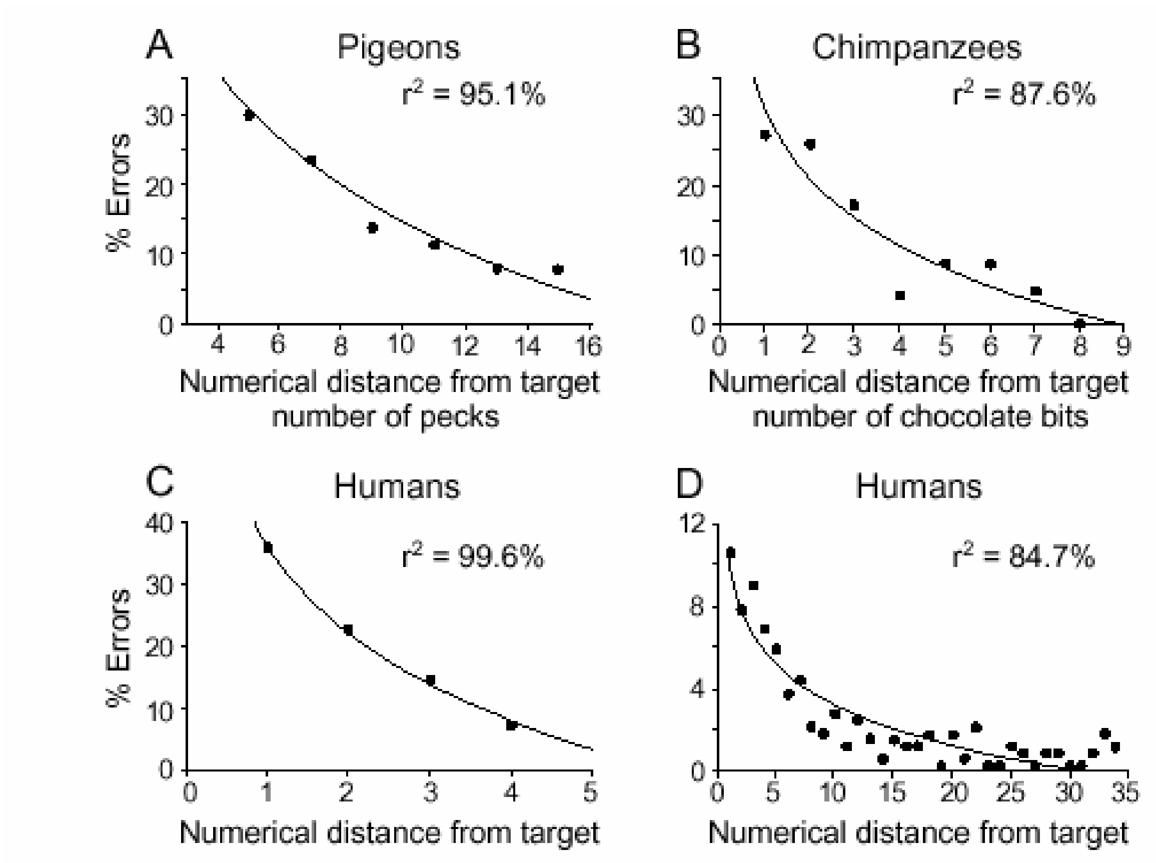


Figure 3 : These data show the actual number of lever presses made by rats after they had learned that a certain number (4, 8, 16, or 24) was required for a reward (part A). Notice that in this situation, the average number matches the required number quite well, but that the data vary from trial to trial. The variability increases with increasing average number. As a matter of fact, the increase in variability is a linear function of the average (and required) number, as shown in part B. Very similar data are found with humans, when they are prevented from counting the actual number and have to rely on rough estimates. Figure copied from Whalen et al. (1999).

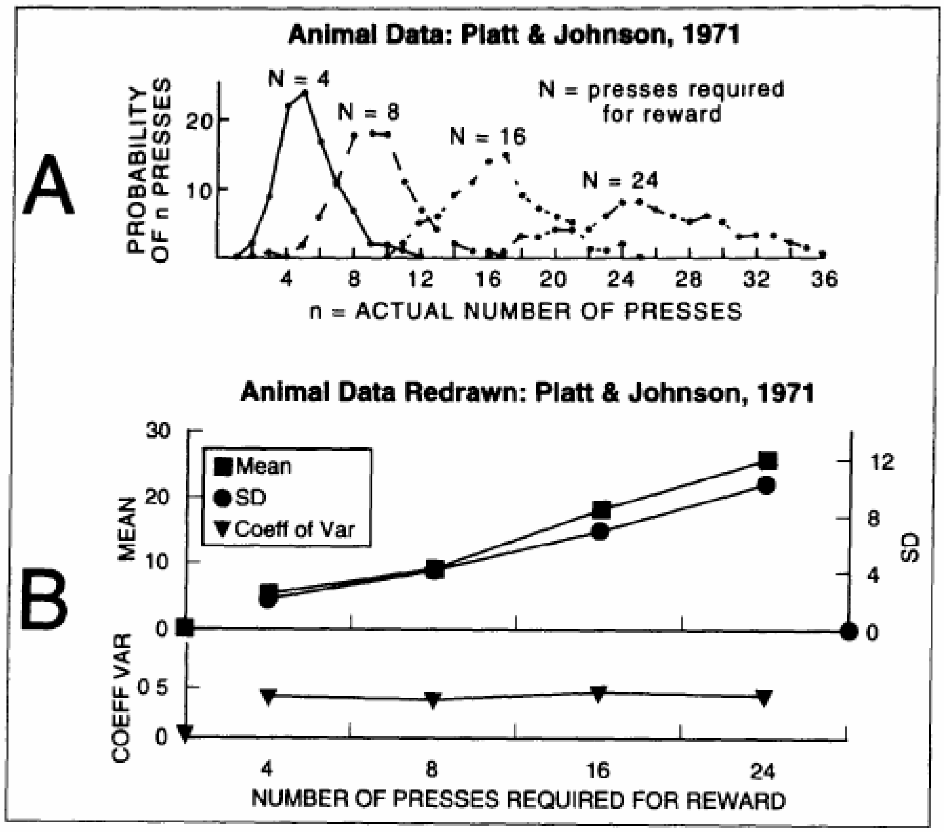


Figure 4 : Coltheart et al.'s (2001) dual-route cascaded model of visual word recognition and reading aloud. This model is exemplary of many traditional models of visual word recognition, based on localist representations. First, the letters of the presented words are identified. These letter representations then activate entries in the orthographic lexicon, and are converted simultaneously into their most likely sounds (phonemes). The phonemes feed into a phonological lexicon, which contains the spoken representations of all known words. Reading aloud of words occurs through a combination of direct grapheme-phoneme conversions and the activation of known word forms in the lexicons. Lexical decision is based on activation within the orthographic and/or the phonological lexicon. Notice that although the model contains a third route through the semantic system, this route is not believed to be fast enough to influence word naming or lexical decision times. For this reason, it has not yet been implemented in the working, computational model.

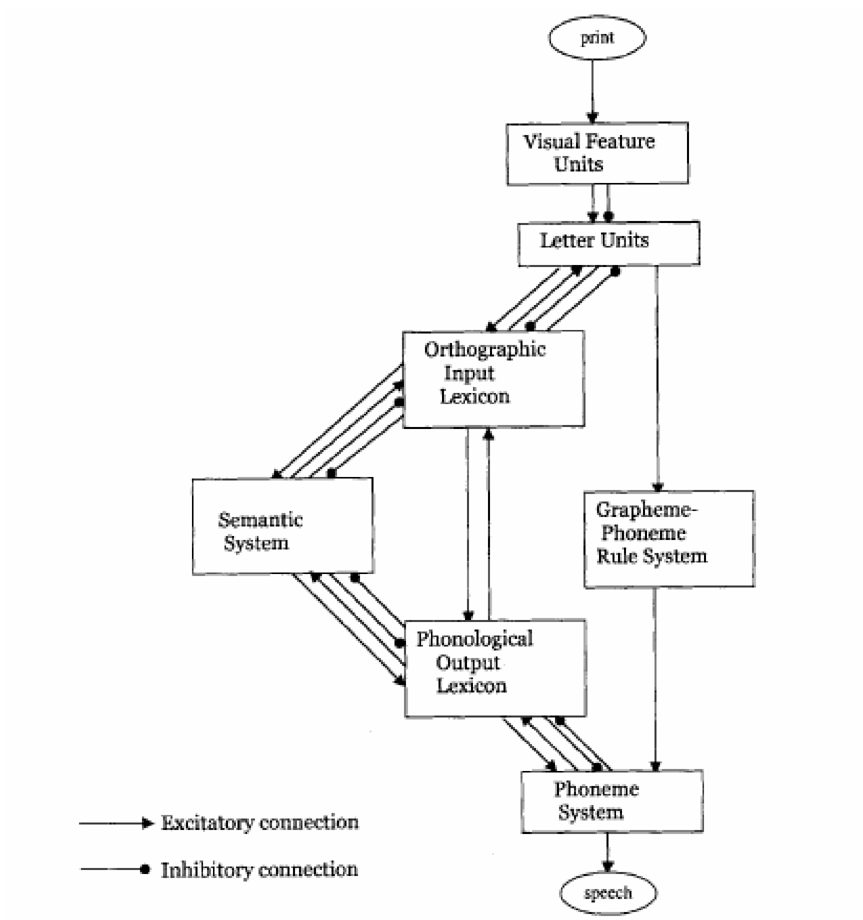


Figure 5 : Seidenberg and McClelland's (1989) triangular model of visual word processing (as implemented by Plaut et al., 1996). In this model, there is no longer a lexicon, where all known word forms are stored in dedicated (localist) units. Instead, information about words is stored in collections of units in the orthographic, the phonological, and the meaning layers that are co-activated. The individual units are activated (to a different extent) by many different words. In this model, the activation of the meaning of words is thought to be central in word processing. However, this part of the model has not yet been implemented, and does not seem necessary to simulate the basic findings of word naming and lexical decision.

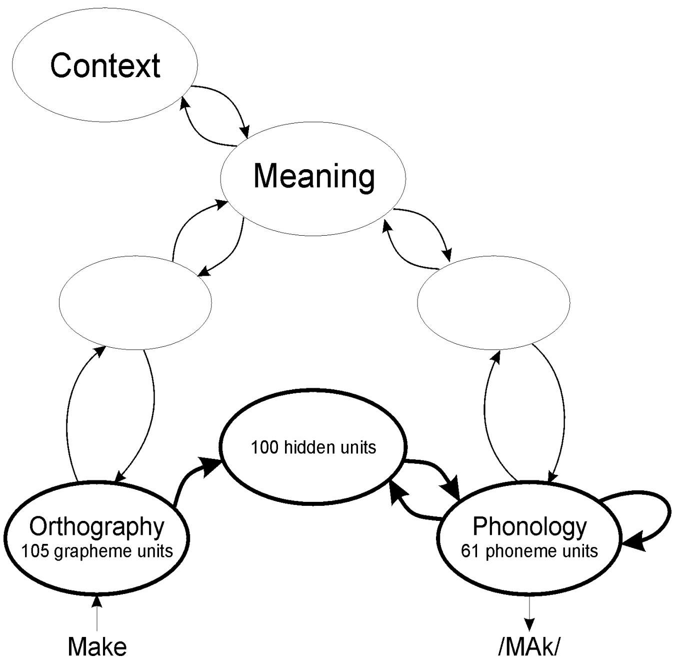


Figure 6 : Reading times for arabic numerals ranging from 0 to 99 in a short-term memory task. Circles indicate the observed data; lines indicate the predicted times on the basis of the logarithm of number magnitude, number frequency, and the number of syllables in the number name. Figure copied from Brysbaert (1995).

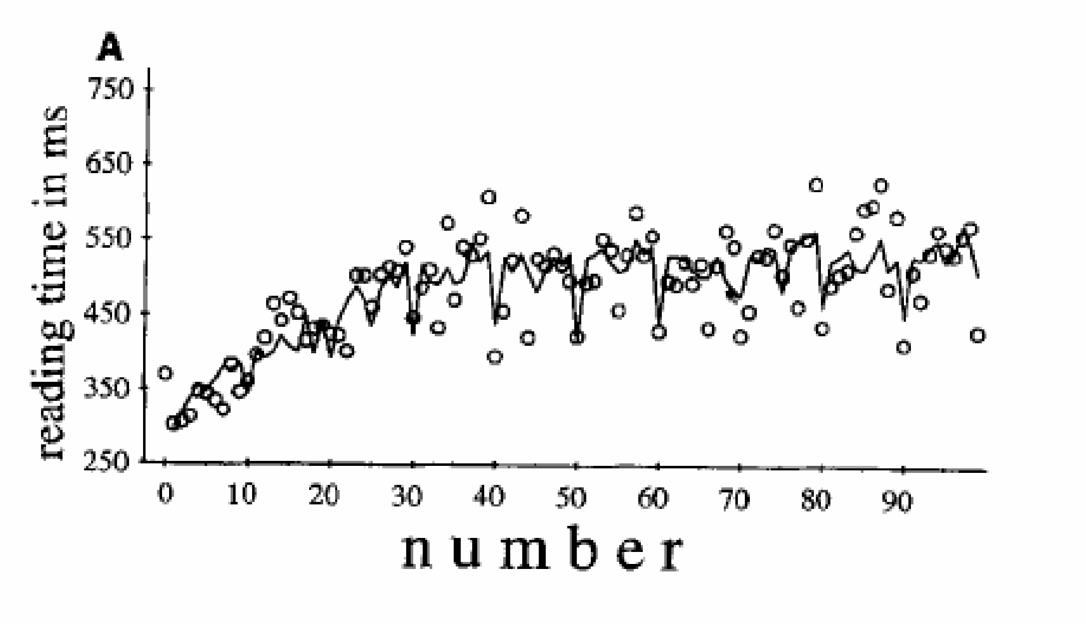


Figure 7 : Time mathematically literate adults need to indicate whether a two-digit arabic numeral is larger or smaller than a fixed standard of 65. Figure copied from Dehaene et al. (1990).

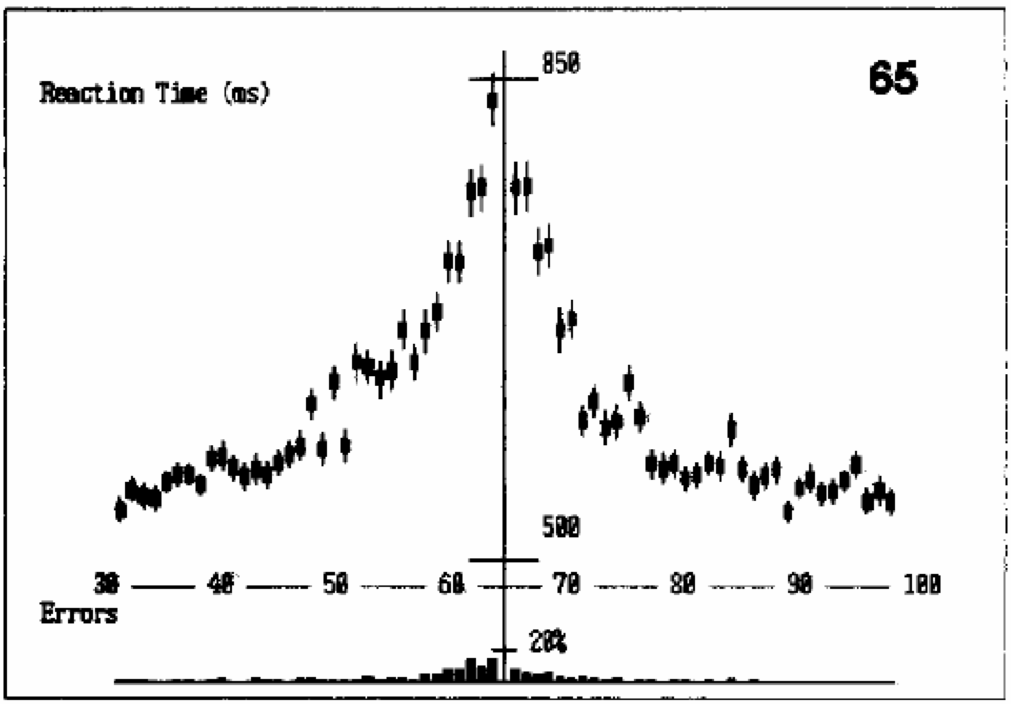


Figure 8 : Time participants need to name an arabic numeral as a function of the value of the preceding prime. Naming latencies are fastest when prime and target have the same value (e.g., 9-9). They are slightly slower when the prime is one unit less than the target (e.g., 8-9) or one unit more (e.g., 10-9). Reaction times are again slower when the distance between prime and target is 2, and when it is 3 (at which point the priming effect for tachistoscopically presented primes levels off). The extra priming effect observed when prime and target have the same value (identity priming) is only present when prime and target are displayed in the same modality (e.g., prime and target in arabic notation). When prime and target are presented in different formats (e.g., prime is verbal, target is arabic), the net priming effect reduces to what can be expected solely on the basis of the distance between prime and target. Data from Reynvoet & Brysbaert (1999).

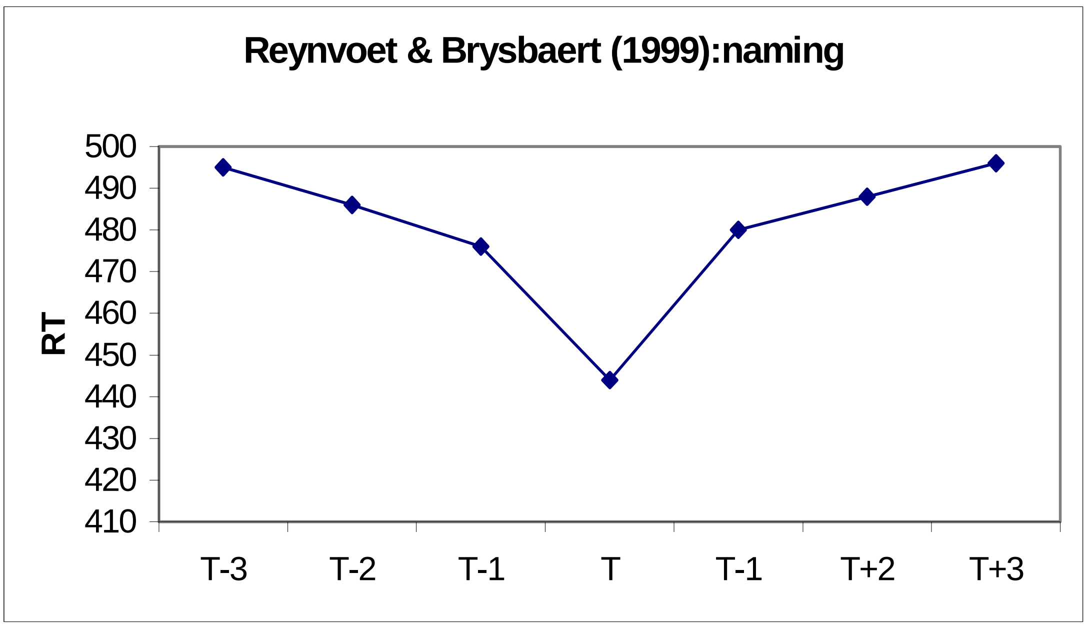


Figure 9 : Figure illustrating the findings that in Western cultures (1) small numbers are preferentially associated with left-hand responses and large numbers with right-hand responses, and (2) that odd numbers are preferentially associated with left-hand responses and even numbers with right-hand responses. The figure shows the results of an experiment in which participants had to indicate whether a presented arabic numeral ranging from 1 to 8 was odd or even, by pressing with the left or the right hand. The figure shows the differences in RT of right hand responses minus that of left hand responses. When left-hand responses were faster than right hand responses, this difference score is positive, which was the case for the small numbers. For the numbers 5-8, the right hand responses were faster than the left hand responses, giving rise to negative difference scores. The difference scores in general were also more negative for the even numbers (2, 4, 6, and 8) than for the odd numbers, indicating that the right-hand responses were faster for these numbers. Data from Nuerk et al. (in press)

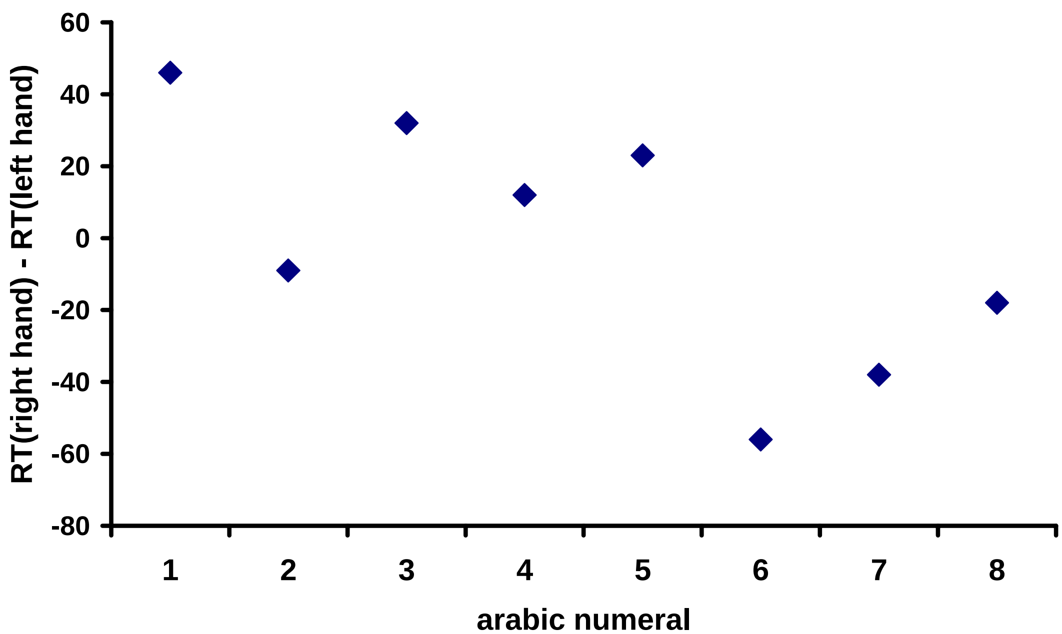
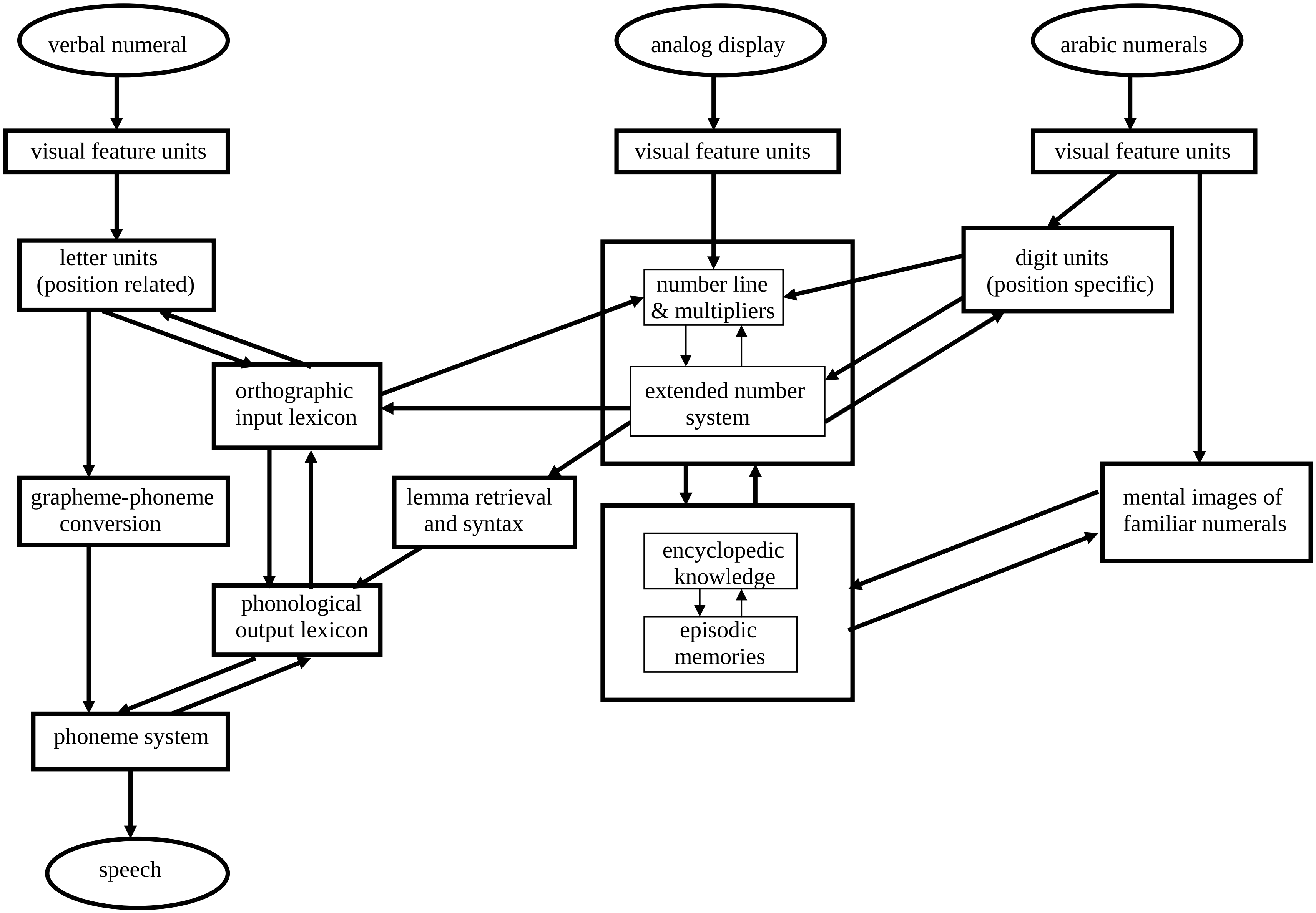


Figure 10 : A model of number recognition in verbal, analog, and arabic format. Note that the three boxes with visual feature units refer to the same perceptual processes. This box has been drawn anew for each input format to increase the clarity of the graph. Also note that the verbal system is the same as in Figure 4. This is a choice for localist representations (against the model presented in Figure 5). Other noteworthy choices are: (1) verbal numerals do not activate representations in the extended number system directly, (2) arabic numerals do, but this is probably limited given the laborious parsing that is needed for numerals of more than 2 digits, (3) there is no feedback from the number line, (4) there is no lexicon for arabic numerals, but (5) there is a store of percepts of arabic numerals similar to the images we have of other visual pictorial stimuli. This store (6) can activate associated memories in semantic and episodic memory.


verbal numeral
analog display
arabic numerals
visual feature units
visual feature units
visual feature units
letter units
(position related)
number line
& multipliers
digit units
(position specific)
orthographic
input lexicon
extended number
system
grapheme-phoneme
conversion
lemma retrieval
and syntax
mental images of
familiar numerals
encyclopedic
knowledge
episodic
memories
phonological
output lexicon
phoneme system
speech